Fish Growth Model with Feed Quality Factor in Wastewater Oxidation Pond

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ABSTRACT

The present paper describes the fish growth model which is including new parameters, i.e. feed quality \( q \) are determined based on nitrogen-carbon ratio. The model is analyzed in form of differential equation model of fish growth in wastewater oxidation ponds views from the aspect of anabolism and catabolism individually. Function \( q \) is proportional to food assimilation efficiency \( b \), a percentage of assimilated feed for catabolism, and rate of feed intake. Fish growth models are constructed, be applied to growth Nile tilapia \((Oreochromis niloticus)\). Fish growth model will decrease if nitrogen-carbon ratio which is produced by fish during limited nitrogen have greater than nitrogen-carbon ratio in fish, otherwise will increase if nitrogen-carbon ratio which is produced by fish during limited nitrogen have smaller than nitrogen-carbon ratio in fish. Decreasing growth, also influenced by the function \( b \) are relatively small and biochemical oxygen demand (BOD) values are enough high. Overall the results of simulations and observations, the parameter \( q \) has fair influence to the fish growth model with condition that \( b \) function has greater, and BOD values close to \( \text{BOD}_{\text{max}} \) values.

Keywords: Fish Growth Models, Oxidation Pond, Feed Quality, Bod, Food Assimilation Efficiency

1. INTRODUCTION

The effective fish growth influenced by the temperature, size of the fish, and feed consumption (Handeland et al., 2008). The temperature itself can also increase the feed consumption of fish (Ursin, 1967). Fish growth is including the production of ammonia and organic matter, oxygen consumption, and the effect of temperature as the energy received (Jørgensen, 1976). Fish growth model adapted from a model developed by the Oregon State University (OSU) DAST (Bolte et al., 1994), which is differential equation for the fish growth model was modified to include the feed uptake on the feed quality and feed digestibility.

The culture fish growth in pond from the aspects of weight changes and considering of anabolism and catabolism aspects, nowadays many researchers to develops related to this topic. Jamu and Piedrahita (1995), make modifications to the differential equation for the fish growth by adding feed quality \( q \) coefficient as a factor influencing parameters on energy intake (energy assimilation) or anabolism in a pond freshwater. The parameter \( q \) is a nutritional food fish viewed from the nitrogen-carbon ratio in the feed or carbon-nitrogen ratio of fish produced when nitrogen limited. Besides the feed quality, the factors involved in the rate of feed intake is feed consumption coefficient \( h \), the relative feeding level \( f \), temperature \( T \), and \( \delta \) as a factor of dissolved oxygen (DO). Furthermore, Yi (1997), developed a bioenergetics growth model for Nile Tilapia \((Oreochromis niloticus)\) cultured in fertilized pond by synthesis process, and Yi (1997) use the model to evaluate the effect of different factors such as the size of the body, temperature, DO, unionized-ammonia, and feed availability.

In contrast, Dampin et al. (2012), conducted research on the rate of growth bioenergetics Nile Tilapia \((Oreochromis niloticus)\) in wastewater oxidation pond with daily ratio \( \text{feed intake} \) which consists of temperature, DO, unionize-ammonia, relative feeding, biochemical oxygen demand (BOD), and \( h \) feed consumption coefficient. The differential equations of fish growth has been added new function i.e. food assimilation efficiency \( b \).

In this paper, presented added a new parameter into models of fish growth on differential equations Dampin et al. (2012), involving the function of feed consumption efficiency \( b \) and feed quality \( q \) (Jamu and Piedrahita, 1995) in a model of fish growth in wastewater oxidation pond.

Furthermore, the data of Nile tilapia \((Oreochromis niloticus)\) in wastewater oxidation pond is used for object the study (Dampin et al., 2012). The object of this study is expected to provide an acceptable description of the model of fish growth in the oxidation pond with the addition of the \( q \) parameter. Therefore, numerical simulations of fish growth model to be done in order to revealed the influence of parameter which is being added.

2. FISH GROWTH MODEL

In general, growth model is equated with growth rate to distinguish between the average intake energy as well as reduced energy. Both energies had been written by Bolte et al. (1994) in differential equation below:

\[
\frac{dW}{dt} = HW^n - kW^m. \tag{1}
\]
where,
\[ W : \text{fish weight (g)}, \]
\[ t : \text{time (d)}, \]
\[ H : \text{coefficient of anabolism (g}^{1-m} \text{d}^{-1}), \]
\[ m : \text{exponent of fish weight for anabolism}, \]
\[ k : \text{coefficient of catabolism (g}^{1-n} \text{d}^{-1}), \]
\[ n : \text{exponent of fish weight for catabolism}. \]

Enberg et al. (2008) describes that von Bertalanffy has been indicated the part of \( kW^n \) as catabolism. As a broader interpretation, the section represents all the energy (mass) is missing. More specifically, Bertalanffy assumes that \( n = 1 \), resulting the von Bertalanffy growth model (VBGM) represented by equation (2):

\[ \frac{dW}{dt} = HW^n - kW. \]  

(2)

Lester et al. (2004) to merge VBGM and Roff model, in this article called Lester Growth Model (LGM), the assumption that is scale exponents for anabolism and catabolism is the same, i.e., \( m = n = 2/3 \). This refers to the linear growth of adult fish length (Enberg et al., 2008), differential equation model below:

\[ \frac{dW}{dt} = HW^{2/3} - kW^{2/3}. \]  

(3)

Furthermore, Ursin (1967) reiterated that fish growth rate increased by anabolism process, which consists of the feed rate or the feed assimilation every day, and fish growth rate decreased by catabolism process in feeding and fasting catabolism. Fish weight increased by anabolism process, which consists of the feed intake that available for fish assimilation and catabolism. Activity. Instead of that, fish weight decreased because catabolism and fish activity. In equation (1) is written (Dampin et al., 2012):

\[ \frac{dW}{dt} = (1-a)b \frac{dR}{dt} - kW^*, \]  

(5)

where, \( a \) is the percentage of feed assimilation is used for catabolism, and \( b \) is food assimilation efficiency.

The parameter \( b \) which is explained by Nath et al. (1994) as food assimilation efficiency on feed intake that available for energy metabolism and usually not constant. Dampin et al. (2012) give new functionality of feed assimilation efficiency on fish weight, below:

\[ b = \alpha [c (1 + (e^{-c(T - Tc)}))] \]  

(6)

the value of \( b \) in the interval \( d-c \leq b \leq d \) that shows the minimum and maximum \( b \). \( c \) shows the average change of \( b \) at minimum and maximum interval values, while \( Tc \) is the time period of the average shift change \( b \).

Assumed that parameter \( k \) of the catabolism coefficient, exponentially increase within temperature (Ursin, 1967 and Sperber et al., 1977). Nath et al. (1994) modifying the coefficient \( k \) in the catabolism by looking at the value of the minimum temperature (\( T_{min} \)) (fish can’t survive), and given the coefficient \( k \) in equation (7) the following:

\[ k = k_{min} \exp[j(T - T_{min})]. \]  

(7)

where, \( k_{min} \) is coefficient of catabolism on \( T_{min} \) and \( j \) is indicates the temperature in the catabolism (constant).

Dampin et al. (2012) describes that the level of feed fish in wastewater oxidation pond depends on the fish weight and environmental around factors. Therefore, daily ratio will depend on it means, so that feed intake of rate can be assumed that:

\[ \frac{dR}{dt} = \tau \delta \beta fhW^*, \]  

(8)

where, \( \tau \) is temperature (Brett, 1969), \( \delta \) is dissolved oxygen (DO) (Bolte et al., 1994), \( v \) is unionize-ammonia (Bolte et al., 1994), \( \beta \) is biochemical oxygen demand (BOD) and \( f \) is relative feeding level (see, Dampin et al., 2012), and \( h \) coefficient of feed consumption.

Fish growth is influenced by feed quality \( (q) \), and the function \( q \) is given below (Jamu and Piedrahita, 1995):

\[ q = \frac{N/C_F}{K_q(N/C_Z)} \quad \text{if } Q^*_{c,e} > N/C_F \]

\[ q = 1 \quad \text{if } Q^*_{c,e} \leq N/C_F \]  

(9)

where,

\[ Q^*_{c,e} = K_q(N/C_Z), \text{ or important nutrients from food or} \]
\[ N/C_F = \text{carbon-nitrogen ratio produced fish}, \]
\[ K_q = \text{efficiency gross weight of fish}, \]
\[ N/C_Z = \text{carbon-nitrogen ratio in fish}. \]

### 3. RESULTS AND DISCUSSION

The results of fish growth model by adding parameter of the feed quality \( (q) \) in the wastewater oxidation ponds as bellows.

#### a. Construction of Model

Fish growth rate increases due to feed intake of rate in anabolism process and decrease in catabolism process. The weight of fish increased in anabolism process, is proportional to the percentage of feed assimilation is used for catabolism \( (a) \), food assimilation efficiency \( (b) \), and feed intake of rate. In contrast, fish weight decreased by catabolism through fish activity. In equation model of fish growth in wastewater oxidation ponds (Dampin et al., 2012), is added \( q \) parameter as a new parameter in anabolism process. In mathematics, a differential equation for fish growth with \( q \) parameter in wastewater oxidation ponds defined as follows:

\[ \frac{dW}{dt} = (1-a)q \frac{dR}{dt} - kW^*, \]  

(10)
with \( \frac{dR}{dt} \) is feed intake of rate (Eq.8), \( b \) is food assimilation efficiency (Eq.6), and \( q \) is feed quality (Eq.9).

Fish growth model of VBGM when applied to equation 10, as below:

\[
\frac{dW}{dt} = \left( (1-a) bq \delta \nu f \right) h W^{\frac{2}{3}} - k W^{\frac{1}{3}}.
\]  

And if fish growth model of LGM when applied to equation 10, we will have:

\[
\frac{dW}{dt} = \left( (1-a) bq \delta \nu f \right) h W^{\frac{2}{3}} - k W^{\frac{1}{2}}.
\]  

b. Growth parameters

The fish growth model with feed quality (\( q \), (Eq.10), is influenced by anabolism and catabolism factors. Anabolism factors, influenced the rate of feed intake depends on the percentage of the feed assimilation is used for catabolism (\( a \)), food assimilation efficiency (\( b \)), feed quality (\( q \)), and feed intake of rate. The value \( a \) parameter is assumed to be 0.53 (Bolt et al., 1994). The parameter \( b, q \), and feed intake of rate is described in the next subsection.

Catabolism factor, growth rate is inhibited by released energy of fish, which is equal to \( k \). In equation (7) is used \( k_{\text{min}} \) and \( j \) coefficient respectively of 0.0133 and 0.0132 (Nath et al., 1994), while the value of \( T_{\text{min}} \) can be seen in Table 1.

1. Food Assimilation Efficiency and Feed Quality Parameter

It is assumed that food assimilation efficiency (\( b \)) constantly of 0.62 (Nath et al., 1994) or 0.7108 (Kapetsky and Nath, 1997), and is not constant according to the equation (6). The minimum parameter value of assimilation feed approximately 5-20% (efficiency assimilation of terrestrial herbivore) and maximum parameter values of assimilation feed approximately 80% (taken from the highest assimilation efficiency of carnivores in general). For value of \( d = 0.9, c = 0.7 \) or 0.05 and \( t = 130 \) days on the function \( b \) (Dampin et al., 2012).

Parameter of \( q \) is measured from the nitrogen-carbon ratio in feed quality (\( q \)) parameter on fish cultured pond Nila tilapia (Oreochromis niloticus) of 9.5 or range of 7.10-10.55 (Beristain, 2005) in feed ratio. While, the ratio of nitrogen-carbon which is produced by fish of 10-20 sourced from carbohydrates (Asaduzzaman et al., 2010).

2. Feed intake of rate parameters

The rate of feed intake depends on temperature factor (\( \tau \)), dissolved oxygen (DO) concentration, Unionize-Ammonia (A), Biochemical Oxygen Demand (BOD), relative feeding level (\( f \)), daily feed consumption coefficient (\( h \)). As for other factors, such as it is assumed to be in reasonable limits and didn’t have fish growth affect significantly. Value parameters on the rate of feed intake can be seen in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{min}} )</td>
<td>Minimum temperature</td>
<td>15°C</td>
<td>Dampin et al. (2012)</td>
</tr>
<tr>
<td>( T_{\text{opt}} )</td>
<td>Optimum temperature</td>
<td>33°C</td>
<td>Dampin et al. (2012)</td>
</tr>
<tr>
<td>( T_{\text{max}} )</td>
<td>Maximum temperature</td>
<td>41°C</td>
<td>Dampin et al. (2012)</td>
</tr>
<tr>
<td>( \text{DO}_{\text{crit}} )</td>
<td>Critical dissolved oxygen</td>
<td>1.0 mg/l</td>
<td>Yi (1997)</td>
</tr>
<tr>
<td>( \text{DO}_{\text{min}} )</td>
<td>Minimum dissolved oxygen</td>
<td>0.3 mg/l</td>
<td>Yi (1997)</td>
</tr>
<tr>
<td>( A_{\text{crit}} )</td>
<td>Critical unionize-ammonia</td>
<td>0.6 mg/l</td>
<td>Dampin et al. (2012)</td>
</tr>
<tr>
<td>( A_{\text{max}} )</td>
<td>Maximum unionize-ammonia</td>
<td>1.84 mg/l</td>
<td>Minggawati and Saptono (2012)</td>
</tr>
<tr>
<td>( \text{BOD}_{\text{crit}} )</td>
<td>Critical Biochemical Oxygen Demand</td>
<td>20 mg/l</td>
<td>Dampin et al. (2012)</td>
</tr>
<tr>
<td>( \text{BOD}_{\text{max}} )</td>
<td>Maximum Biochemical Oxygen Demand</td>
<td>40 mg/l</td>
<td>Dampin et al. (2012)</td>
</tr>
<tr>
<td>( f )</td>
<td>Relative feeding level</td>
<td>0.97</td>
<td>Cho (2004)</td>
</tr>
</tbody>
</table>

c. Research object

As object of this paper is used data from observations that have been conducted by The Leam Phak Bia Environmental Research and Development Study Project under the Royal Initiative Petchaburi Province of Thailand (Dampin et al., 2012). The data were taken from the averages total length and weight of fish Nila tilapia (Oreochromis niloticus), cultured in cages at a density of one fish per square meter respectively 7.0 ± 0.33 cm and 5.99 ± 0.75 g at beginning observation. During 48 weeks without feeding, observations indicate that Nila tilapia is gradually growing with an average total length of 28.3 ± 1.85 cm and the total weight of the average 433.33 ± 86.40 g. Fish weight is measured every 4 weeks as shown in Table 2.
4. NUMERICAL SIMULATION

Numerical simulation of equation (10) using the Runge-Kutta method of order 4 with the parameters that have been defined to investigate and evaluate growth rate and influence of feed quality which is added to fish growth model. In simulation, using equation VBGM (Eq.11) and LGM (Eq.12) fish growth. However, during simulation is also presented the other parameters, catabolism and anabolism to exponents respectively, i.e., $m = 0.67$ (Nath et al., 1994) and $n = 0.81$ (Ursin, 1967).

Moreover, the simulation model of fish growth with feed quality ($q$) also conducted some simulations with related parameters, i.e., parameters of food assimilation efficiency ($b$) and BOD are dynamic. Because, food assimilation efficiency process (function of $b$) depend on feed quality of resources, and feed that is not to be assimilated into feces. As a result, the water quality will be reduced. For BOD parameters, roles as a determinant of quality water can affect the condition of the fish to survive or not directly.

This paper only presented LGM simulation models. Because of simulations model indicate the most excellent results. But above all, the model involves fish growth of feed quality ($q$) factor increased quite save when the nitrogen-carbon ratio produced fish ($N/C_2$) is smaller than the nitrogen-carbon ratio in the feed ($N/C_0$). Instead of the growth decreased when the carbon-nitrogen ratio produced fish larger than nitrogen-carbon ratio in the feed. Furthermore, value of BOD 10-20 mg/l growth increases and decreases when value of BOD 25-30 mg/l. Function of $b$ great also be a good support for the growth of all, when the function of $b$ is constant at 0.62 and 0.7108, see Figure 1.

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Fish weight (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$5.57 \pm 0.75$</td>
</tr>
<tr>
<td>4</td>
<td>$24.46 \pm 5.11$</td>
</tr>
<tr>
<td>8</td>
<td>$65.42 \pm 12.44$</td>
</tr>
<tr>
<td>12</td>
<td>$113.61 \pm 21.73$</td>
</tr>
<tr>
<td>16</td>
<td>$157.44 \pm 27.96$</td>
</tr>
<tr>
<td>20</td>
<td>$195.92 \pm 37.95$</td>
</tr>
<tr>
<td>24</td>
<td>$243.00 \pm 46.04$</td>
</tr>
<tr>
<td>28</td>
<td>$273.22 \pm 49.48$</td>
</tr>
<tr>
<td>32</td>
<td>$312.76 \pm 56.60$</td>
</tr>
<tr>
<td>36</td>
<td>$338.12 \pm 59.93$</td>
</tr>
<tr>
<td>40</td>
<td>$343.88 \pm 65.79$</td>
</tr>
<tr>
<td>44</td>
<td>$400.81 \pm 78.93$</td>
</tr>
<tr>
<td>48</td>
<td>$433.33 \pm 86.40$</td>
</tr>
</tbody>
</table>

Source: Dampin et al. (2012)
Equation model of fish growth, when function of $b$ is not constant (used Eq.6), fish growth depends on the minimum and maximum limits of change $b$ in interval $d-c \leq b \leq d$. Simulations indicate decreased growth is poor when BOD 25-30 mg/l, N/C$_F$ > N/C$_Z$ ratio, and value of $c$ in function of $b$ changed quite large. While the condition return to change to good, $c$ have smaller values which is affected in food assimilation efficiency ($b$) increased, see figure 2. Thus, the function $b$ is dependent on value of $c$ which is quite small, see Figure 3.

**Figure 1.** Fish growth model LGM with function of $b$ is constant: (a) N/C$_F$ > N/C$_Z$ ratio and BOD 10-20 mg/l; (b) N/C$_F$ < N/C$_Z$ ratio and BOD 10-20 mg/l; (c) N/C$_F$ > N/C$_Z$ ratio and BOD 25-30 mg/l; (d) N/C$_F$ < N/C$_Z$ ratio and BOD 25-30 mg/l

5. CONCLUSION

Fish growth model with feed quality factor in wastewater oxidation ponds increased quite save when ratio of the nitrogen-carbon produced by fish larger than nitrogen-carbon ratio in feed. It is different, when the condition of carbon-nitrogen ratio produced by fish smaller than nitrogen-carbon ratio in feed, and fish growth will decreases than before. But good conditions, growth does not involve feed quality (no effect) or function $q = 1$. The decreasing growth, as well as influenced function of $b$ (food assimilation efficiency), are relatively small, and Biochemical Oxygen Demand (BOD) values are quite high. Unfriendly conditions (decreasing growth), can be overcome by reducing the value of $c$ in function of $b$ that result in function of $b$ is high. Thus, when BOD 25-30 mg/l (quite large), feed quality ($q$) will take effect when the value of the function of $b$ is great.
REFERENCES


