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On the Inequality of the Union of power Sets and Power set of Union

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ABSTRACT

The present work gives the necessary and sufficient condition for
P(AUB) $\subseteq$ [P(A) U P(B]). Where A and B are any two finite sets; and P(A), P(B) and P(AUB) are the power set of A, B and (AUB) respectively.

Keywords: *Power set, set, union*

1. INTRODUCTION

Unlike most areas of mathematics, which arise as a result of the cumulative efforts of many mathematicians, sometimes over several generations, set theory is the creation of a single individual, George Cantor, around the turn of the 20th century. In 1847, the Czech mathematician Bernhard Bolzano considered the idea of set, which he described this way:

…an embodiment of the idea or concept which we conceive when we regard the arrangement of its part as a matter of indifference.

The year 1897 saw the appearance of the first of the several paradoxes that arise within set theory once infinite sets are allowed. The first paradox was discovered by Cesare Burali-Forti and Cantor himself discovered a paradox in1899. The most devastating paradox was discovered by Bertrand Russell in 1902.

In 1908, Ernst Zermelo produced what looked like an acceptable solution: a set of axioms for set theory that appeared to avoid the paradoxes. A short while later, Abraham Fraenkel proposed amendment to Zarmelo's theory, and the mathematical community was a able to breathe a collective sigh of relief [1].

The power-set axiom was one of the axioms developed; and it guarantees the existence of a power set of a set [2].

From the literature, it is a known fact that, for any two finite sets A and B, it follows that [P(A) U P(B)] $\subseteq$ P(AUB) and that the converse may or may not be true. That is in general, the converse is not true. In the literature a counter example is usually given to show that P(AUB) $\nsubseteq$ [P (A)UP(B)] [1].

In this work a study of such a counter-example is made and the following was discovered:

The necessary and sufficient condition needed for P(AUB) $\subseteq$ [P (A)UP(B)].

2. MATHEMATICAL ANALYSIS

Let A and B be any two finite sets then, from the literature, [P (A)UP(B)] $\subseteq$ P(AUB) but P(AUB) $\subseteq$ [P (A)UP(B)] is not generally true. For example, if A = {1, 2} and B = {3} then P(AUB) $\nsubseteq$ [P (A)UP(B)]. Under what conditions is P(AUB) $\nsubseteq$ [P (A)UP(B)]? First, the following theorem is given thereafter the description of the conditions on A and B; such that P(AUB) $\nsubseteq$ [P (A)UP(B)] is then given.

2.1 Theorem

For any two finite sets A and B, P(AUB) = [P (A)UP(B)] if and only if A $\subseteq$ B or B $\subseteq$ A

2.2 Proof

Since P(AUB) $\subseteq$ [P (A)UP(B)] for any two finite sets A and B, whether or not; A $\subseteq$ B or

B $\subseteq$ A. Then it suffices to only show that P(AUB) $\subseteq$ [P (A)UP(B)] if and only if A $\subseteq$ B or B $\subseteq$ A.

Suppose A $\subseteq$ B or B $\subseteq$ A then A U B = B or A U B = A, this imply

P(AUB) = P(B) $\subseteq$ [P (A)UP(B)] or P(AUB) = [P (A)UP(B)].
That is A $\subseteq$ B or B $\subseteq$ A imply
P(AUB) $\subseteq$ [P (A)UP(B)].

Conversely, suppose P(AUB) $\subseteq$ [P (A)UP(B)].

Then for all X $\in$ P(AUB), X $\in$ P(A) U P(B), that is X $\subseteq$ AUB implies X $\subseteq$ A or X $\subseteq$ B.

In particular, let X = AUB then by the hypothesis X = AUB $\subseteq$ A or X = AUB $\subseteq$ B which imply B $\subseteq$ A or A $\subseteq$ B. That is P(AUB) $\subseteq$ [P (A)UP(B)] imply A $\subseteq$ B or B $\subseteq$ A.

As an application of the above theorem, the following example is considered:

Let A = {1, 2, 3} and B = {2, 3}, that is B $\subseteq$ A; then P(AUB) = P(A) and P(A) U P(B) = P(A). Therefore, P(AUB) = [P (A)UP(B)].

2.3 Establishing the conditions for which P(AUB) $\nsubseteq$ [P(A) U P(B)]

From the above theorem, P(AUB) $\subseteq$ [P (A)UP(B)] imply A $\subseteq$ B or B $\subseteq$ A. The contra positive of this statement is: if A $\nsubseteq$ B and B $\nsubseteq$ A then P(AUB) $\nsubseteq$ [P (A)UP(B)] (In fact, A $\nsubseteq$ B and B $\nsubseteq$ A if and only if P(AUB) $\nsubseteq$ [P (A)UP(B)]. So, the conditions required for P(AUB) $\nsubseteq$ [P (A)UP(B)] are A $\nsubseteq$ B and B $\nsubseteq$ A. Suppose A $\nsubseteq$ B and B $\nsubseteq$ A, then P(AUB) $\nsubseteq$ [P (A)UP(B)] can also be justified by showing there exists an element say Z, such that Z $\in$ P(AUB) but Z $\notin$ [P(A) U P(B)]. The fact that A $\nsubseteq$ B and B $\nsubseteq$ A, makes it possible for Z $\subseteq$ (AUB) such that Z $\nsubseteq$ A and Z $\nsubseteq$ B, this imply Z $\in$ P(AUB) but Z $\notin$ P(A) and Z $\notin$ P(B) thus Z $\in$ P(AUB) but Z $\notin$ [P(A) U P(B)]. Hence it has now been seen that if A, B and Z are as described above then P(AUB) $\nsubseteq$ [P (A)UP(B)].

3. RESULT AND DISCUSSIONS

In the above result, it has been verified that if A and B are any two finite sets, then P(AUB) $\subseteq$ [P (A)UP(B)] is not generally true.

If A $\nsubseteq$ B and B $\nsubseteq$ A then P(AUB) $\nsubseteq$ [P (A)UP(B)], and obviously both A and B in this case are each non-empty. If Z $\subseteq$ AUB such that Z $\nsubseteq$ A and Z $\nsubseteq$ B, then Z $\in$ P(AUB) but Z $\notin$ [P(A) U P(B)]. Then it is also obvious, that Z is a non-empty and Z cannot be a singleton set, that is Z consist of at least two elements. For simplicity Z can be taken as AUB.

4. CONCLUSION

The presented result has shown that, for any two finite sets A and B,
P(AUB) $\subseteq$ [P (A)UP(B)] is not always true. The result has established the conditions needed for P(AUB) $\subseteq$ [P (A)UP(B)] and P(AUB) $\nsubseteq$ [P (A)UP(B)].

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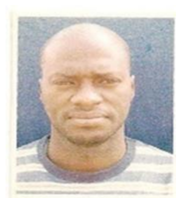
Authors Profile



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