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New Approach for Determining the Smoothing Constant (α) of a Single Exponential Smoothing Method

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ABSTRACT

In this research work, a heuristic algorithm has been developed for deriving the exponential constant (α) of the single exponential smoothing method. The approach bound the constant as $0 < \alpha < 1$. This is as against the arbitrary choice of the constant in the open interval as $0 \le \alpha \le 1$, which is the existing approach. Thus, the mathematical approach of determining the constant proposed in this research will guide analysts and users of single exponential smoothing model to derive same exponential constant for a given problem rather than the arbitrary choice which may results into different users or analysts choosing different exponential constants for the same problem. The determination of the constant proposed in this research is based on the number of historical data. The approach was tested empirically on thirty (30) documented problems and results were compared, graphically, with that obtained through the arbitrary choices of the exponential constants employed on same problems. The Exponential constants derived were found to be suitable for both smoothing time series data and for forecasting.

Keywords: *Single exponential, Arbitrary, Heuristic, Smoothening, Forecasting*

1. INTRODUCTION

Planning is one of the functions of management whether strategic, middle or operational level management. If uncertainties cloud the planning horizon, decision makers will find it difficult to plan effectively. Therefore forecasts help decision makers by reducing some of the uncertainties, thereby enabling them to develop more meaningful plans. A forecast is a statement about the future value of a variable (Stevenson, 2009). Forecasting is a method or technique for estimating or projecting into the future based on historic data which can contain certain amount of random variation (Sanjoy, 2011; Prajakta, 2004;). Krajewski *et al* (2007) pointed out that forecasts are useful for both managing processes and managing value chains. At the value chain level, a firm needs forecasts to coordinate with its customers and suppliers, while at the process level, output forecasts are needed to design the various processes throughout the organization. Forecasting arises in many real life situations, and there are many methods available for the purpose of forecasting. It is important that planners and decision makers select forecasting method that is appropriate for particular situations (Pritibhushan, 2012; Rao, 2012). There are many different types of forecasting techniques among which the single exponential smoothing method. Exponential smoothing is a procedure for continually revising a forecast in the light of more recent experience. It assigns exponentially decreasing weights as the observation gets older (Gardner and Diaz-Saiz, 2006; Gregory and Warren, 1999; Taylor, 2004). In other words, recent observations are given relatively more weight in forecasting than the older observations. Prajakta (2004) asserts that to produce smoothed data or to make forecast, this model has received widespread acceptance among American business firms that employ sales forecasts for decision making, planning and control. Exponential smoothing is generally the most accurate of the time series forecasting model, which are extensions of the running average model. Exponential smoothening model can be single or double exponential (Sridharan, Ivan and Annamalai, 2004; Snyder *et al*, 1999). In this research however, the single exponential smoothing model was considered. A lot of forecasting methods have been elaborated and studied in the field of forecasting.

Accuracy and control of forecasts is a vital aspect of forecasting, as a result, forecasters need to minimize forecast errors. However the complex nature of most real-world variables makes it difficult to correctly predict forecast values of those variables on a regular basis. When making periodic forecasts, it is important to monitor forecast errors to determine if the errors are within reasonable bounds. If they are not, it will be necessary to take corrective action (s). Forecast error is the difference that occur between the actual value and the predicted value for a given time period (Stevenson, 2009; Ledolter and Box, 1976).

The single exponential smoothing method is suitable for short range forecast usually one month into the future. For is successful application, an exponential constant α , needs to be determined which presently is subjective but within $0 \le \alpha \le 1$. The model assumes that the data fluctuates around a reasonably stable mean (no trend or consistent pattern of growth) (Prajakta, 2004; Forbes, Snyder and Shami,2012; Collopy and Armstrong, 1992). The equation to calculate a single (simple) exponential smoothing is:

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

where:

F_t *is forecast at period t*

F_{t-1} *is forecast for the immediate past period*

A_{t-1} *is the actual value of the immediate past period*

α *is the exponential smoothing constant;* $0 \le \alpha \le 1$

The accuracy of forecasting of this technique, simple exponential smoothing model depends on the smoothing constant. The Smoothing constant represents a percentage of the forecast error. The value of the smoothing constant, α, is selected by the analyst using the method. The choice of the smoothing constant plays an important role in determining how responsive the forecast is to the historical data. A low constant, α indicates that the forecast would not be as responsive to the observation as can be provided with a higher value of α since it will allow the time series to be swaged quickly by the most recent observation. This implies that forecast values vary with varying α values. Stevenson (2009) stated that “selecting a smoothing constant is basically a matter of judgment or trial and error and so commonly used values of α range from 0.05 to 0.50”. Rao described determining the choice of smoothing constant through the rule of thumb that α < 0.5; typically, α = 0.2 or α = 0.3 work well. Marzena and Toporowski (2012) argued that when smoothing constant is close to 0 it is used for smoothing out unwanted cyclical and irregular components of a time series and if close to 1, it is good enough for forecasting. Chiang (2005) affirmed that as a guide for selecting the smoothing constant, α values close to 0 should be selected if the series have small variations and values close to 1 should be selected if the forecast values appear to depend on recent changes in actual values. Brown states that the smoothing constant should be selected between 0.7 and 0.9. Actual study of time series, however, gives no empirical support to this assertion and no theoretical reasons seem to be available for discussion. The supposition that α ought to be picked in this range appears “strange”. Typical α values range from 0.01 to 0.40, but a satisfactory α can generally be determined by trial-and-error modeling *(*on computer) to see which value minimizes forecast error (Thomas, 2008; Xie, Hong and Wohlin, 1997). In their own view, Krajewski *et al* (2007) stated that realistically more values of α for exponential smoothing should be experimented to determine the smoothing constant. Chiang (2005) also suggested that usually the Mean Squared Error (MSE) or Mean Absolute Deviation (MAD) can be used as the criterion for selecting an appropriate smoothing constant. For instance, by assigning α value from 0.1 to 0.99, the value of the exponential constant that produces the smallest Mean Squared Error (MSE) should be selected. Cho (2003) asserts that in order to find the optimal values of α, simply choose the value that gives the smallest sum of square errors (SSE) by grid search. The grid search chooses a combination for the parameter by employing a method of trial and error. The grid values start with zero and end with one, incrementing by *0.01*. Thus the grid generates values for each parameter ranging from 0, 0.01, 0.02,. . . 1.00 This involves conducting a grid search to evaluate a wide range of possible values. Salim *et al* (2010) stated that “the optimal smoothing constant can be obtained by an exhaustive grid search between the values 0.1 and 1.0 in step sizes of 0.1. William and Friedhelm (2007), suggested that a pattern search which is better than a grid search can be used. The pattern search strategy itself consists of conducting a series of trail evaluations, where the expression is evaluated successively with particular set of α values. The strategy then uses these trial results to decide what to do next. The basic notion underlying this strategy involves moving from one set of trials to another, and when desirable results are obtained, the moves towards “better” values are made in increasingly larger step sizes. Thus, the research for an improved solution is guided by the successes (or failures) obtained in previous function evaluations. The limitations are that a solution is not guaranteed in a finite time and, any solutions discovered by the procedure are not guaranteed to be the minimum (or maximum).

Sanjoy (2011) affirmed that the trial and error method can be used to find minimum value of exponential smoothing constant and argued that no research has been conducted to determine the optimal value of the exponential smoothing constant and that the selection of the smoothing constant is crucial in estimating future forecasts- this has always been chosen subjectively. In Addition, if an analyst is considering several values of α, a forecast using each value could be prepared. Then a performance measure for error could be used to determine which of the values of α considered has the lowest value of the selected performance measure. This method does not aid decision makers and analysts. Therefore, depending on the choice of the value of α, forecasts produced from single exponential smoothing model may or may not be good forecast and hence planning based on the forecast values can be impaired.

The need to derive α value that can produce good forecast values and good smoothening of a time series data is therefore necessary, hence prompted this research work. Therefore, the research developed a heuristic method for determining α values that can produce good forecasts and smoothening of time series data. The aim is to improve on the existing exponential smoothing model used for short term forecast such that the model produces good forecasts and smoothing of time series data. This way, analysts and users using simple exponential smoothing model can derive the value of α mathematically (objectively) rather than the arbitrary (subjective) choices which is the case in practice now. Thus, this improvement will enhance the suitability of the model for producing forecasts based on historic data.

2. METHDOLOGY

The equation to calculate a simple exponential smoothing is as in eq. (1) was used to derive the criteria for the mathematical determination of the α value.

Problems (documented historic data) solved using arbitrary choices of α values with their respective forecasting results were used for the purpose of making comparism with the results obtained using α value derived from the new approach and applied on same documented problems. The documented problems used were divided into two sets- training data and testing data. This is in order to verify the performance of the proposed approach for obtaining the α value towards deriving better forecast and smoothening of time series data that contains seasonality. Graph plots of these comparative results were performed to depict the performances of these α values. Forecasts are estimates and therefore cannot be perfect, implying that errors come into the forecast values. Therefore, it is necessary to determine whether the forecasted errors are tolerable or are beyond the reasonable bounds (Golamreza, 2002).

Thirty (30) documented problems were experimented. These were considered sufficient to draw conclusion on the proposed heuristic approach to determining the exponential constant. However, in this paper the experimental results of ten (10) problems are presented.

3. DISCUSSION OF RESULTS AND FINDINGS

The Proposed Heuristic Development of the α value

In this paper, the heuristic development of α value considered it to be a function of the number of historical data, n; number of operands in the single exponential formula (equ 1); the upper (extreme) value1 in $0 \le \alpha \le 1$; and the one-period time lag in the exponential model. These inputs were therefore used to mathematically determine suitable α value for forecasting as follows:

$$\alpha = upper\ extreme\ value - \frac{number\ of historical\ data - 1\ (i.e\ time\ lag)}{number\ of operands * number\ of\ historic\ data}$$

i.e. $\alpha = 1 - \left(\frac{n-1}{3n}\right)$

This value is determined for using single exponential smoothing for forecasting.

Similarly, employing the same line of argument the α value that would be suitable for smoothening time series data was mathematically developed as follows:

$$\alpha = \frac{number\ of historical\ data - 1\ (i.e\ time\ lag)}{number\ of operands * number\ of\ historic\ data} - 0$$

$$= \left(\frac{n-1}{3n}\right) - 0$$

$$= \left(\frac{n-1}{3n}\right)$$

$where\ 0\ is\ the\ lower\ extreme\ value\ of\ 0 \le \alpha \le 1$

With these developments therefore, α can now closed bounded as $0 < \alpha < 1$.

This condition eliminates the possibility of perfect forecasts and perfect smoothing, which is impractical in real life. This is because forecast is an estimate and must therefore be associated with some elements of errors from the actual observation. Also no time series can perfectly be smoothen without the presence of some elements of factors that may still influence variability in historic data.

Discussion of Results of Experiments

Numerical experiments of different α-values, as used with the documented problems were performed. The proposed α-values for both forecasting and smoothing were used on the same problem. The results of the computations were plotted on graphs for visualizing the performances of the α-values for both smoothing and forecasting. These were carried out for each of the ten documented problems.

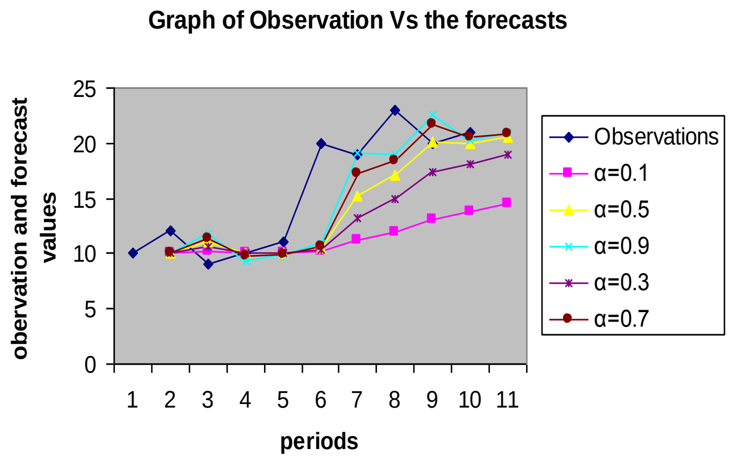

Figure 4.1 Graphs of documented sales data, forecasts and smoothing values of existing and the proposed α-values for problem set 1

For this problem, the arbitrary values of α used were 0.10 (for smoothing), 0.50 and 0.90 (for forecasting). The values derived from the proposed approach are 0.3 and 0.70 for smoothing and making forecast respectively. The graphs produced by 0.10 (arbitrarily selected) and 0.30(determined mathematically) tend to smoothen this time series data. The forecasts produced by 0.90 (arbitrarily selected) and 0.70 (determined mathematically) as can be seen from their plots are good forecasts. This can be inferred from the closeness of the plots to the actual observation. The difference between the forecast produced by 0.90 and 0.70 are not significant. In fact beyond period 8 the forecast values from both plots coincide. This indicates that the 0.70 value derived mathematically produces forecasts as can be obtained with 0.90, thereby justifying the claim that the higher the value of α used for forecasting the better the forecast.

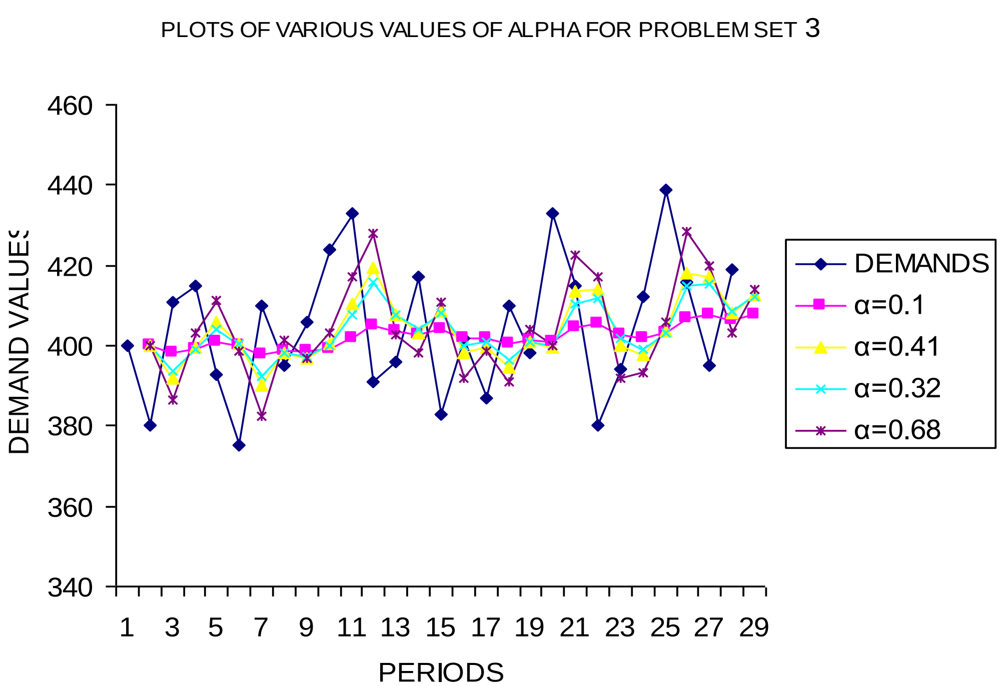

Figure 4.2 Graphs of documented sales data, forecasts and smoothing values of existing and the proposed α-values for problem set 2

In this problem α values were arbitrarily selected as 0.10 (for smoothing) and 0.41 for forecasting. Using the proposed the values were obtained as 0.32 and 0.68 for smoothing and forecasting respectively. The plot of the time series data has high fluctuations. The plots associated with 0.10 and 0.32 smoothen it. Though plot of 0.10 seem to smoothen the data better than that associated with 0.32 however, the difference is not too significant. Therefore this 0.32 value derived mathematically can be used also. The plot produced by 0.68 value is closer to the actual data than the 0.41 plot. This indicates that the proposed value produced forecast better than the arbitrarily selected value.

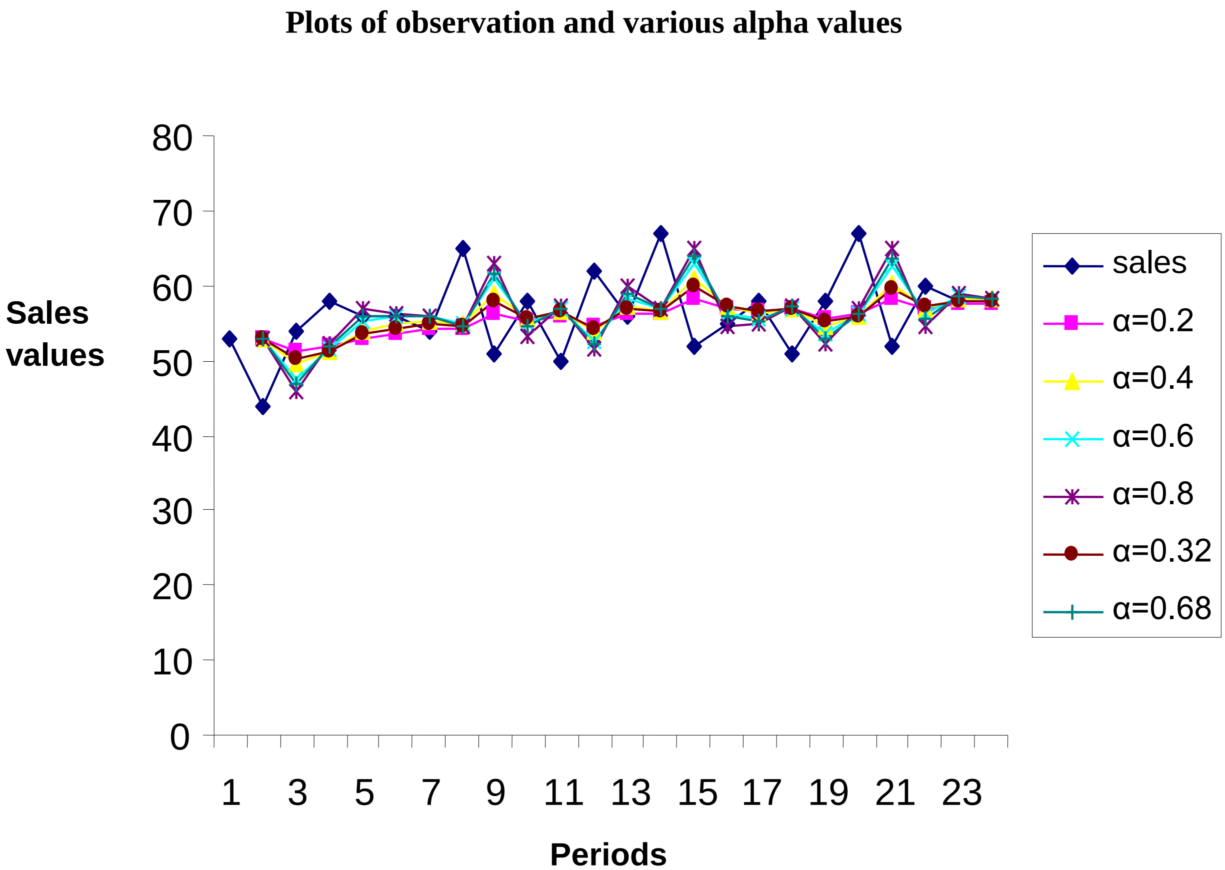

Figure 4.3 Graphs of documented sales data, forecasts and smoothing values of existing and the proposed α-values for problem set 3

In this problem set, arbitrary values were selected as 0.20, 0.40 (for smoothing) and 0.60 0.80 (for forecasting). The values derived from proposed approach were 0.32 (for smoothing) and 0.68 (for forecasting). The plots of produced by 0.20 and 0.32 smoothens the data appreciably with very minimal difference. The plots of 0.80 and 0.68 produced good forecasts for the data with almost the same accuracy as indicated by their closeness to the plot of the actual observation.

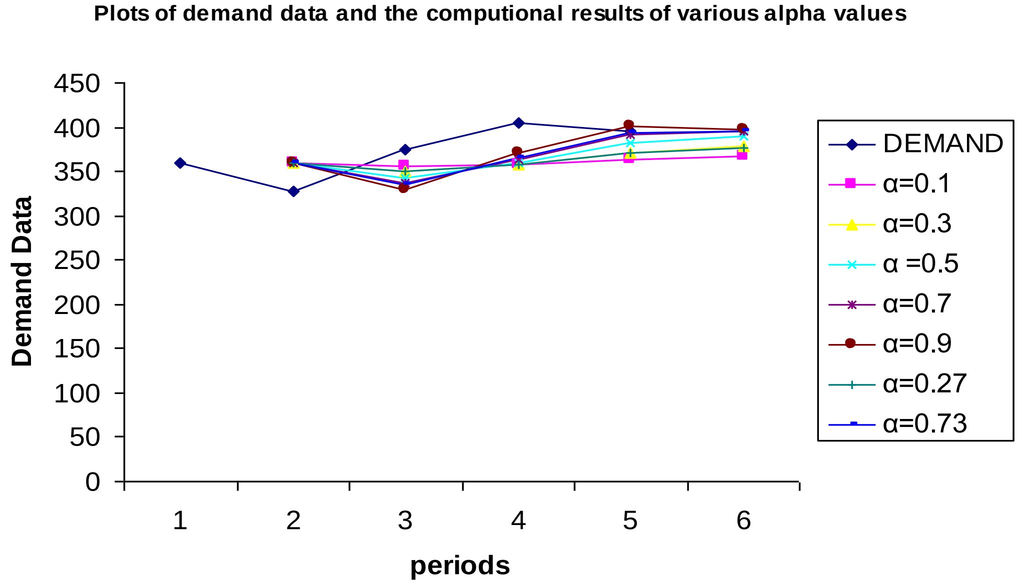

Figure 4.4 Graphs of documented sales data, forecasts and smoothing values of existing and the proposed α-values for problem set 4

For this problem set, 0.10, 0.30 (for smoothing) and 0.50, 0.70, 0.9(for forecasting) were the arbitrary used while the 0.27 (for smoothing) and 0.73 (for forecasting) were derived from the mathematical approach proposed. The plot of the observation indicates no much seasonal or cyclical fluctuations. The plots produced by 0.10, 0.27 and 0.30 smoothen the data with less significant difference. The plots produced by 0.90 and 0.73 are closer to the actual data plot. This indicate that good forecasts were produced by both the arbitrarily high value selected and the value derived from the proposed approach. In fact beyond period 4, the forecast by the proposed approach is almost the same with actual observation while for the 0.90 value the forecasts are above the actual observation. This implies that the propose value performs better than the high value selected.

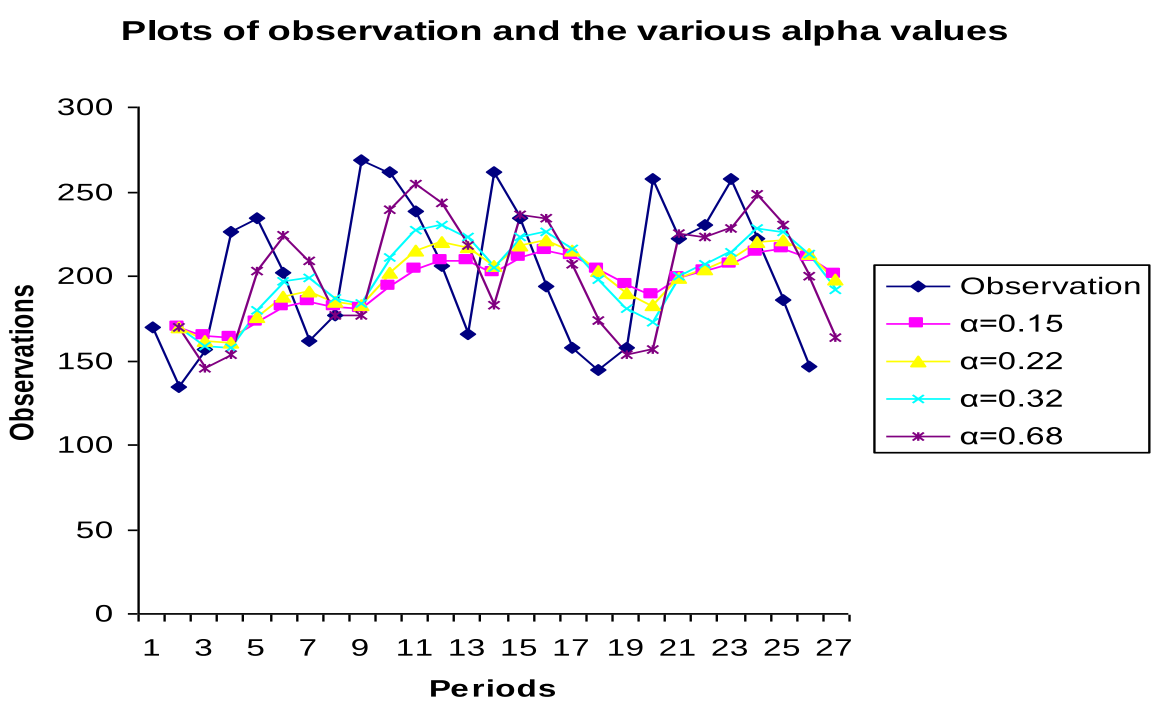

Figure 4.5 Graphs of documented sales data, forecasts and smoothing values of the existing and proposed α-values for problem 5

In this problem, the arbitrary values of α were 0.15 and 0.22 while the values obtained from the propose approach were 0.32 (for smoothing) and 0.68 for forecasting. The plots of the arbitrary values served for smoothing the time series data. The plot of

0.32 also smoothen the data, though as not as produced by the arbitrary values, it still indicates that the value is also suitable for smoothing this data. This is evident from the plot which has no significant difference from the plots of 0.15 and 0.22. The plot of 0.68 produced is closer to the actual observation. This indicates its suitability for producing forecasts for this problem.

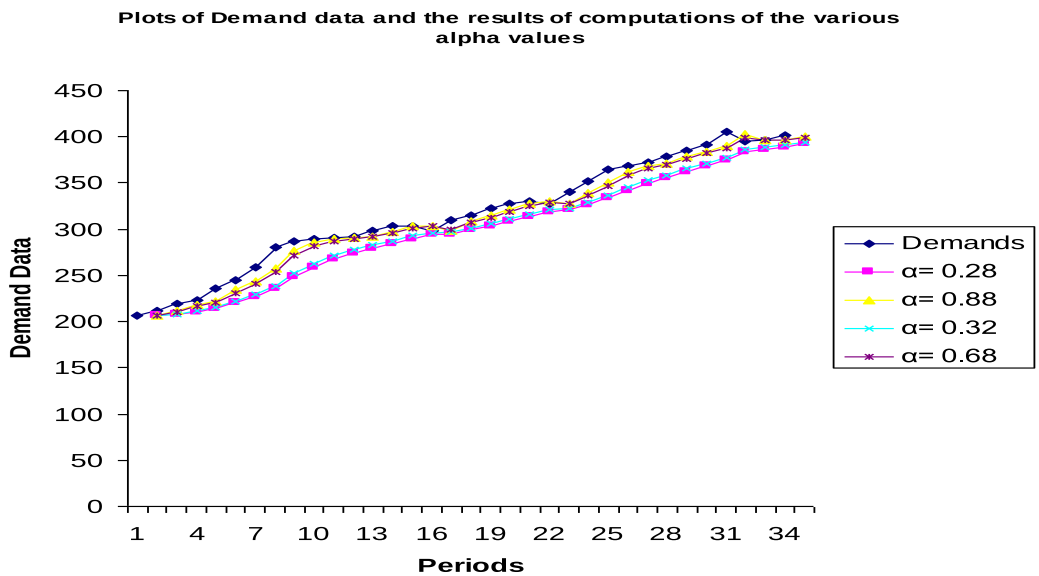

Figure 4.6 Graphs of documented sales data, forecasts and smoothing values of the existing and proposed α-values for problem 6

The plot of the demand data indicates that the data is almost free of seasonal or cyclical factors. Thus, the arbitrary values of α selected, 0.28 for smoothing produced almost the smoothing effect as the value derived from the proposed approach. The plot of 0.88 selected arbitrarily produced almost the same forecast values produced by the plot of 0.68 values derived proposed approach. These indicate that the values of α derived from the proposed approach are suitable for smoothing and forecasting for this problem.

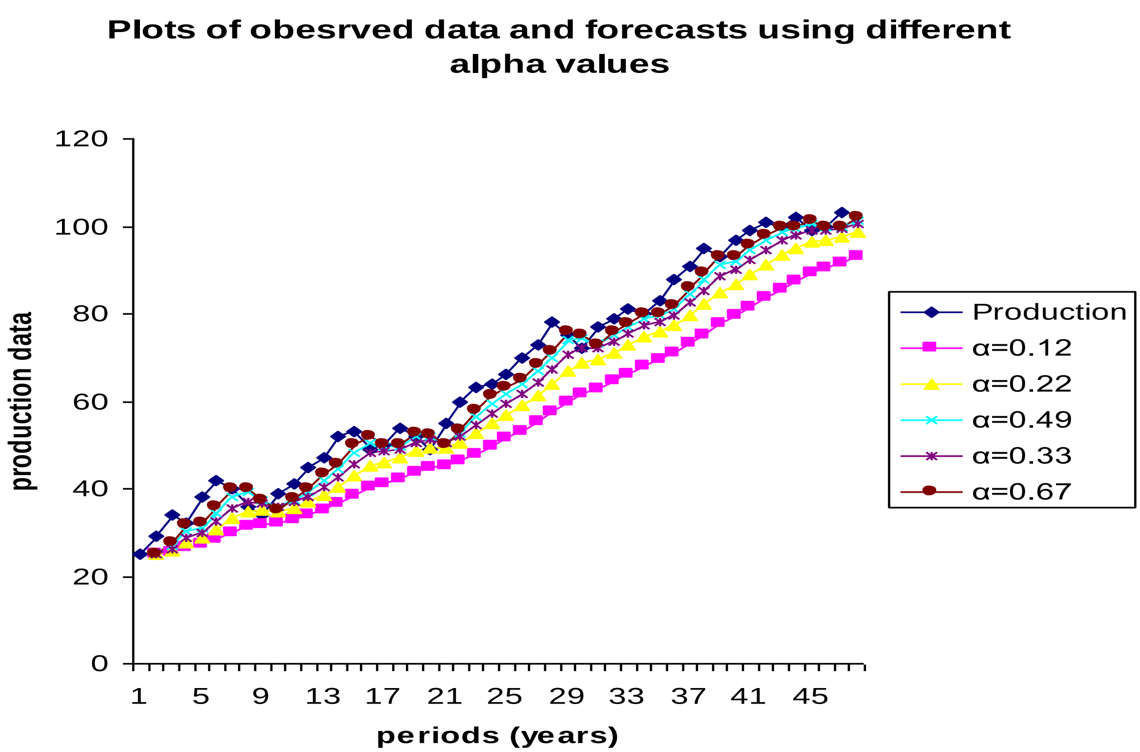

Figure 4.7 Graphs of documented sales data, forecasts and smoothing values of the existing and proposed α-values for problem 7

The plot of the production data shows little fluctuation in the series, implying that little or no seasonal or cyclical factors in time series data. The plots of the values of 0.12 and 0.22 (arbitrarily selected) smoothen the data. The value of 0.33 (derived from the proposed approach) also smoothen the data with almost the same smoothing effect as that of 0.22 value. The values of 0.49 (arbitrary selection) and 0.67 (derived from the proposed approach) were used for producing forecasts to this problem. The plots of the 0.67 value are closer to the actual data than the plot of the 0.49 value. This indicates that the 0.67 value produced better forecast than the arbitrarily selected value. Therefore the proposed approach produced values that are suitable for both smoothing and forecasting for this problem.

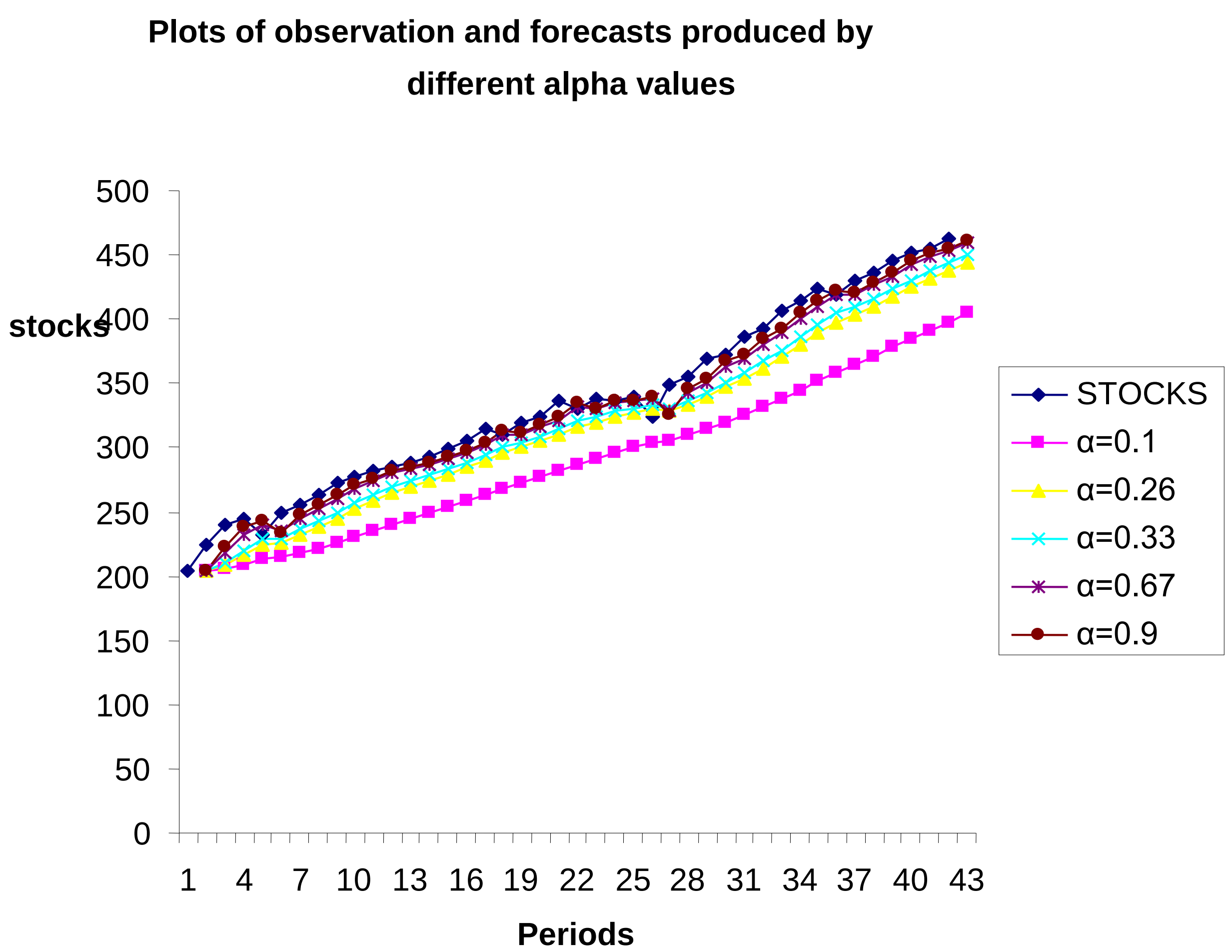

Figure 4.8 Graphs of documented Stock data, forecasts and smoothing values of the existing and proposed α-values for problem 8

For this problem, the arbitrary values selected as 0.10 and 0.26 smoothen the time series data, as can be seen from their plots. The plot of the 0.33 value derived from the proposed approach also smoothen the data with almost the same smoothing effect with the 0.26 value. The values of 0.90 (selected arbitrarily) and 0.67 derived from the proposed approach were used to produce forecasts for the problem. The plots of the two values produced almost the forecast values for the problem. Thus, the values derived from the proposed approach imply that both are suitable for smoothing and forecasting.

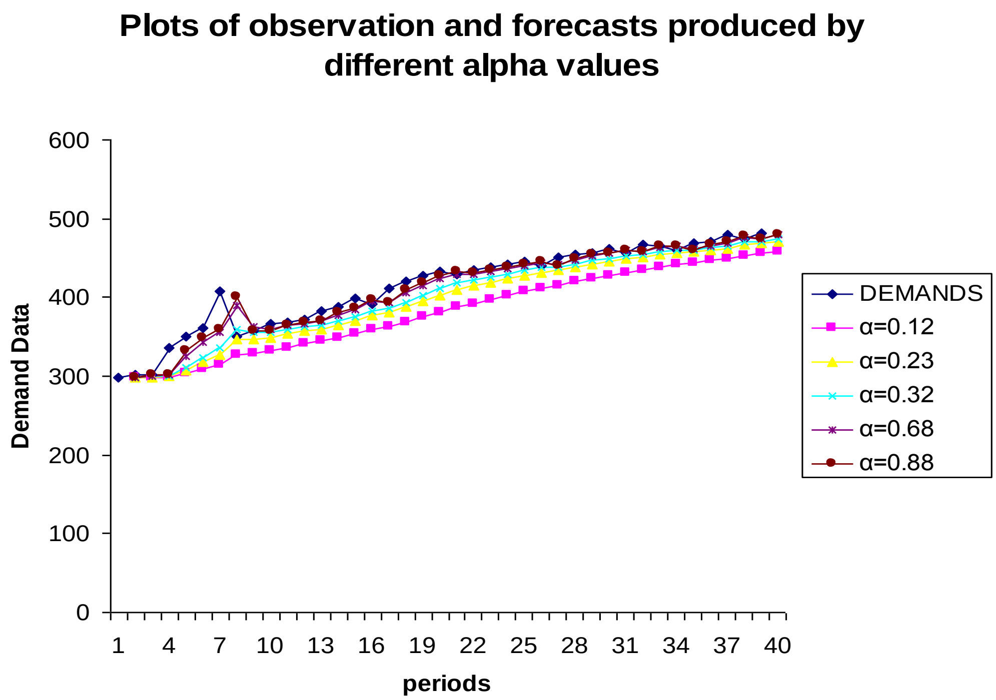

Figure 4.9 Graphs of documented Demand data, forecasts and smoothing values of the existing and proposed α-values for problem 9

The plot of the observation (demand data) has little fluctuation, indicating that the data less seasonal or cyclical factors. The arbitrarily selected values for smoothing were 0.12 and 0.23 and their respective plots exhibit good smoothing of the data. The value derived from the proposed approach for smoothing was 0.32 and its plot also indicate good smoothing of the data, with almost the same smoothing effect with the 0.23value.Values used for forecasting were 0.88 (arbitrarily chosen) and 0.68 (derived from the proposed approach. Both values produced good forecasts for the problem. This can be observed from the closeness of the plots to the plot of the actual observation. Thus the values derived from the proposed approach clearly demonstrate that both values 0.32 and 0.68, are suitable for smoothing and making forecasts respectively.

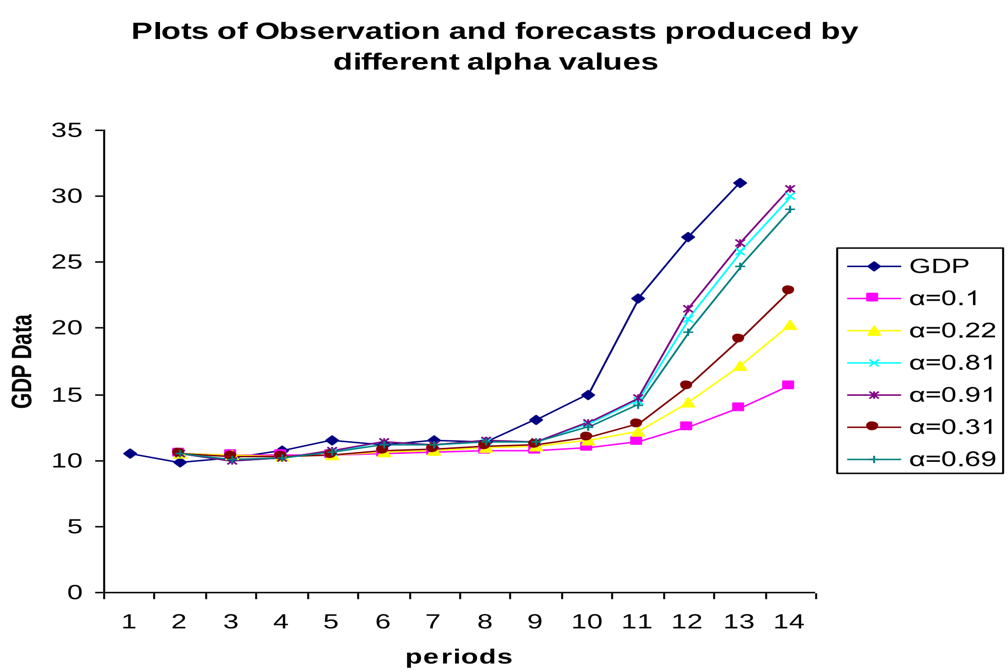

Figure 4.10 Graphs of documented GDP data, forecasts and smoothing values of the existing and proposed α-values for problem 10

The plot of the observed data (GDP) shows an upward increasing trend with no fluctuations. Thus, all the values 0.10, 0.22, 0.81, 0.91(arbitrary values), 0.31 and 0.69 (derived from the proposed approach) were used for forecasting. The forecasts produced by 0.69, 0.81 and 0.91 were good forecasts with almost the same effect, especially period 1 to 11. The values 0.10, 0.22 and 0.31 produced poor forecasts to the problem. This is expected since lower values are meant to smoothen the time series data from factors that affect time series data.

4. CONCLUSION

The exponential smoothing constants, derived from the proposed approach, produced good forecasts and smoothing for all the problems explored. This research has therefore provided a better approach to determining appropriate exponential smoothing constant which can now be bounded as $0 < \alpha < 1$ for a single exponential smoothing model(method). The mathematical method developed in this research is therefore recommended for use when single exponential smoothing method is the choice for short term forecast. The research can be extended to the determination of smoothing constants for higher exponential smoothing models such as double exponential and is therefore recommended for further research.

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