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Spectral Computational Based Methods for Solutions of Second Order Parabolic Partial Differential Equations

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ABSTRACT

In this work, new computational methods are developed for solutions of second order partial differential equation using a spectral global trigonometric function at step length$k \geq 2$. Accuracy, Consistency, Stability & Convergence properties of the methods are tested and found suitable for solutions of second order PDE when implemented on some problems which had been solved using existing finite difference method.

Key Words; *Trigonometry, Taylor Series, Time, & Space Difference*

1. INTRODUCTION

Differential equation constitutes a large and very important aspect of modern mathematics. The introduction of calculus led to the discovery of differential equations as an area of great theoretical and practical applications in physical, natural and engineering research works, as at then, and till today. Problems in various fields of sciences, engineering, economics, medicine, sociology, celestial mechanics, spaceship cyclotron, atomic weapons, applied kinematics and energy matters; to mention but a few; can be model as differential equation. A differential equation is an equation containing a function and its derivative with respect to the independent variables.

A little less than century ago; the Schrodinger equation became the key opening the door to the application of Partial Differential Equations in quantum chemistry for small atomic and molecular systems and, the system is fast growing complex [Glowinski & Neittaanmiki, 2008]. Partial Differential Equations are used to model problems involving functions of several variables and are either solved by hand, or computer. They can be used to describe a wide variety of phenomena such as sound, heat, electrostatics, electrodynamics, fluid flow, elasticity or quantum mechanics [Richard et. al. 2004].

In recent years, some promising approximate analytical methods are proposed, such as Exponential Function Method, Homotopy Perturbation Method, Adomian Decomposition Method and Variational Iteration Method [VIM] and so on.

Variational Iteration Method (VIM) is arguably considered as the most effective and convenient method for both weakly and strongly non-linear Partial Differential Equations [He, 2007]**.** The method was firstly considered by Inokuti, Sekina and Mura [1978]. He [1999, 2000, & 2006] developed the method for solving linear, non-linear, and boundary value problems of partial differential equations. This method has been shown to be effective, easy and accurate in solving a large class of linear and non-linear problems with components converging rapidly to accurate solution. Noor et. al.[2007] used the method to suggest a wide class of iterative methods for solving the non-linear equations of the form $f(x) = 0$. Over the years, VIM was applied to Klien-Gordon equation [Yusufoglu, 2008], Helmholt equation [Odibat , 2008], differential algebraic equation [Soltanian et. al. 2008], epidermics and prey - predator models [Rafei et. al. 2007] , non-linear boundary value problems [Momani et. al. 2006], a Neural functional differential with proportional delays (Chem. et. al. 2010), solve Korteweg-de Vries (KdV) equation [Mohyud-din et. al. 2010], modified VIM was used to solve non-linear PDE [Olayiwola et. al [2011]], MVIM and Homotopy Analysis Method were used to solve Schrodinger equation[Behzadi, 2011], Modified initial guess VIM is used in solving variable coefficient fourth order parabolic equations [Olayiwola et. al, 2010] and so on.

However, the exponential growth in speeds and memory of digital computers had been at the origin of an explosive development of numerical methods. Several areas of mathematics such as differential geometry have benefited from their interaction with Partial Differential Equations. Mathematical modeling of many physical system leads to partial differential equation in various fields of physics and engineering. Non linear differential equation in engineering and applied mathematics has been a topic of intensive research for many years. Partial differential equation system is applied in study of mechanic system and field of another science such as description of wave propagation [Aytac and Ibrahim, 2008], dynamical behavior of structures such as beam and plates under the action of moving load [Oni and Awodola, 2009] .

DERIVATION OF THE METHODS

The development of the numerical methods for solution of partial differential equation of the form $U = U(x,t), \dots\dots\dots\dots\dots\dots\dots..(1.1)$

Equation (1.1) is defined in terms of parabolic partial differential equation in (1.2);

$\frac{\partial U(x,t)}{\partial t} = \frac{\partial^2 U(x,t)}{\partial x^2}$ ……………………………… (1.2)

where t is the time,
and x is the distance coordinate;

; $y(0,t) = \varphi_0(t)$ *and* $y(1,t) = \varphi_1(t); t \geq 0;$ *and* $\Phi = |a \leq x \leq b| \times |t \geq 0|$.

The grid point $(x,t) = (x_n, t_n)$ *where* $x = a + nh, t = nk;\ h = \frac{b-a}{n}$, *n is an integer*.

Equation (1.2) is a sample of diffusion equation and how one solve it numerically has to begin with outlining some of its basic properties in the case of one space dimension and to more than one space dimension qualitatively and, which can be generally apply to parabolic partial differential equation. Equation (1.2) is first degree in time (t) and thereby requires one initial condition and since it is second degree in space (x) and, therefore, requires two boundary conditions. For details see Fatunla (1988), Lambert(1973), Jain(1984), Jain-et-al(1984).

In one space dimension the boundary conditions are typically applied at two endpoints. Hence, the parabolic partial differential equation is a mixture of an ODE - IVP and a two point ODE-BVP and, generally requires numerically implicit methods. (Ancona, M. G., 2002).

Assuming the theoretical solution of equation (1.2) is in form of the basis spectra function (1.3)

$$y_{(x_{m+i},t_{n+j})} = aCos(x_{m+i},t_{n+j}) + bSin(x_{m+i},t_{n+j}), \ldots\ i,j = 0,1,2,3 \ldots \qquad (1.3)$$

and time (t) and space (x) are independent variables.

By considering time (t) - as an independent variable and, assume space (x) to be stable then the equation (1.3) becomes

$y_{(x_m,t_{n+j})} = aCos(x_m,t_{n+j}) + bSin(x_m,t_{n+j}), \ldots\ \ldots\ldots\ldots (1.4)$

At $j = 0;\quad y_{(x_m,t_n)} = aCos(x_m,t_n) + bSin(x_m,t_n) \ldots\ldots (1.5)$

At $j = 1;\quad y_{(x_m,t_{n+1})} = aCos(x_m,t_{n+1}) + bSin(x_m,t_{n+1}), (1.6)$

At $j = 2;\quad y_{(x_m,t_{n+2})} = aCos(x_m,t_{n+2}) + bSin(x_m,t_{n+2}), \ldots (1.7)$

Subtract equation (1.5) from equation (1.6) and simplify to obtain equation (1.8)

$y_{(x_m,t_{n+1})} - y_{(x_m,t_n)} = aCos(x_m,t_{n+1}) + bSin(x_m,t_{n+1}) - aCos(x_m,t_n) + bSin(x_m,t_n),$ $y_{(x_m,t_{n+1})} - y_{(x_m,t_n)} = a(Cos(x_m,t_{n+1}) - Cos(x_m,t_n)) + b(Sin(x_m,t_{n+1}) - Sin(x_m,t_n)) \ldots. (1.8)$

Simplify equation (1.8) to have equation (1.9)

$y_{(x_m,t_{n+1})} - y_{(x_m,t_n)} = a[-2sin(x_m, \frac{(2t_n+h)}{2}\sin(x_m, \frac{h}{2})] + b[2Sin(x_m, \frac{h}{2})\cos(x_m, \frac{(2t_n+h)}{2}] \ldots. (1.9)$

By setting $(x_m, t_n) = (0,0)$ then equation (1.9) becomes

$$y_{(m,n+1)} - y_{(m,n)} = a\left[-2sin\frac{h}{2}sin\frac{h}{2}\right] + b\left[2sin\frac{h}{2}cos\frac{h}{2}\right]$$

$y_{(m,n+1)} - y_{(m,n)} = -2sin\frac{h}{2}\left[asin\frac{h}{2} - bcos\frac{h}{2}\right]$……………………………….(1.10)

Subtract equation (1.6) from equation (1.7) and simplify to obtain equation (1.11)

$y_{(x_n,t_{n+2})} - y_{(x_n,t_{n+1})} = aCos(x_n,t_{n+2}) + bSin(x_n,t_{n+2}) - aCos(x_n,t_{n+1}) + bSin(x_n,t_{n+1})$ $y_{(x_m,t_{n+2})} - y_{(x_m,t_{n+1})} = a\left(Cos(x_m,t_{n+2}) - Cos(x_m,t_{n+1})\right) + b(Sin(x_m,t_{n+2}) - Sin(x_m,t_{n+1})) \ldots\ldots (1.11)$

Simplify equation (1.11) to have equation (1.12)

$y_{(x_m,t_{n+2})} - y_{(x_m,t_{n+1})} = a[-2sin(x_m, \frac{(2t_n+3h)}{2}\sin(x_m, \frac{h}{2})] + b[2Sin\left(x_m, \frac{h}{2}\right)\cos\left(x_m, \frac{(2t_n+3h)}{2}\right] \ldots\ldots (1.12)$

By setting $(x_m, t_n) = (0,0)$ in (1.12) and simplify as follows

$$y_{(m,n+2)} - y_{(m,n+1)} = a\left[-2sin\frac{3h}{2}sin\frac{h}{2}\right] + b\left[2sin\frac{h}{2}cos\frac{3h}{2}\right]$$

$$y_{(m,n+2)} - y_{(m,n+1)} = -2sin\frac{h}{2}\left[asin\frac{3h}{2} - bcos\frac{3h}{2}\right] \ldots\ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots..(1.13)$$

Subtract equation (1.11) from equation (1.13) and simplify to obtain equation (1.14)

$$y_{(m,n+2)} - 2y_{(m,n+1)} + y_{(m,n)} = -2sin\frac{h}{2}\left[asin\frac{3h}{2} - bcos\frac{3h}{2}\right] + 2sin\frac{h}{2}\left[asin\frac{h}{2} - bcos\frac{h}{2}\right]\ldots$$

$$y_{(m,n+2)} - 2y_{(m,n+1)} + y_{(m,n)} = -2sin\frac{h}{2}\left[a[sin\frac{3h}{2} - sin\frac{h}{2}]\right] + 2bsin\frac{h}{2}\left[cos\frac{3h}{2} - cos\frac{h}{2}\right]\ldots \quad .$$

$$y_{(m,n+2)} - 2y_{(m,n+1)} + y_{(m,n)} = -2sin\frac{h}{2}\left[\left[a[sin\frac{3h}{2} - sin\frac{h}{2}]\right] + b\left[cos\frac{3h}{2} - cos\frac{h}{2}\right]\right]\ldots(1.14)$$

Simplify equation (1.14) as follows and obtain equation (1.15)

$$y_{(m,n+2)} - 2y_{(m,n+1)} + y_{(m,n)} == -2sin\frac{h}{2}\left[\left[a[sin\frac{2h}{2}Cos\frac{4h}{2}]\right] - 2b\left[Sin\frac{4h}{2}Sin\frac{2h}{2}\right]\right]$$

$$y_{(m,n+2)} - 2y_{(m,n+1)} + y_{(m,n)} = -2sin\frac{h}{2}Sinh\left[[a[Cos2h]] + b[Sin2h]\right]\ldots\ldots\ldots(1.15)$$

Take the first derivative of equation (1.4) with respect to time t as an independent as follows;

$$f_{(x_m,t_{n+j})} = y'_{(x_m,t_{n+j})} = -aSin(x_m,t_{n+j}) + bCos(x_m\,,t_{n+j}).\,(1.16)$$

$$At\ \ j = 0\,;\ f_{(x_m,t_n)} = y'_{(x_m,t_n)} = -aSin(x_m,t_n) + bCos(x_m\,,t_n),\ldots\ldots\ldots\ldots(1.17)$$

$$At\ \ j = 1;\ \ f_{(x_m,t_{n+1})} = y'_{(x_m,t_{n+1})} = -aSin(x_m,t_{n+1}) + bCos(x_m\,,t_{n+1}),\ldots\ldots(1.18)$$

$$At\quad j = 2, f_{(x_m,t_{n+2})} = y'_{(x_m,t_{n+2})} = -aSin(x_m,t_{n+2}) + bCos(x_m\,,t_{n+2}),\ldots\ .(1.19)$$

Add equations (1.17) and (1.18) to obtain equation (1.20)

$$f_{(m,n+1)}+f_{(m,n)} = [-aSin(x_m,t_{n+1}) + bCos(x_m\,,t_{n+1})] + [-aSin(x_n,t_n) + bCos(x_n\,,t_n)]$$

$$f_{(m,n+1)}+f_{(n,m)} = \left[-[aSin(x_m,t_{n+1}\right) + aSin(x_m,t_n)]$$

$$+b[Cos(x_m\,,t_{n+1}) + Cos(x_m\,,t_n)]\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1.20)$$

Simplify equation (1.20) and obtain equation (1.21)

$$f_{(m,n+1)} + f_{(m,n)} = -a\left[2sin[x_m,\frac{(2t_n + k)}{2}]cos[x_m,\frac{k}{2}]\right] + b[2cosx_m,\frac{k}{2}cos[x_m,\frac{(2t_n + k)}{2}].$$

$$f_{(m,n+1)} + f_{(m,n)} = -2cos[x_m,\frac{k}{2}]\left[asin[x_m,\frac{k}{2}] - bcos[x_m,\frac{k}{2}]\right]\ldots(1.21)$$

Adopting equilibrium or quasi – equilibrium phenomena; in which the solution in one place depends on the solution everywhere else; then we can equate equations (1.10) and (1.21) and simplify to have equation (1.22) [Richard et. al, 2004]

$$\frac{y_{(m,n+1)} - y_{(m,n)}}{f_{(m,n+1)} + f_{(m,n)}} = \frac{-2\sin[x_m,\frac{k}{2}]\left[\mathrm{asin}[x_m,\frac{k}{2}] - bcos[x_m,\frac{k}{2}]\right]\ldots}{-2\cos[x_m,\frac{k}{2}]\left[asin[x_m,\frac{k}{2}] - bcos[x_m,\frac{k}{2}]\right]\ldots} = tan[x_m,\frac{k}{2}] \approx tan\frac{k}{2}\ldots\ldots\ldots\ldots(1.22)$$

By adopting Taylor series expansion for $tan\frac{k}{2}\ as\ follows$

$$tan\frac{k}{2} = h\left\{\frac{1}{2} + \frac{h^2}{24} + \frac{h^4}{240} + \cdots\ldots\ldots\ldots\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.23)$$

to a sufficiently small value of k; as k approaches zero then, $tan\frac{k}{2} \approx \frac{k}{2}$ and equation (1.22) becomes

$$\frac{y_{(m,n+1)} - y_{(m,n)}}{f_{(m,n+1)} + f_{(m,n)}} = \frac{k}{2} \ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots.. (1.24)$$

$$y_{(m,n+1)} - y_{(m,n)} = \frac{k}{2}[f_{(m,n+1)} + f_{(m,n)}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (1.25)$$

$$y_{(m,n+1)} = y_{(m,n)} + \frac{k}{2}[f_{(m,n+1)} + f_{(m,n)}] \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1.26)$$

Equation (1.26) is similar to Trapezodial Rule which is of one –step implicit method. [Lambert,1973]

$$y_{(m,n+2)} - y_{(m,n+1)} = a[-2sin[x_m, \tfrac{(2t_n+3k)}{2}\sin[x_m, \tfrac{3k}{2}] + b[2Sin[x_m, \tfrac{3k}{2}]cos[x_m, \tfrac{2t_n+3k)}{2}].$$

$$y_{(m,n+2)} - y_{(m,n+1)} = -2\sin\left[x_m, \frac{3k}{2}\right]\left[asin[x_m, \frac{(2t_n+3k)}{2}\right] - bCos[x_m, \frac{2t_n+3k}{2}]] \ldots\ldots\ldots (1.27)$$

$$f_{(m,n+2)} + f_{(m,n+1)} = a\left[-2sin[x_m, \frac{2t_n+3k}{2}]cos[x_m, \frac{3k}{2}]\right] + b\left[2cos[x_m, \frac{k}{2}]cos[x_m, \frac{3k}{2}]\right]$$

$$f_{(m,n+2)} + f_{(m,n+1)} = -2cos[x_m, \frac{3k}{2}]\left[asin[x_m, \frac{2t_n+3k}{2}]\right] - \left[bcos[x_m, \frac{2t_n+3k}{2}]\right] \ldots (1.28)$$

Following the conditions leading to equation (1.22) to have;

$$\frac{y_{(m,n+2)} - y_{(m,n+1)}}{f_{(m,n+2)} + f_{(m,n+1)}} = \frac{-2\sin\left[x_m, \frac{3k}{2}\right]\left[asin[x_m, \frac{(2t_n+3k)}{2}\right] - bCos[x_m, \frac{2t_n+3k}{2}]}{-2cos[x_m, \frac{3k}{2}]\left[asin[x_m, \frac{2t_n+3k}{2}]\right] - \left[bcos[x_m, \frac{2t_n+3k}{2}]\right]} = \tan\left(x_m, \frac{3k}{2}\right) \ldots\ldots (1.29)$$

Adopting Taylor series to expand $tan[x_m, \frac{3k}{2}] \approx tan\frac{3k}{2}\ as\ follows$

$$tan\frac{3k}{2} = h\left\{\frac{3}{2} + \frac{27h^2}{24} + \frac{243h^4}{240} + \cdots \ldots\ldots\ldots..\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (1.30)$$

to a sufficiently small value of k; as k approaches zero $tan\frac{3k}{2} \approx \frac{3k}{2}$; adopting this assumption in equation (1.29) and simplify to obtain

$$y_{(m,n+2)} - y_{(m,n+1)} = \frac{3k}{2}[f_{(m,n+2)} + f_{(m,n+1)}$$

$$y_{(m,n+2)} = y_{(m,n+1)} + \frac{3k}{2}\left[f_{(m,n+2)} + f_{(m,n+1)}\right] \ldots\ldots\ldots.. (1.31)$$

$$y_{(m,n+2)} = y_{(m,n)} + \frac{k}{2}[f_{(m,n+1)} + f_{(m,n)}] + \frac{3k}{2}\left[f_{(m,n+2)} + f_{(m,n+1)}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.32)$$

$$y_{(m,n+2)} = y_{(m,n)} + \frac{k}{2}[3f_{(m,n+2)} + 4f_{(m,n+1)} + f_{(m,n)}] \ldots\ldots\ldots\ldots (1.33)$$

By reconsidering equation (1.2) and theoretical solution (1.3) for the second derivative in terms of time-space independent variables

$$y_n = aCosx_n, t_n + bSinx_n, t_{m}, \ldots \qquad \ldots. \qquad (1.3)$$

$$f_{(n),x} = y_n'' = -(aCosx_n, t_n + bSinx_n, t_n) = -y_n \qquad \ldots\ldots \qquad (1.32)$$

$$f_{(n+1),x} = y_{n+1}'' = -(aCosx_{n+1}, t_{n+1} + bSinx_{n+1}, t_{n+1}), = -y_{n+1} \quad \ldots \quad \ldots\ldots\ldots. (1.33)$$

$$f_{(n+2),x} = y_{n+2}'' = -\left(aCosx_{n+2}, t_{n+2} + bSinx_{n+2,}t_{n+2}\right), = -y_{n+2} \quad \ldots\ldots\ldots.. \qquad (1.34)$$

Subtract equation (1.32) from equation (1.33)

$$y_{n+1}'' - y_n'' = -(aCosx_{n+1}, t_{n+1} + bSinx_{n+1}, t_{n+1}) + \left(aCosx_n, t_n + bSinx_{n,}t_n,\right)$$

$$y_{n+1}'' - y_n'' = -a(Cosx_{n+1}, t_{n+1} - Cosx_n, t_n) - b\left(Sinx_{n+1,}t_{n+1} - Sinx_n, t_n,\right) \ldots\ldots. \quad (1.35)$$

$$y_{n+1}'' - y_n'' = -a\left(Cos(x_n + \mathrm{h})(t_n + \mathrm{k}) - Cosx_{n,}t_n\right) - b\left(Sin(x_n + \mathrm{h})(t_n + \mathrm{k}) - Sinx_{n,}t_n,\right) \ldots\ldots. \quad (1.36)$$

Adopting h = k in equation (1.36) and simplify to obtain equation (1.37)

$$y''_{n+1} - y''_n = -[-2a\sin\frac{(x_n t_n + x_n h + t_n h + h^2 + x_n t_n)}{2}\sin\frac{(x_n t_n + x_n h + t_n h + h^2 - x_n t_n)}{2}]] + b[2sin\frac{(x_n t_n + x_n h + t_n h + h^2 - x_n t_n)}{2}]cos\frac{(x_n t_n + x_n h + t_n h + h^2 + x_n t_n)}{2}] \dots\dots\dots\dots\dots\dots\dots\dots. (1.37)$$

$Setting\ (x_n,\ t_n) = (0,0)$ then $f_{(n+1)x} - f_{(n)x} = 2sin\frac{h^2}{2}\left[asin\frac{h^2}{2} - bcos\frac{h^2}{2}\right]$.(1.38)

By adding equations (1.32) and (1.33) to obtain (1.39)

$$f_{(n+1),x} + f_{(n),x} = -a(Cos(x_n + h)(t_n + h) + Cosx_n t_n) - b(Sin(x_n + h)(t_n + h) + Sinx_{n,}t_{n,}) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (1.39)$$

Simplify equation (1.39) to get equation (1.40)

$$f_{(n+1),x} + f_{(n),x} = -\{[2acos\frac{(x_n t_n + x_n h + t_n h + h^2 + x_n t_n)}{2}cos\frac{(x_n t_n + x_n h + t_n h + h^2 - x_n t_n)}{2}] + b[2sin\frac{(x_n t_n + x_n h + t_n h + h^2 + x_n t_n)}{2}cos\frac{(x_n t_n + x_n h + t_n h + h^2 - x_n t_n)}{2}]\} \dots.. (1.40)$$

Setting $(x_n,\ t_n) = (0,0)$ then

$$f_{(n+1),x} + f_{(n),x} = -2cos\frac{h^2}{2}\left[acos\frac{h^2}{2} + bsin\frac{h^2}{2}\right]$$

$$f_{(n+2),x} + f_{(n+1),x} = -(aCosx_{n+2}t_{n+2} + bSinx_{n+2}t_{n+2}) + (aCosx_{n+1}t_{n+1} + bSinx_{n+1}t_{n+1},) \dots\dots\ (1.41)$$

Simplifying equation (1.41) to get

$$f_{(n+2),x} + f_{(n+1),x} = -\{[2acos\frac{(x_n t_n + 2x_n h + 2t_n h + 4h^2 + (x_n t_n + x_n h + t_n h + h^2)}{2}cos\frac{(x_n t_n + 2x_n h + 2t_n h + 4h^2 - (x_n t_n + x_n h + t_n h + h^2)}{2}] + b[2sin\frac{(x_n t_n + 2x_n h + 2t_n h + 4h^2) + (x_n t_n + x_n h + t_n h + h^2)}{2}cos\frac{(x_n t_n + 2x_n h + 2t_n h + 4h^2) - (x_n t_n + x_n h + t_n h + h^2)}{2}].. (1.42$$

$Setting\ (x_n,\ t_n) = (0,0)$ in equation (1.42) then

$$f_{(n+2),x} + f_{(n+1),x} = -2cos\frac{3h^2}{2}\left[acos\frac{5h^2}{2} + bsin\frac{3h^2}{2}\right] \dots\dots\dots\dots\dots (1.43)$$

Add equations (1.40) and (1.43) to get

$$f_{(n+2),x} + 2f_{(n+1),x} + f_{(n),x} = \begin{Bmatrix} -((aCosx_{n+2}t_{n+2} + bSinx_{n+2}t_{n+2}) \\ +(aCosx_{n+1}t_{n+1} + bSinx_{n+1}t_{n+1},)) \end{Bmatrix} + \begin{Bmatrix} (-((aCosx_{n+1}t_{n+1} + bSinx_{n+1}t_{n+1}) \\ +(aCosx_n t_n + bSinx_n t_n,)) \end{Bmatrix} ..(1.44)$$

$$f_{(n+2),x} + 2f_{(n+1),x} + f_{,xn} = -2a\left[cos\frac{3h^2}{2}cos\frac{5h^2}{2} + cos\frac{h^2}{2}cos\frac{h^2}{2}\right] - 2b\left[cos\frac{3h^2}{2}sin\frac{5h^2}{2} + cos\frac{h^2}{2}sin\frac{h^2}{2}\right]$$

$$f_{(n+2),x} + 2f_{(n+1),x} + f_{,xn} = -2Cos\frac{h^2}{2}\left[aCos\frac{2h^2}{2} + bSin\frac{2h^2}{2}\right] \dots\dots\dots. (1.45)$$

$$y_{(n+1),x} - y_{n,x} = a(Cos(x_{n+1},t_{n+1}) - Cos(x_n,t_n)) + b(Sin(x_{n+1,}t_{n+1}) - Sin(x_n,t_n)) \dots 1.46$$

$$y_{(x_{n+2},t_{n+2})} - y_{(x_{n+1},t_{n+1})} = a(Cos(x_{n+2},t_{n+2}) - Cos(x_{n+1},t_{n+1}))$$

$$+b(Sin(x_{n+2},t_{n+2}) - Sin(x_m,t_{n+1})) \dots\dots\dots\dots\dots. (1.47)$$

Add equation (1.46) and equation (1.47) and simplify to obtain;

$$y_{(,n+2),x} - 2y_{(,n+1),x} + y_{(n),x} = -2sin\frac{h^2}{2}[[a[Cosh^2]] + b[Sinh^2]] \dots\dots\dots\dots\dots\dots(1.48)$$

Divide equation (1.48) by equation (1.45) and adopt the condition stated in (1.22) and (1.26) to obtain the new spectra computational based scheme in equation (1.49);

$$y_{n+2} = 2(y_n + \frac{k}{2}[f_{n+1,t} + f_{n,t}]) - y_n + \frac{h^2}{4}[f_{n+2,x} + 2f_{n+1,x} + f_{n,x}]$$

$$y_{n+2} = y_n + k\{[[f_{n+1,t} + f_{n,t}] + \frac{\mathrm{h}^2}{4}(f_{n+2,x} + 2f_{n+1,x} + f_{n,x})\} \ldots\ldots..(1.49)$$

Implementation of Equation (1.49)

Numerical Example 1

Sample problem; $\frac{\partial^2(x,t)}{\partial x^2} = \frac{\partial(x,t)}{\partial t}, for\ 0 \le x \le 1\ and\ t \ge 0\ where\ f(0,t) = 1,$

$f(0,t) = 1 + x\ and\ \frac{\partial f(x,t)}{\partial x}_{x=1} = 0$

$whose\ theoretical\ solution\ is\ f(x,t) = Cosxt + Sinxt$

[K. A. Stroud and Dexter J. Booth, (2003)]

Numerical Example 2

Consider the heat equation $\frac{\partial u}{\partial t}(x,t) - \frac{\partial^2 u}{\partial x^2}(x,t) = 0, \quad 0 < x < 1,\ 0 < t$

With the boundary conditions $u(0,t) = u(1,t)0, \quad 0 < t$

And initial conditions $u(x,0) = \sin(\pi x), \quad 0 \le x \le 1$

Theoretical solution to this problem is $u(x,t) = e^{-\pi^2 t}\sin(\pi x)$ [Richard L et. al (2004)]

Tables and Graphs for problem 1 are shown below;
Table 1

x	Yexact	Ynum	Error
0	0.00000000000000000000	0.00995000083332917000	-0.00995000083332917000
0.1	0.00999983333416667000	0.01977461196746320000	-0.00977477863329653000
0.2	0.03998933418663420000	0.04924899934898780000	-0.00925966516235360000
0.3	0.08987854919801110000	0.09840865226273250000	-0.00853010306472141000
0.4	0.15931820661424600000	0.16735823198118700000	-0.00804002536694098000
0.5	0.24740395925452300000	0.25596455897848600000	-0.00856059972396303000
0.6	0.35227423327509000000	0.36254609853366500000	-0.01027186525857500000
0.7	0.47062588817115800000	0.48174568638085400000	-0.01111979820969600000
0.8	0.59719544136239200000	0.60480291586661200000	-0.00760747450421995000
0.9	0.72428717437014300000	0.72515226864763300000	-0.00086509427749004200

GRAPHICAL RESULT OF PROBLEM 1 h = 0.01

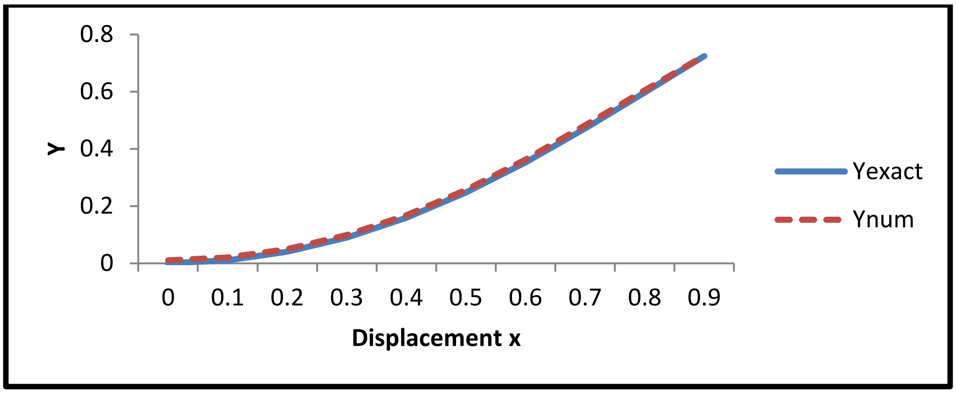


Table 2

x	Yexact	ynum	error
0	0	8.00E-05	-8.00E-05
0.02	8.00E-05	0.00016	-7.99E-05
0.04	0.00032	0.0004	-7.98E-05
0.06	0.00072	0.000799	-7.95E-05
0.08	0.00128	0.001359	-7.91E-05
0.1	0.002	0.002079	-7.86E-05
0.12	0.00288	0.002958	-7.80E-05
0.14	0.00392	0.003997	-7.72E-05
0.16	0.005119	0.005196	-7.64E-05
0.18	0.006479	0.006554	-7.55E-05

h = 0.02

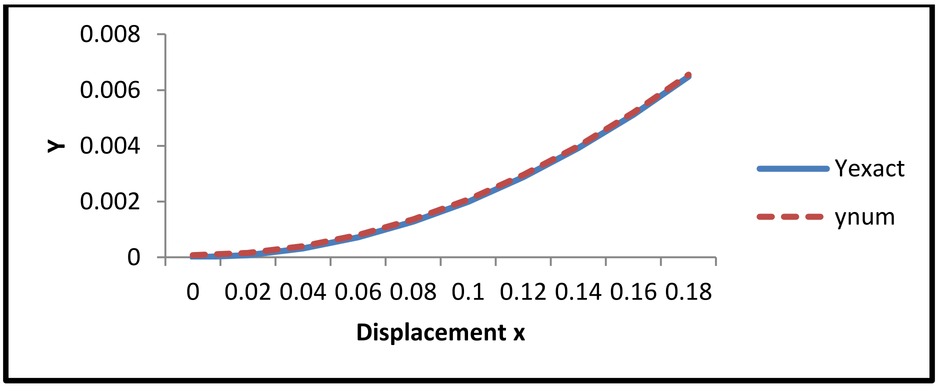


Table 3 for h = 0.3

x	Y exact	Y num	Error
0	0.00000000000000000000	0.00026987850001640200	-0.00026987850001640200
0.03	0.00026999996355000100	0.00053945310968662600	-0.00026945314613662500
0.06	0.00107999766720151000	0.00134817376769503000	-0.00026817610049352000
0.09	0.00242997342803717000	0.00269601980741446000	-0.00026604637937729000
0.12	0.00431985070234793000	0.00458291959512375000	-0.00026306889277582000
0.15	0.00674943048316609000	0.00700869467668452000	-0.00025926419351843000
0.18	0.00971829947805945000	0.00997298132908143000	-0.00025468185102198000
0.21	0.01322571211092770000	0.01347512899799130000	-0.00024941688706360000
0.24	0.01727044643615930000	0.01751407459015740000	-0.00024362815399809800
0.27	0.02185063412217700000	0.02208819084210310000	-0.00023755671992610100

Graph for h = 0.03

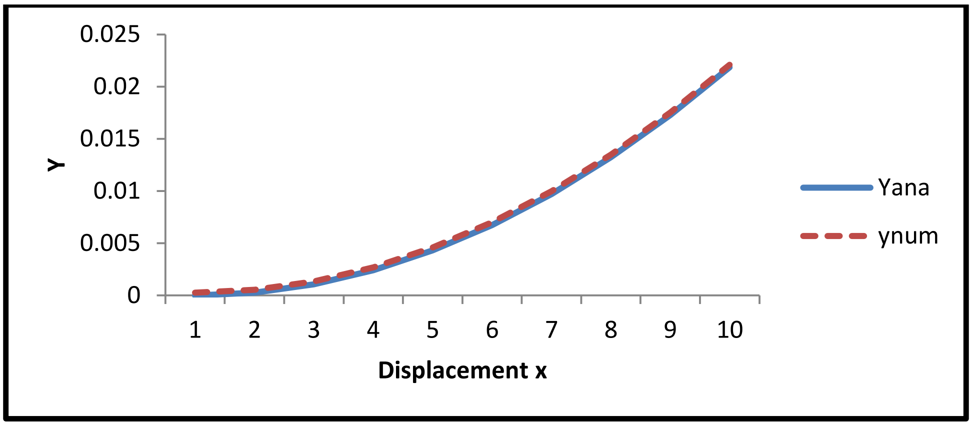


Table 4 for h = 0.04

x	Y exact	Y num	Error
0	0.00000000000000000000	0.00063948800021845300	-0.00063948800021845300
0.04	0.00063999972693336800	0.00127769498080537000	-0.00063769525387200200
0.08	0.00255998252376912000	0.00319229407866958000	-0.00063231155490046000
0.12	0.00575980093646390000	0.00638314764544542000	-0.00062334670898152000
0.16	0.01023888155558320000	0.01084978541320790000	-0.00061090385762470200
0.2	0.01599573367465370000	0.01659104019192970000	-0.00059530651727600000
0.24	0.02302726191487910000	0.02360452687268370000	-0.00057726495780460000
0.28	0.03132788385151410000	0.03188594275244790000	-0.00055805890093380000
0.32	0.04088845473236180000	0.04142814706208440000	-0.00053969232972259800
0.36	0.05169500300082170000	0.05221995189074760000	-0.00052494888992590100

Graph of h = 0.04

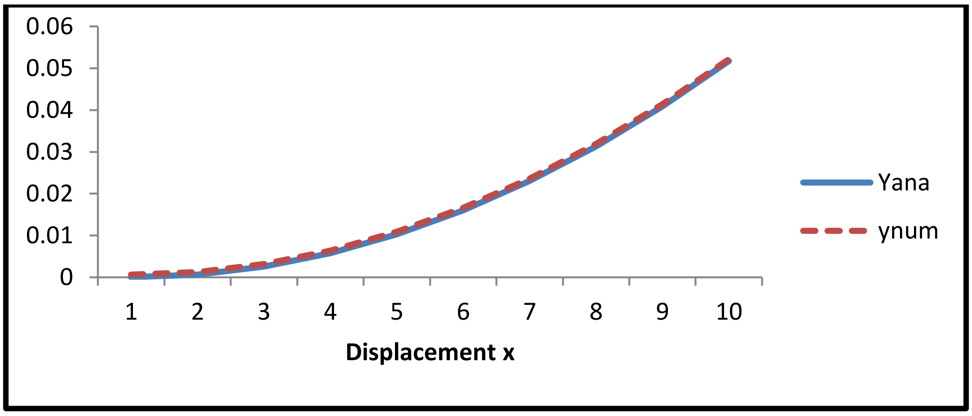
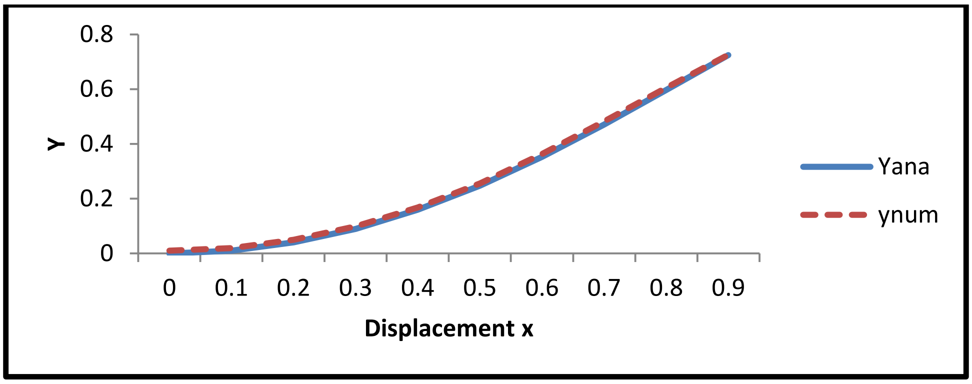


Table 5 for h = 0.05

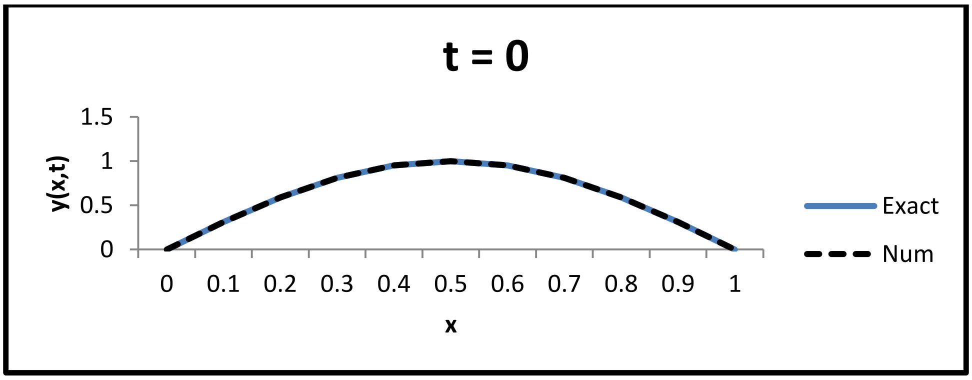
x	Y exact	Y num	Error
0	0.00000000000000000000	0.00995000083332917000	-0.00995000083332917000
0.1	0.00999983333416667000	0.01977461196746320000	-0.00977477863329653000
0.2	0.03998933418663420000	0.04924899934898780000	-0.00925966516235360000
0.3	0.08987854919801110000	0.09840865226273250000	-0.00853010306472141000
0.4	0.15931820661424600000	0.16735823198118700000	-0.00804002536694098000
0.5	0.24740395925452300000	0.25596455897848600000	-0.00856059972396303000
0.6	0.35227423327509000000	0.36254609853366500000	-0.01027186525857500000
0.7	0.47062588817115800000	0.48174568638085400000	-0.01111979820969600000
0.8	0.59719544136239200000	0.60480291586661200000	-0.00760747450421995000
0.9	0.72428717437014300000	0.72515226864763300000	-0.00086509427749004200

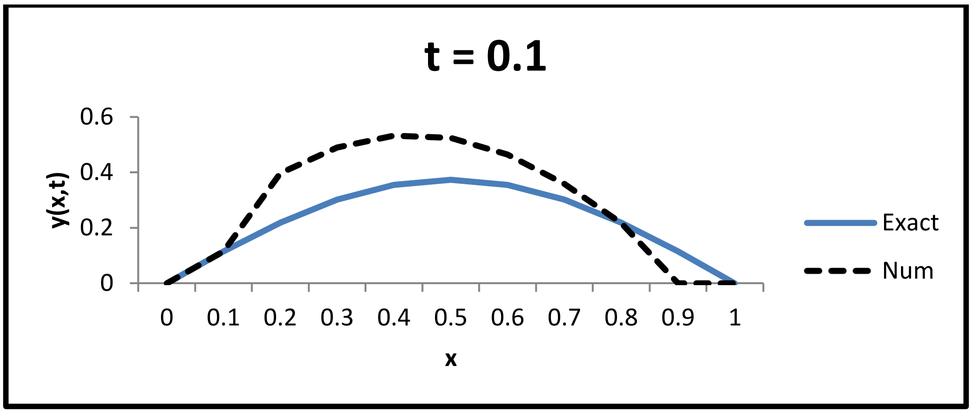
h = 0.05

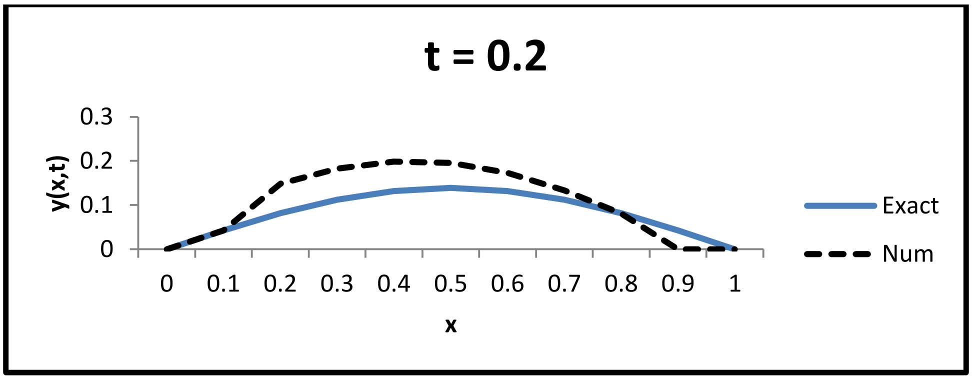


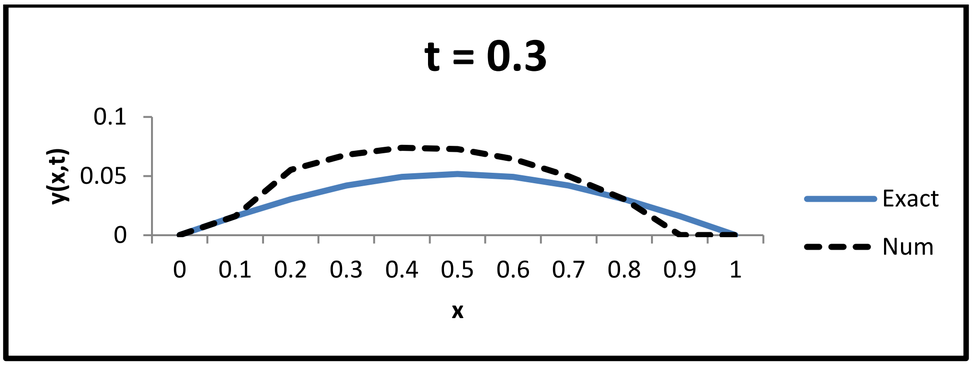
Tables and Graphs for problem 2 are shown below

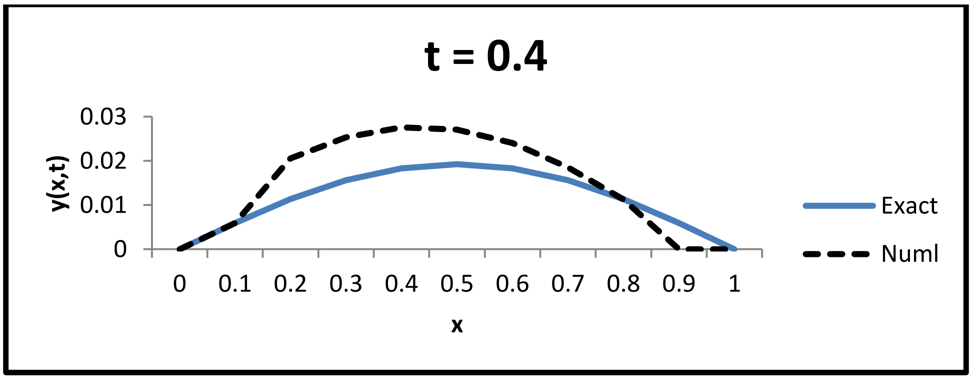
h = 0.1 and k = 0.1 are defined to be constants.
At t = 0.0, t = 0.1, t =0.2, t = 0.3, t = 0.4, t = 0.5; the graphs are depicted below

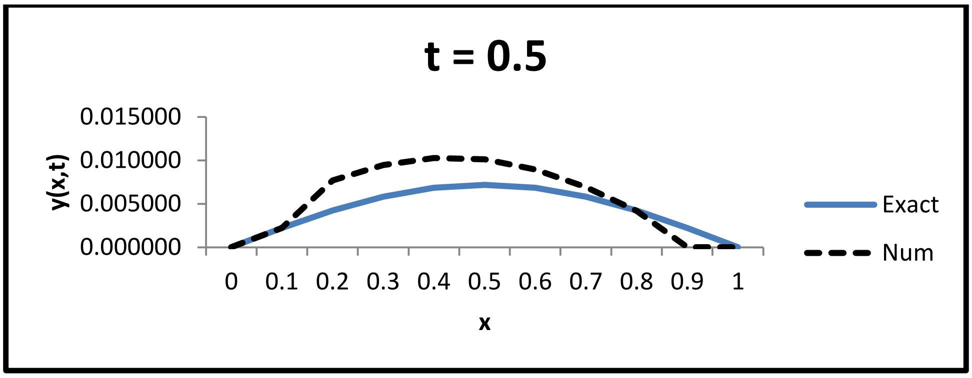



t = 0.1
y(x,t)
0.6
0.4
0.2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
x
Exact
Num


t = 0.2
y(x,t)
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
x
Exact
Num


t = 0.3
y(x,t)
0.1
0.05
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
x
Exact
Num


t = 0.4
y(x,t)
0.03
0.02
0.01
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
x
Exact
Numl



DISCUSSION

It could be seen that the movement along x direction yield a very close results compare to the exact solutions as the values of h = 0.1, 0.2, 0.3, 0.4, 0.5 the method converges speedily as the computed error tends to zero. Also the deviation noted between exact solutions and numerical solutions while varying the values of t are very negligible.

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