

# On the Thermodynamics of Black Holes in Special Relativity

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## ABSTRACT

A black hole is defined as an object with an escape velocity of the speed of light. There is one common circumstance in which a black hole is typically formed. This is after the death of a star. After the star can no longer support its mass via nuclear fusion, it begins to collapse in on itself. This results in a large amount of mass being compressed into a tiny volume. However, I propose an alternative method for black hole creation. In this paper, a relativistic black hole is defined as an apparent black hole caused by the effects of special relativity. When Lorentz contraction and mass dilation are applied, not only do black holes appear to be formed, but Hawking Radiation also appears to be affected.

### 1. DERIVATION OF SCHWARZSCHILD RADIUS USING NEWTONIAN MECHANICS

The escape velocity of an object is calculated by setting kinetic energy and the gravitational potential energy equal to each other.

Given the formula for kinetic energy:

$$E_k = \frac{mv^2}{2}$$

And the gravitational potential energy of a body is:

$$U = \frac{GMm}{R}$$

The escape velocity can be derived by:

$$\frac{mv^2}{2} = \frac{GMm}{R}$$

Which when rearranged for escape velocity:

$$v_e^2 = \frac{2GM}{R}$$

This means that the radius for an object to have an escape velocity  $v$  is:

$$R = \frac{2GM}{v^2}$$

When  $v=c$

$$R = \frac{2GM}{c^2}$$

### 2. DERIVATION OF LENGTH CONTRACTION EQUATION

This shows that an object can be compressed to a point that its escape velocity is the speed of light, making it a black hole.

Given the formula for Lorentz contraction:

$$l_r = l_o \sqrt{1 - \frac{v^2}{c^2}}$$

Rearranging for  $v$ :

$$v = \frac{c \sqrt{l_o^2 - l_r^2}}{l_o}$$

This formula yields the velocity that needs to be reached to contract to a specified length.

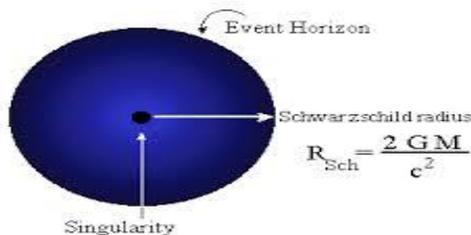
$$l_r = \frac{2GM}{c^2} = r_s$$

$$v = \frac{c \sqrt{l_o^2 - r_s^2}}{l_o}$$

### 3. CALCULATING THE LORENTZ FACTOR

This can then be applied to the Lorentz factor to calculate the relativistic effects a body would undergo traveling at this velocity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



$$v = \frac{c \sqrt{l_o^2 - r_s^2}}{l_o}$$

$$\gamma = \frac{l_o}{r_s}$$

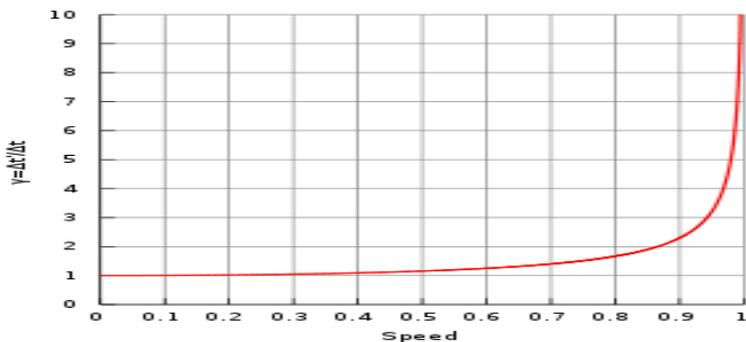
#### 4. APPLICATION TO HAWKING RADIATION

This relativistic change factor can be applied to Hawking Radiation:

Time to evaporate:

$$t_{ev} = \frac{M^3 5120 \pi G^2}{\hbar c^4}$$

$$t_{ev} = \frac{M^2 2560 G l_o}{\hbar c^2}$$



#### 5. CONCLUSION

This shows how the thermodynamic properties of a black hole are altered by it being created by relativistic means, even if the masses are equivalent.

#### REFERENCES

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