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Bending Analysis of Smart Material Plates Using Higher Order Theory

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ABSTRACT

A smart material is one which is having a molecular structure responds in a particular and controlled way to influences upon it. They have one or more properties that can be dramatically altered. The most of the materials used everyday have physical properties which cannot be significantly altered, example, if oil is heated it will become a little thinner, where as smart material with variable viscosity may turned from a fluid which flows easily to a solid. A variety of smart materials are already exists and are being researched extensively. These include piezoelectric materials, magneto-rheostatic materials, electro-rheostatic materials, and shape memory alloys. In the present paper, an attempt is made to analyze piezoelectric materials using the higher order shear deformation theory for more accurate results. In this paper analytical procedure is developed to find the bending characteristics of piezoelectric laminated material plates which are subjected to electromechanical loading. The results predicted in this work are compared with the other theories.

Keywords: *smart material, piezoelectric materials, magneto-rheostatic materials, electro-rheostatic materials, shape memory alloys*

1. INTRODUCTION

Piezoelectric materials are materials that produce a voltage when stress is applied. Since this effect also applies in the reverse manner, the voltage across the sample will produce stress within the sample. Structures made from smart materials can be bent, expanded or contracted when a voltage is applied. An analytical solution for cross-ply composite laminates integrated with piezoelectric fiber-reinforced composite (PFRC) actuators under bidirectional bending is presented by Kant et al [1]. A higher order shear and normal deformation theory (HOSNT12) is used by them to analyze smart materials subjected to electromechanical loading. Two piezoelectric actuators are symmetrically surface bonded on a cross-ply composite laminate by Chi-Sheng Lin, et al [2]. They applied electric voltage with the same amplitude and opposite sign to the two symmetric piezoelectric actuators, resulting in the bending effect on the laminated plates. The analytical solution of the flexural displacement of the simply supported composite plate subjected to bending moment is solved by them using the plate theory. Ray et al [3] presented the three dimensional elasticity solutions for intelligent plates. They perfectly bonded with distributed polyvinylidene fluoride (PVDF) piezoelectric layers at the top and bottom and found the static displacement for various span to depth ratios. Then Mallik et al [4, 5] developed exact and finite element solutions for PFRC activated composites. Further Kant et al [6] developed analytical solutions for the cylindrical flexure of piezoelectric plates based on higher order shear deformation theory. They analyzed a unidirectional composite plate attached with distributed actuator and sensor layers under mechanical and electrical loading conditions. An exact three dimensional state space is developed by Senthil et al [7] for the static cylindrical bending of simply supported laminated plates with embedded piezoelectric actuators. They compared the displacements and stresses with those obtained by first order shear deformation theory (FSDT). Senthil et al [8] analyzed then generalized plane quasistatic deformations of linear piezoelectric laminated plates by the Eshelby – Stroh formulation. Further Aditi Chattopadhyay et al [9] developed a refined higher order laminate theory to analyze smart materials. They used refined displacement field which accounts transverse shear stresses through the thickness. Geometrically nonlinear analysis of piezolaminated smart composite laminated plates is presented by Sudhakar et al [10]. In order to access the accuracy of results a critical comparison is made by Paolo Bisegna et al [11] in a completely analytical way. In this paper a complete and simple analytical solution is presented using higher order shear deformation theory. The theoretical model presented by Kant et al [12] has incorporated laminate deformations which accounts for effects of transverse shear deformation and transverse normal strain/stress. They have obtained the solutions in closed from using Navier's technique by solving the boundary value problem. Ray et al [13] has presented an exact analysis of a piezoelectric plate under cylindrical bending. He has derived the solutions for deformations, stresses, electrical potential and electric displacement of a simply supported plate subjected to sinusoidal mechanical loading. K. Chandrashekhara et al [14] presented a formulation for modeling composite laminated plates integrated with piezoelectric sensors and actuators. Their formulation is based on first order shear deformation theory which is applicable for both thin and thick plates. Numerical results presented them indicates, as feedback gain increases damping also increases. Using a multi-layer higher-order finite element approach Mannini et al [15] has developed a finite element model for composite laminated plates. Two active layers has been considered by them one on the top and the other at bottom of the elastic substrates. Robbins DH et al [16] investigated static and dynamic interaction between beam and a bonded piezoelectric actuator using four displacement–based finite elements. To accurately represent piezoelectric actuation of beam structure they developed and compared two equivalent single-layer models and two equivalent multi-layer models. Tauchert TR et al [17]

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studied cylindrical bending of a composite laminated plate constructed with both orthotropic and isotropic thermoelastic layers. They have obtained a benchmark solution for both thermal and electric-potential surface loadings by employing an exact analytical formulation. Saravanos DA et al [18] presented an analysis for composite laminated plate with piezoelectric sensors and actuators. For quasi-static and dynamic analysis of smart composite structures they have developed finite-element formulations.

2. FORMULATION OF HSDT

In formulating the higher-order shear deformation theory, a composite plate of $0 \le x \le a$; $0 \le y \le b$ attached with PFRC actuator and is simply supported along four sides of the plate is considered.

The displacement components u (x, y, z, t), v(x, y, z, t) and w (x, y, z, t) at any point in the plate are expanded in terms of the thickness coordinate. In this work the in-plane displacements are expanded as cubic functions of the thickness coordinate. The displacement field w (x, y, z) is assumed as constant through the plate thickness and thus setting $\varepsilon_z = 0$ is expressed as [12]:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\}$$

…… (1)

Where the parameters u_o, v_o and w_o denote the displacements of a point (x, y) on the midplane. The functions θ_x, θ_y are rotations of the normal to the midplane about y and x–axes, respectively. The parameters u_0^*, v_0^*, θ_x^* and θ_y^* are the corresponding higher order deformation terms representing higher-order transverse cross sectional deformation modes and are also defined at the midplane.

Constitutive Relations for Smart Materials

The constitutive relation for smart materials of a single piezoelectric layer couples the elastic and electric fields are [6]:

$$\{\sigma\} = [Q]\{\varepsilon\} - [e]\{E\}$$

…… (2)

The electric field intensity vector E related to electrostatic potential $\psi(x,y,z)$ in the L^{th} layer is given by:

$$E_x^L = -\frac{\partial\psi(x,y,z)^L}{\partial x}, \quad E_y^L = -\frac{\partial\psi(x,y,z)^L}{\partial y}, \quad E_z^L = -\frac{\partial\psi(x,y,z)^L}{\partial z}$$

…… (3)

Where σ, Q, ε, e and E are stress vector, elastic constant matrix, strain vector, piezoelectric constant matrix and electric field intensity vector respectively.

Eq. 2 can be represented in two components of stresses. One is elastic stress component (es) and other is piezoelectric stress component (pz) and written as [6]:

$$\{\sigma\} = \{\sigma\}^{es} - \{\sigma\}^{pz}$$

……. (4)

Where

$$\{\sigma\}^{es} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{54} & Q_{55} \end{bmatrix}^L \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix},$$

$$\{\sigma\}^{pz} = \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{33} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \end{bmatrix}^L \begin{Bmatrix} -\dfrac{\partial\psi(x,y,z)}{\partial x} \\ -\dfrac{\partial\psi(x,y,z)}{\partial y} \\ -\dfrac{\partial\psi(x,y,z)}{\partial z} \end{Bmatrix}^L$$

…… (5)

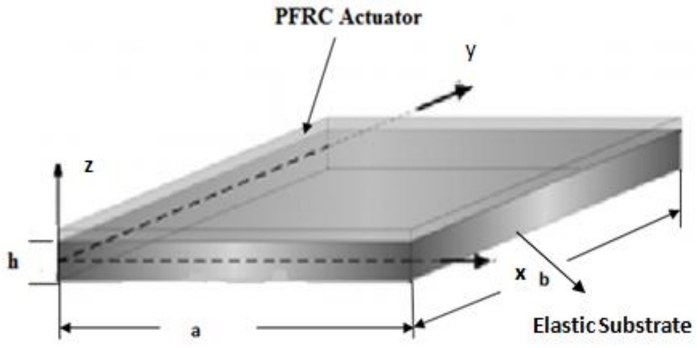

Fig. 1. Elastic substrate attached with PFRC actuator.

Source: Kant T [1]

Using Hamilton's principle for the total potential energy, the equations of equilibrium obtained as [1]:

$$\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0,$$

$$\delta v_0 : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0,$$

$$\delta w_0 : \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = I_1 \ddot{w}_0$$

$$\delta\theta_x : \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2\ddot{u}_0 + I_3\ddot{\theta}_x + I_4\ddot{u}_0^* + I_5\,\ddot{\theta}_x^*$$

$$\delta\theta_y : \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I_2\ddot{v}_0 + I_3\ddot{\theta}_y + I_4\ddot{v}_0^* + I_5\,\ddot{\theta}_y^*$$

$$\delta u_0^* : \frac{\partial N_x^*}{\partial x} + \frac{\partial N_{xy}^*}{\partial y} = I_3\ddot{u}_0 + I_4\ddot{\theta}_x + I_5\ddot{u}_0^* + I_6\,\ddot{\theta}_x^*$$

$$\delta v_0^* : \frac{\partial N_y^*}{\partial y} + \frac{\partial N_{xy}^*}{\partial x} = I_3\ddot{v}_0 + I_4\ddot{\theta}_y + I_5\ddot{v}_0^* + I_6\,\ddot{\theta}_y^*$$

$$\delta\theta_x^* : \frac{\partial M_x^*}{\partial x} + \frac{\partial M_{xy}^*}{\partial y} - Q_x^* = I_4\ddot{u}_0 + I_5\ddot{\theta}_x + I_6\ddot{u}_0^* + I_7\,\ddot{\theta}_x^*$$

$$\delta\theta_y^* : \frac{\partial M_y^*}{\partial y} + \frac{\partial M_{xy}^*}{\partial x} - Q_y^* = I_4\ddot{v}_0 + I_5\ddot{\theta}_y + I_6\ddot{v}_0^* + I_7\,\ddot{\theta}_y^*$$

…… (6)

Upon substitution of displacements in equations of equilibrium and rewritten in matrix form it is obtained as:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} & S_{19} \\ S_{21} & S_{22} & S_{24} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} & S_{29} \\ S_{31} & S_{32} & S_{34} & S_{34} & S_{35} & S_{36} & S_{37} & S_{38} & S_{39} \\ S_{41} & S_{42} & S_{44} & S_{44} & S_{45} & S_{46} & S_{47} & S_{48} & S_{49} \\ S_{51} & S_{52} & S_{54} & S_{54} & S_{55} & S_{56} & S_{57} & S_{58} & S_{59} \\ S_{61} & S_{62} & S_{64} & S_{64} & S_{65} & S_{66} & S_{67} & S_{68} & S_{69} \\ S_{71} & S_{72} & S_{74} & S_{74} & S_{75} & S_{76} & S_{77} & S_{78} & S_{79} \\ S_{81} & S_{82} & S_{84} & S_{84} & S_{85} & S_{86} & S_{87} & S_{88} & S_{89} \\ S_{91} & S_{92} & S_{94} & S_{94} & S_{95} & S_{96} & S_{97} & S_{98} & S_{19} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \\ U_{mn}^* \\ V_{mn}^* \\ X_{mn}^* \\ Y_{mn}^* \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} - V_t \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{Bmatrix}$$

….. (7)

The elastic substrate is attached with distributed actuator layer of PFRC. Thickness of the PFRC layer is small as compared to thickness of the substrate. Electro-static potential in the actuator layer is assumed to be linear through the thickness of the PFRC layer as follows:

$$\psi_{mn}(z) = \left(\frac{V_t}{t_p}\right) z - \left(\frac{V_t h}{2t_p}\right)$$

...... (8)

Eq. (8) represents the linear variation of through thickness electrostatic potential in the PFRC layer.

3. RESULTS AND DISCUSSIONS

The material properties of graphite/epoxy used for each orthotropic layer of the substrate are [12]:

$$\frac{E_1}{E_2} = 25,\ \frac{G_{12}}{E_2} = 0.5,\ \frac{G_{23}}{E_2} = 0.2,\ E_2 = E_3 = 10^6\ \ N/cm^2$$
$$G_{12} = G_{13}\ \ and\ \mu_{12} = \mu_{23} = \mu_{13} = 0.25$$

Material properties for PFRC layer are [1]:

C_{11} = 32.6 GPa, C_{12} = C_{21} = 4.3 GPa; C_{13} = C_{31} = 4.76 GPa; C_{22} = C_{33} = 7.2 GPa; C_{23} = 3.85 GPa; C_{44} = 1.05 GPa; C_{55} = C_{66} = 1.29 GPa; e_{31} = - 6.76 C/m^2; g_{11} = g_{22} = 0.037E - 9 C/V m;

g_{33} = 10.64E - 9 C/V m.

The results are normalized using the following factors:

$$\overline{w}\left(\frac{a}{2},\frac{b}{2},0\right) = \frac{100E_2}{q_o S^4 h} w,$$

$$\overline{\tau_{xz}}\left(0,\ \ \frac{b}{2},\ \ 0\right) = \frac{\tau_{xz}}{q_0 S}$$

E_2 is transverse Young's modulus of the elastic orthotropic layer**.** Two laminate configurations are taken into consideration i.e., two layered symmetric [0/90]$_s$ and anti-symmetric [90/0]$_{as}$, laminated plates.

Variation of non dimensional transverse displacement against aspect ratio of two layered symmetric substrate with and without applied voltage at top of PFRC actuator is shown in Fig. 2. As the thickness of composite plate increases the actuating effect is decreasing, it may be because of decrease of intensity factor. In the present work the percentage variation of transverse displacement ($\overline{w}$) with Mallik[4, 5] for aspect ratio 10 is 4.87% and for 100 it is 4.65% when there is no applied voltage at top of PFRC actuator. Where as, for aspect ratio 10 it is 8.16% and for 100 it is 6.0% variation when 100 V is applied at top of the actuator. Non dimensional transverse shear stress against thickness of symmetric substrate is presented in Fig. 3 to demonstrate actuating effects on the laminate. From Fig. 3 it is observed that top layer of laminate [0/90]$_s$ is affected maximum because of piezoelectric stress coefficient is effective only in x-direction. Fig. 4 demonstrates the non dimensional transverse shear stress against thickness of anti-symmetric substrate. The percentage variation of shear stress obtained in the present paper with Mallik [4, 5] is less than 10%. Top layer is effected maximum upto midplane.

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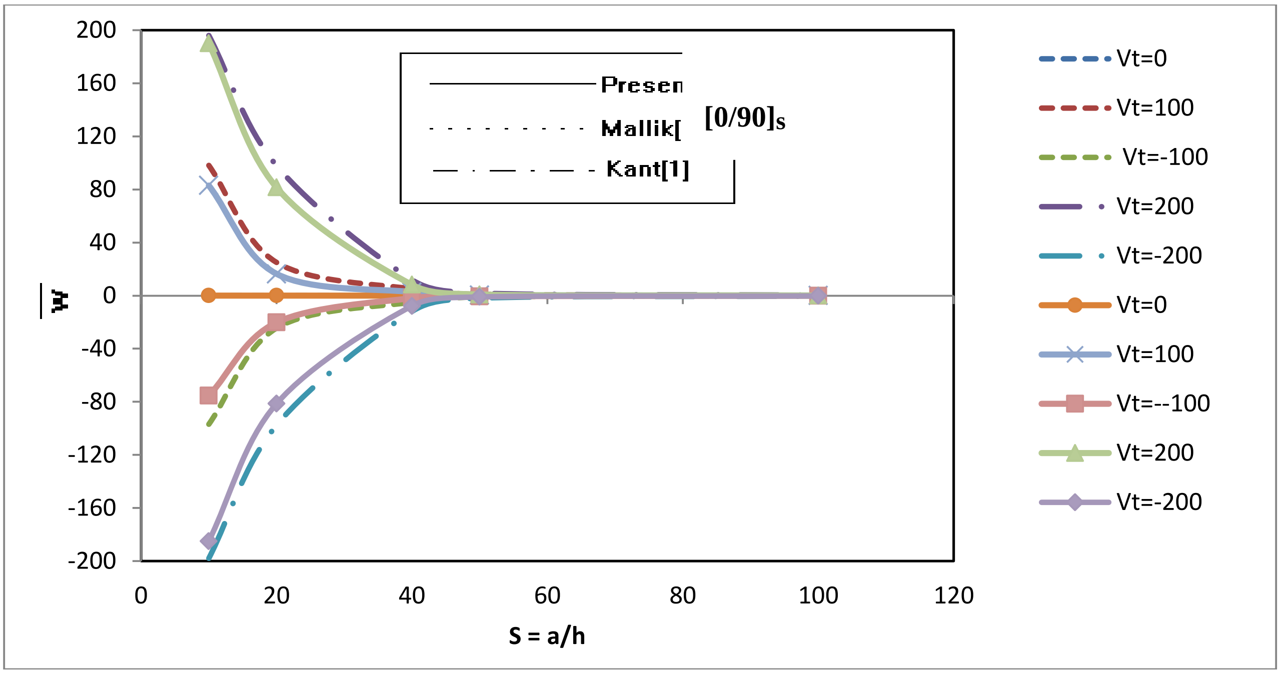

Fig.2. Variation of non dimensional transverse displacement ($\overline{W}$) against aspect ratio (S = a/h) without and with applied sinusoidal electric voltages at top of the PFRC actuator surface.

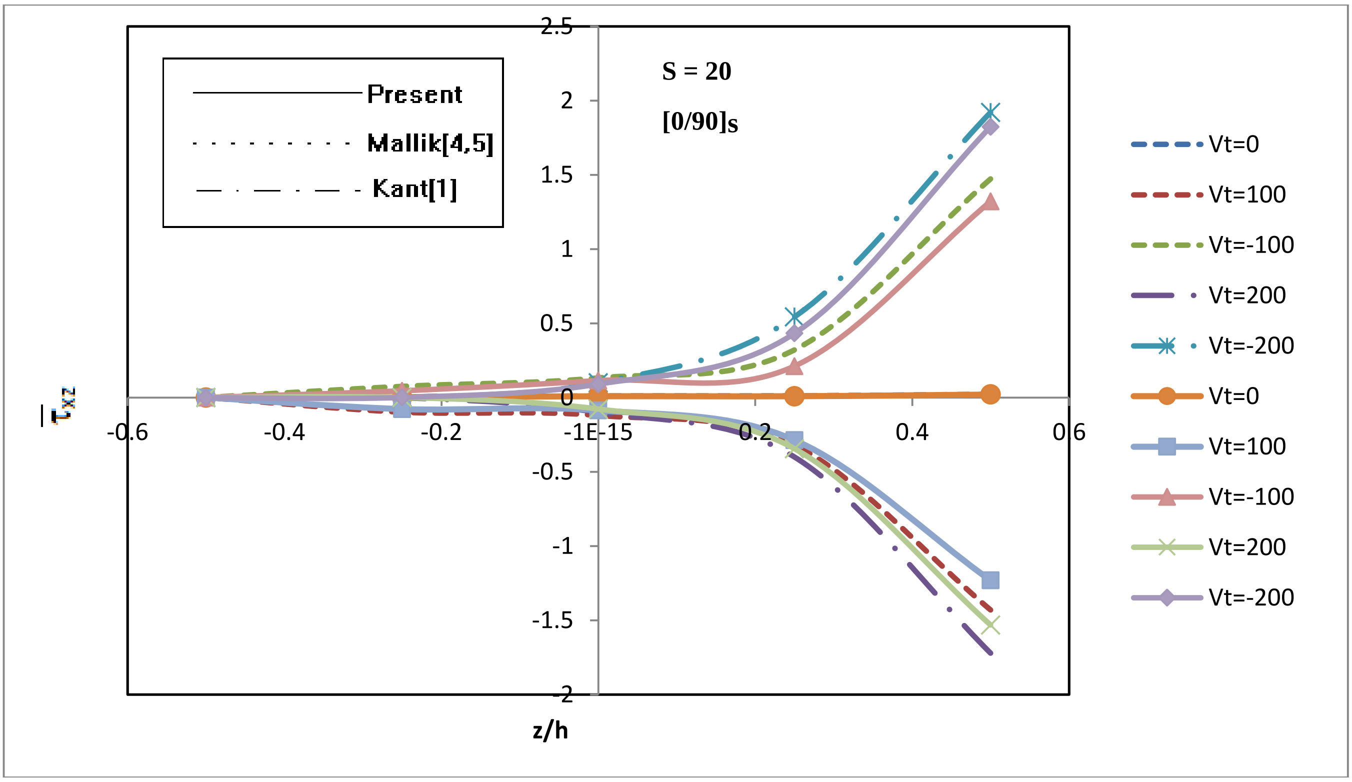

Fig.3. Non dimensional transverse shear stress ($\overline{\tau}_{xz}$) against thickness of symmetric substrate without and with applied sinusoidal electric voltages at top of the PFRC actuator surface

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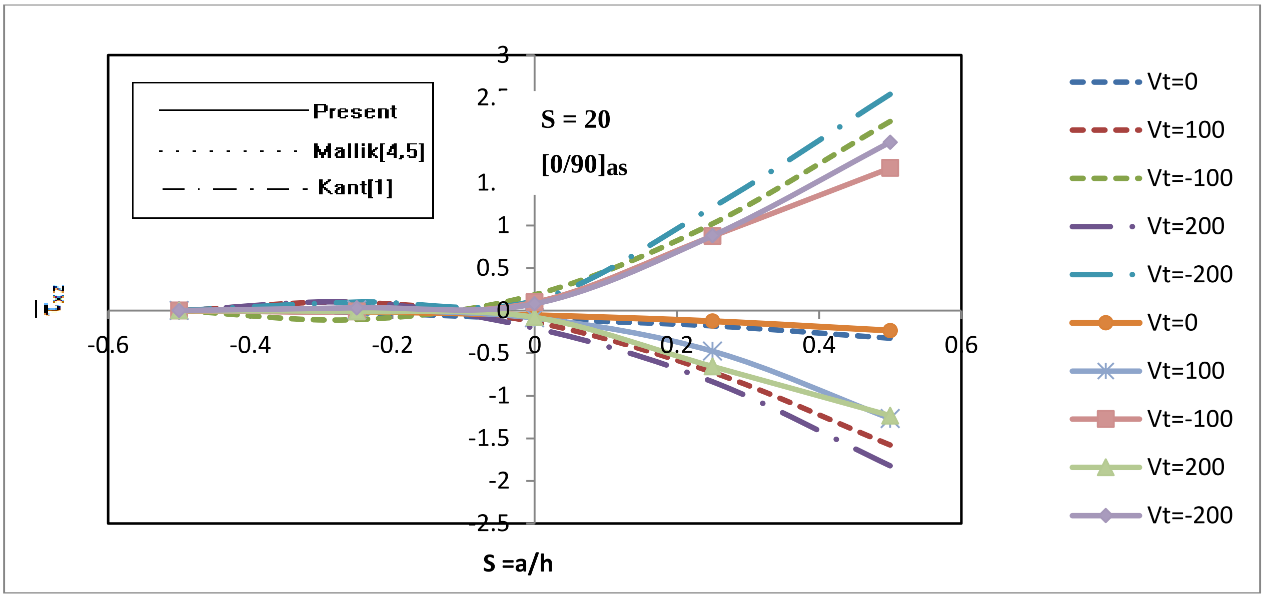

Fig.4. Non dimensional transverse shear stress ($\bar{\tau}_{xz}$) against thickness of anti-symmetric substrate without and with applied sinusoidal electric voltages at top of the PFRC actuator surface

4. CONCLUSIONS

Analytical procedure is developed in this paper for bending analysis of piezoelectric laminated plates subjected to electromechanical loading. The non dimensional transverse displacement and shear stresses are obtained for various voltages, aspect ratios and thickness coordinates. Programs have been developed for all these models to find the displacements and stresses for various materials. It is concluded from the results that, the present model is in close agreement with 3D exact solutions. From the study it is found that, shear deformation effects has the impact on piezoelectric laminates, and cannot be ignored while modeling them.

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