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Universal Relationship between Discharge and Depth for a Given Specific Energy in Triangular Channels

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ABSTRACT

For all open channel cross-sectional shapes, the discharge versus depth of flow curve for triangular cross-sections depends on the Specific Energy, and thus to tackle triangular open channel problems where the discharge is known but flow depth is required, separate curves have to be plotted for the different Specific Energies that may be of relevance in the analysis. Alternatively, the depth is obtained by iteration. Either of this is time-consuming. Previous work has obtained a graphical relationship between the discharge and the flow depth which is applicable to all rectangular open channels irrespective of the Specific Energy, thus allowing the flow depth for any rectangular channel to be determined for any given specific energy and discharge, without iteration. A similar graphical relationship that is applicable to triangular channels is developed here.

Keywords: *Unit Discharge, Specific Energy, Triangular Section*

1. INTRODUCTION

Analysis of a rapid change of flow in an open channel often involves the assumption that energy is conserved. Examples are transitions in open channels such as at a sudden change in the bottom elevation and/or the width of the channel. This would involve first obtaining the Specific Energy of flow after the transition from the transition characteristics and the Specific Energy of flow before the transition. With the specific Energy of flow after the transition known, the problem then is often to determine after the transition, the depth of flow for the given discharge and Specific Energy.

There are two approaches to facilitate the determination of depth for a given discharge and Specific Energy. One is to obtain a curve of Specific Energy versus flow depth curve for the given discharge. The other is to obtain a curve of discharge versus flow depth corresponding to the given Specific Energy. However these curves differ with the given Specific Energy or flow discharge as the case may be, and also with cross-sectional shapes. Since in a given analysis, even for a given cross-section, many Specific Energies or discharges would be of interest, many curves would need to be produced thus making the analysis tedious.

This can only be avoided by iteration (see Henderson, 1989; Streeter, 1971) which is also time-consuming. To avoid these problems, Aiyesimoju (2007) presented a single specific energy curve applicable to all triangular channels at all discharges. However the presented curve shows Specific Energies approaching infinity as flow depth approaches zero or infinity. Practical curves must have a limited range, thus they have to leave out flow depths are either very small or very large. Aiyesimoju (2011) presented discharge versus flow depth curves for any given Specific Energy applicable to all rectangular channels. This covers all the entire range of flow depth of interest from zero to infinity thus avoiding the problem just highlighted for Specific Energy curves. The paper demonstrates that although either of the universal discharge or Specific Energy curves can be used to solve specific problems, the discharge curves give a better intuitive feel for sudden width change problems whereas the universal Specific Energy curves are similarly better for sudden bed elevation change problems. For instance the discharge curve confirms the well-known fact that when channel width suddenly contracts, flow depth decreases for subcritical flow and increases for supercritical flow. Such exercise with universal Specific Energy curves shows that the result would depend on the specific case.

Other workers in this area have been mostly concerned with open channel flow phenomena governed by the specific force, rather than the specific energy. For example Alhamid and Negm (1996) considered rapidly varied flow in sloping and rough rectangular channels. Mossa (1999) focused on the oscillating characteristics that accompany hydraulic jumps in rectangular channels. Beirami and Chamani (2006) studied sequent depth ratio for hydraulic Jumps in sloping channels. Jan and Chang (2009) focussed on hydraulic jumps in an inclined rectangular chute contraction. Bose et al. (2012) analyzed free surface profiles of undular hydraulic jumps. Chanson (2012) was concerned with momentum considerations in hydraulic jumps and bores. Fu and Jin (2013) developed a mesh-free method boundary condition technique for open channel flow simulations. Gaudio et al. (2011) studied friction factor and von Kármán's κ in open channels with bed-load. Castro-Orgaz and Hager (2010) involved spatially-varied open channel flow equations with vertical inertia. Zare and Doering, (2011) investigated forced hydraulic jumps below abrupt expansions. Heller et al. (2005) as well as Jan and Chang (2009) were concerned with the hydraulics of energy dissipators. Abderrezzak et al. (2011) focused on division of critical flow at three-branch open-channel intersections. Liu and Zhu (2004) as well as Kara et al. (2012) were concerned with turbulence in open channels.

The aim of this paper is to obtain a discharge versus flow depth curve for any given Specific Energy, which is applicable to all triangular channels.

2. SPECIFIC ENERGY BASED FORMULATION

Given a depth of flow y in an open channel, the specific energy E is given by (Chow [1])

$$E = y + \frac{V^2}{2g}$$

where V is flow velocity and g is acceleration due to gravity. This can be written as

$$E = y + \frac{Q^2}{2gA^2} \tag{1}$$

where Q is the discharge and A is the flow cross-sectional area. In triangular channels,

$$y = cb \tag{2}$$

where b is the channel breadth at the water surface for flow depth y and c is a constant for a given triangular shape. This implies

$$A = \frac{by}{2}$$

which implies (from Eq. 2)

$$A = \frac{y^2}{2c}$$

Substituting this in Eq. 1 yields

$$E = y + \frac{2c^2Q^2}{gy^4} \tag{3}$$

This can be rewritten as

$$Q^2 = \frac{Egy^4}{2c^2} - \frac{gy^5}{2c^2}$$

Thus for a given triangular section (and hence a given c), different values of E result in different Q-y curves.

Let there be a scaling parameter p. Dividing through Eqn. 3 by the scaling parameter yields

$$\frac{Q^2}{p} = \frac{Egy^4}{2c^2p} - \frac{gy^5}{2c^2p}$$

This implies that

$$\frac{Q^2}{p} = \frac{Egp^3}{2c^2}\left(\frac{y}{p}\right)^4 - \frac{gp^4}{2c^2}\left(\frac{y}{p}\right)^5 \tag{4}$$

Since p is arbitrary, Eqn. 4 is simplified if we choose

$$\frac{Egp^3}{2c^2} = \frac{gp^4}{2c^2} \tag{5}$$

which gives

$$p = E \tag{6}$$

Substituting this in Equation 4 gives

$$\frac{Q^2}{E} = \frac{gE^4}{2c^2}\left(\frac{y}{E}\right)^4 - \frac{gE^4}{2c^2}\left(\frac{y}{E}\right)^5 \tag{7}$$

which simplifies to

$$Q\Big/\sqrt{\frac{gE^5}{2c^2}} = \sqrt{\left(\frac{y}{E}\right)^4 - \left(\frac{y}{E}\right)^5} \tag{8}$$

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If Y is the total channel depth and B is the top width, then from Equation 2, c can be estimated as

$$c = \frac{Y}{B} \tag{9}$$

From Eq. 8, a relationship exists between $Q\Big/\sqrt{\frac{gE^5}{2c^2}}$ and thus a plot of $Q\Big/\sqrt{\frac{gE^5}{2c^2}}$ versus $\frac{y}{E}$ (or unit discharge versus unit depth) can be produced which is universally applicable to all triangular channels. This has been tabulated in Table 1 and plotted in Figure 1.

Table 1 - Dimensionless Depths Versus Dimensionless Discharges

$Q\Big/\sqrt{\frac{gE^5}{2c^2}}$	$\frac{y}{E}$
0.000E+00	0.00
3.960E-04	0.02
1.568E-03	0.04
3.490E-03	0.06
6.139E-03	0.08
9.487E-03	0.10
1.351E-02	0.12
1.818E-02	0.14
2.346E-02	0.16
2.934E-02	0.18
3.578E-02	0.20
4.275E-02	0.22
5.021E-02	0.24
5.815E-02	0.26
6.652E-02	0.28
7.530E-02	0.30
8.444E-02	0.32
9.391E-02	0.34
1.037E-01	0.36
1.137E-01	0.38
1.239E-01	0.40
1.343E-01	0.42
1.449E-01	0.44
1.555E-01	0.46
1.661E-01	0.48
1.768E-01	0.50
1.873E-01	0.52
1.978E-01	0.54
2.080E-01	0.56
2.180E-01	0.58
2.277E-01	0.60
2.370E-01	0.62
2.458E-01	0.64
2.566E-01	0.67
2.684E-01	0.70
2.769E-01	0.73
2.830E-01	0.76
2.854E-01	0.78
2.862E-01	0.80
2.853E-01	0.82
2.822E-01	0.84
2.767E-01	0.86
2.683E-01	0.88
2.561E-01	0.90
2.484E-01	0.91
2.394E-01	0.92
2.288E-01	0.93
2.164E-01	0.94
2.018E-01	0.95
1.843E-01	0.96
1.630E-01	0.97
1.358E-01	0.98
9.801E-02	0.99
0.000E+00	1.00

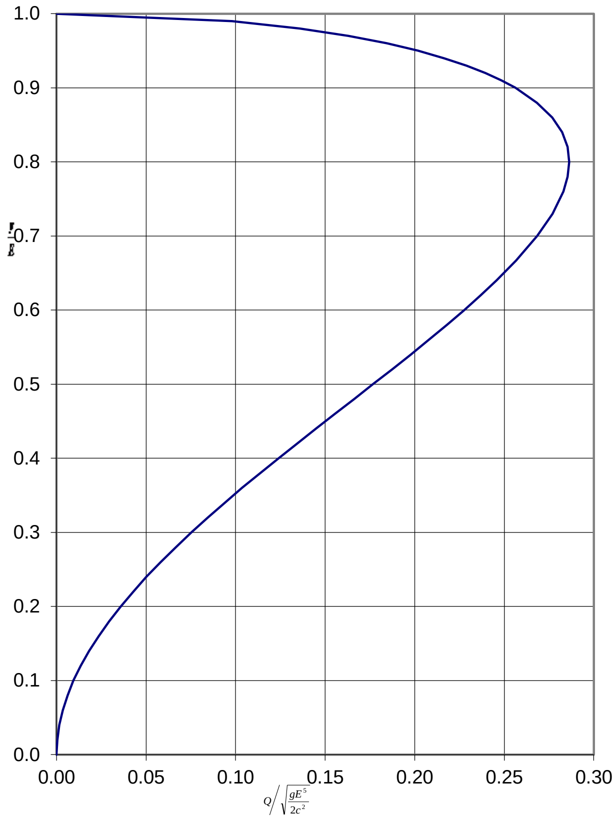

Figure 1 - Dimensionless Depth Vs. Dimensionless Discharge

The following procedure can thus be used to obtain the flow depth without iteration.

1. If triangular shape constant c is not directly known, obtain it from Equation 9.
2. Compute $Q\Big/\sqrt{\frac{gE^5}{2c^2}}$
3. Read off the corresponding $\frac{y}{E}$ from Table 1 or from Figure 1.
4. Use this to obtain the corresponding depth *y*.

Let $y' = \frac{y}{E}$ and $Q' = Q\Big/\sqrt{\frac{gE^5}{2c^2}}$,

then

$$Q' = \left(y'^4 - y'^5\right)^{1/2}$$

For the maximum and minimum values of Q' ,

$$\frac{dQ'}{dy'} = \frac{d}{dy'}\left(y'^4 - y'^5\right)^{1/2}$$
$$= \frac{1}{2}\left(y'^4 - y'^5\right)^{-1/2}\left(4y'^3 - 5y'^4\right)$$
$$= \frac{1}{2}\left(y'^{-2}\left(1-y'\right)\right)^{-1/2} y'^2\left(4y' - 5y'^2\right)$$
$$= \frac{1}{2}\left(1-y'\right)^{-1/2}\left(4y' - 5y'^2\right)$$
$$= \frac{4y' - 5y'^2}{2\sqrt{1-y'}} = 0$$

This implies a maximum value at y' of 0.8 and thus the critical depth y_c is that at which the discharge is maximum for a given Specific Energy is given by

$$\frac{y_c}{E} = 0.8 \qquad (10)$$

which implies a maximum value for y' i.e. $Q\Big/\sqrt{\frac{gE^5}{2c^2}}$ of 16/(25√5) i.e. 0.2862. The depths $0 < \frac{y}{E} < 0.8$ and $0.8 < \frac{y}{E} < 1$ correspond to supercritical and subcritical flows respectively.

3. CRITICAL-DEPTH-BASED FORMULATION

Eliminating E from Equation 8 using Equation 10 gives

$$\frac{cQ}{\sqrt{gy_c^{\,5}}} = \sqrt{\frac{5}{8}\left(\frac{y}{y_c}\right)^4 - \frac{1}{2}\left(\frac{y}{y_c}\right)^5}$$

which simplifies to

$$\frac{cQ}{\sqrt{gy_c^{\,5}}} = \left(\frac{y}{y_c}\right)^2\sqrt{\frac{5}{8} - \frac{1}{2}\frac{y}{y_c}} \qquad (11)$$

Equation 11 gives a relationship between unit discharge $\frac{cQ}{\sqrt{gy_c^{\,5}}}$ and unit depth $\frac{y}{y_c}$ which is applicable to all triangular channels. This has been tabulated in Table 2 and plotted in Figure 2. The depths $0 < \frac{y}{y_c} < 1$ and $1 < \frac{y}{y_c} < 1.25$ correspond to supercritical and subcritical flows respectively. The following procedure can also be used to obtain the flow depth without iteration.

1. Calculate y_c from Equation 10
2. If triangular shape constant c is not directly known, obtain it from Equation 9.
3. Compute $\frac{cQ}{\sqrt{gy_c^{\,5}}}$ and read off the corresponding $\frac{y}{y_c}$ from Table 2 or from Figure 2.
4. Use this to obtain the corresponding depth y.

4. SUMMARY AND CONCLUSION

Normalization of the specific energy equation by a scaling parameter p, and then conveniently setting the scaling parameter as the Specific Energy E, yielded a relationship (Equation 11) between what can be termed unit discharge $Q\Big/\sqrt{\frac{gE^5}{2c^2}}$ and unit depth $\frac{y}{E}$ which is applicable to all triangular channels. This has been tabulated in Table 1 and plotted in Figure 1. $Q\Big/\sqrt{\frac{gE^5}{2c^2}}$ attains a maximum value of 16/(25√5) i.e. 0.2862 at $\frac{y}{E}$ of 0.8 i.e. critical depth.

Table 2 - Dimensionless Depths Versus Dimensionless Discharges (critical depth formulation)

$\frac{c^2Q^2}{gy_c^5}$	$\frac{y}{y_c}$
0.000E+00	0.00
3.137E-04	0.02
1.245E-03	0.04
2.777E-03	0.06
4.895E-03	0.08
7.583E-03	0.10
1.082E-02	0.12
1.460E-02	0.14
1.890E-02	0.16
2.370E-02	0.18
2.898E-02	0.20
3.473E-02	0.22
4.093E-02	0.24
4.756E-02	0.26
5.460E-02	0.28
6.203E-02	0.30
6.983E-02	0.32
7.798E-02	0.34
8.645E-02	0.36
9.524E-02	0.38
1.043E-01	0.40
1.136E-01	0.42
1.232E-01	0.44
1.330E-01	0.46
1.430E-01	0.48
1.531E-01	0.50
1.634E-01	0.52
1.737E-01	0.54
1.842E-01	0.56
1.947E-01	0.58
2.052E-01	0.60
2.157E-01	0.62
2.262E-01	0.64
2.400E-01	0.67
2.570E-01	0.70
2.717E-01	0.73
2.859E-01	0.76
2.949E-01	0.78
3.036E-01	0.80
3.118E-01	0.82
3.195E-01	0.84
3.266E-01	0.86
3.331E-01	0.88
3.388E-01	0.90
3.414E-01	0.91
3.438E-01	0.92
3.460E-01	0.93
3.479E-01	0.94
3.495E-01	0.95
3.509E-01	0.96
3.521E-01	0.97
3.529E-01	0.98
3.534E-01	0.99
3.536E-01	1.00

$\frac{c^2Q^2}{gy_c^5}$	$\frac{y}{y_c}$
3.536E-01	1.00
3.534E-01	1.01
3.528E-01	1.02
3.519E-01	1.03
3.505E-01	1.04
3.486E-01	1.05
3.463E-01	1.06
3.435E-01	1.07
3.401E-01	1.08
3.360E-01	1.09
3.314E-01	1.10
3.260E-01	1.11
3.198E-01	1.12
3.128E-01	1.13
3.048E-01	1.14
2.957E-01	1.15
2.854E-01	1.16
2.738E-01	1.17
2.605E-01	1.18
2.453E-01	1.19
2.277E-01	1.20
2.071E-01	1.21
1.823E-01	1.22
1.513E-01	1.23
1.087E-01	1.24
7.750E-02	1.245
6.943E-02	1.246
6.023E-02	1.247
4.925E-02	1.248
5.046E-02	1.248
0.000E+00	1.250

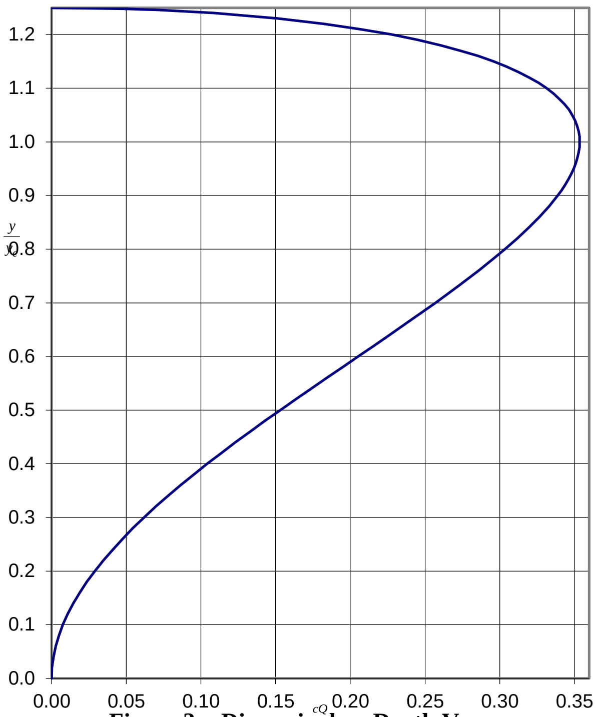

Figure 2 - Dimensionless Depth Vs. Dimensionless Discharge (critical depth formulation)

The equations were reformulated in terms of critical depth in place of Specific Energy which resulted in a relationship between unit discharge $\frac{cQ}{\sqrt{gy_c^5}}$ and unit depth $\frac{y}{y_c}$ which is applicable to all triangular channels. . $\frac{cQ}{\sqrt{gy_c^5}}$ attains a maximum value of $1/(2\sqrt{2})$ i.e. 0.3536 at $\frac{y}{y_c}$ of 1 i.e. critical depth.

These results are consistent with the results that are obtained using the Specific Energy curve method in Aiyesimoju (2007). Finally for any triangular channel with any discharge *Q*, if the specific energy *E* is known, the procedures were established to obtain the flow depth without iteration using the Specific Energy formulation or the critical depth formulation.

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