IJST EXPLORE International Journal of Science and Technology
CENTRE OF PROFESSIONAL RESEARCH PUBLICATIONS
ISSN 2049-7318

International Journal of Science and Technology Volume 3 No. 9, September, 2014

On the Construction of Counter Example for Power Set

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ABSTRACT

This work presents a way of constructing counter-examples to show that P(AUB) ⊈ P (A)UP(B). Where A and B are any two finite sets; and P(A), P(B) and P(AUB) are the power set of A, B and (AUB) respectively.

Keywords: *Construction, counter example, power set, set, union*

1. INTRODUCTION

Unlike most areas of mathematics, which arise as a result of the cumulative efforts of many mathematicians, sometimes over several generations, set theory is the creation of a single individual, George Cantor, around the turn of the 20th century. In 1847, the Czech mathematician Bernhard Bolzano considered the idea of set, which he described this way:

…an embodiment of the idea or concept which we conceive when we regard the arrangement of its part as a matter of indifference.

The year 1897 saw the appearance of the first of the several paradoxes that arise within set theory once infinite sets are allowed. The first paradox was discovered by Cesare Burali-Forti and Cantor himself discovered a paradox in1899. The most devastating paradox was discovered by Bertrand Russell in 1902.

In 1908, Ernst Zermelo produced what looked like an acceptable solution: a set of axioms for set theory that appeared to avoid the paradoxes. A short while later, Abraham Fraenkel proposed amendment to Zarmelo's theory, and the mathematical community was a able to breathe a collective sigh of relief [1].

The power-set axiom was one the axioms developed; and it guarantees the existence of a power set of a set [2].

From the literature, it is a known fact that, for any two finite sets A and B, it follows that

P(A) U P(B)⊆ P(AUB) and that the converse may or may not be true. That is in general, the converse is not true. In the literature a counter example is usually given to show that P(AUB) ⊈ P (A)UP(B) [1].

In [my second paper 3] a study of such a counter-example is made and the following was discovered:

The necessary and sufficient condition needed for P(AUB) ⊆ P(A) U P(B).

We now extend the result in [3] to develop a means of constructing -examples to show that P(AUB) ⊈ P (A)UP(B).

2. MATHEMATICAL ANALYSIS

Let A and B be any two finite sets then, from the literature, P(A) U P(B)⊆ P(AUB) but P(AUB) ⊆P(A) U P(B) is not generally true. For example, if A = {1, 2} and B = {3} then P(AUB) ⊈ P(A) U P(B). How do we construct A and B such that P(AUB) ⊈ P(A) U P(B)?

2.1 How to construct counter example to show that P(AUB) ⊈ P(A) U P(B)

In [3] we established the conditions for P(AUB) ⊆ P(A) U P(B) and P(AUB) ⊈ P(A) U P(B). We discovered that if A ⊆B or B⊆A then P(AUB) ⊆ P(A) U P(B); and if otherwise, that is if A⊈ B and B ⊈ A then

P(AUB) ⊈ P(A) U P(B). Therefore, to construct counter example to show that P(AUB) ⊈ P(A) U P(B), we need to choose A and B, such that A⊈ B and B ⊈ A. The example above actually satisfied A⊈ B and B ⊈ A that is why P(AUB) ⊈ P(A) U P(B). For example, if:

i. A = {1, 2, …, 1 000 000} and B = {2, …, 1 000 001}
or
ii. A = {a} and B = {b}
or
iii. A = {1, 2, …, 1 000 000} and B = {-1 000 001, -1 000 000, …, 0}

then P(AUB) ⊈ P(A) U P(B).

3. RESULT AND DISCUSSIONS

In the above, we have developed a means of constructing counter example of finite sets A and B such that $P(AUB) \not\subseteq P(A)UP(B)$. Moreover, since the empty is a subset of any set then the condition, $A \not\subseteq B$ and $B \not\subseteq A$, imply that neither A nor B can be empty.

4. CONCLUSION

The presented result has shown that, for any two finite sets A and B,

$P(AUB) \subseteq P(A)\ U\ P(B)$ is not always true. The result has established a way to construct counter-examples to show that $P(AUB) \not\subseteq P(A)UP(B)$.

Hence many counter-examples can now be constructed to show that

$P(AUB) \not\subseteq P(A)UP(B)$, and therefore there is no need to stick to some few counter-examples which, perhaps, were obtained by chance or trial and error.

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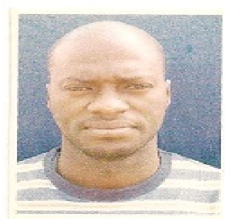
Authors Profile



Ummar Shehu received his MSC from United Kingdom in 2007. The author was a recipient of overseas postgraduate scholarship from Petroleum Technology Development Fund in 2007. He is currently working with the Mathematics Department of Ahmadu Bello University, Nigeria.