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Intuitionistic Fuzzy Sets in Electoral System

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ABSTRACT

In this paper, we give a concise note on intuitionistic fuzzy sets (IFSs) and thereafter, showed how resourceful IFS is in decision making in a typical electioneering process.

Keywords: *electoral system, fuzzy sets, intuitionistic fuzzy sets.*

INTRODUCTION

In [2, 3], intuitionistic fuzzy set (IFS) was introduced as a generalization of fuzzy set earlier proposed in [1]. Intuitionistic fuzzy sets (IFSs) attracted much attention due to it's significant in tackling vagueness, uncertainties or the representation of imperfect knowledge in decision making. There are volume of literatures involving some basic, theory and some applications of IFS (see [4] for account of IFS past, presence, and future). The cardinal aim of this work is to supply a brief note on IFS and straight away shows its application in electoral system.

Precise Note on Intuitionistic Fuzzy Sets

Definition 1: Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x): X \longrightarrow [0,1]$ is the membership function of the fuzzy set A.

Definition 2: Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x) : X \longrightarrow [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e., $\pi_A(x) : X \longrightarrow [0,1]$ and $0 \leq \pi_A \leq 1$ for every $x \in X$. $\pi_A(x)$expresses the lack of knowledge of whether x belongs to IFS A or not.

Definition 3: Let $A \in X$ be IFS, then;

1. $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of indetermacy of the element $x \in A$.
2. $\partial_A(x) = \mu_A(x) + \pi_A(x)\mu_A(x)$ is called the degree of favour of $x \in A$.
3. $\eta_A(x) = \nu_A(x) + \pi_A(x)\nu_A(x)$ is called the degree of against of $x \in A$.

For example, let A be an intuitionistic fuzzy set with $\mu_A(x) = 0.3$ and $\nu_A(x) = 0.4$ then, $\pi_A(x) = 0.3$, $\partial_A(x) = 0.39$, and $\eta_A(x) = 0.52$. It can be interpreted as "the degree that the object x belongs to IFS A is 0.3, the degree that the object x does not belong to IFS A is 0.4, the degree of hesitancy or indetermacy of x belonging to IFS A is 0.3, the degree of favour of x belonging to IFS A is 0.39 and the degree of against of x not belonging to IFS Ax is 0.52".

Basic Operations on Intuitionistic Fuzzy Set

1. [inclusion]
   $A \subseteq B \leftrightarrow \mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)\ \ \forall x \in X.$

2. [complement]
$$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}.$$
3. [union]
$$A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle : x \in X\}.$$
4. [intersection]
$$A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle : x \in X\}.$$
5. [addition]
$$A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)\rangle : x \in X\}.$$
6. [multiplication]
$$A \otimes B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle : x \in X\}.$$

Definition 4 (Similar IFS): Two IFSs A and B are said to be similar or cognate if $\exists \mu_A(x) = \mu_B(x)$ or $\nu_A(x) = \nu_B(x)$.

Definition 5 (Comparable IFS): Two IFSs A and B are said to be equal or comparable if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$.

Definition 6 (Equivalent IFS): Two IFSs A and B are said to be equivalent to each other i.e. A is equivalent to B , denoted by $A \sim B$ if $\exists$ functions $f: \mu_A(x) \to \mu_B(x)$ and $f: \nu_A(x) \to \nu_B(x)$ which are both injection and surjection (i.e. bijection). Then, the functions define a one-to-one correspondence between A and B.

Definition 7(Proper Subset): A is a proper subset of B i.e. $A \subset B$ if $A \subseteq B$ and $A \neq B$. It means $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ but $\mu_A(x) \neq \mu_B(x)$ and $\nu_A(x) \neq \nu_B(x)$ for $x \in X$.

Definition 8 (Dominations): An IFS A is dominated by another IFS B (i.e. $A \preccurlyeq B$), if there exist an injection from A to B. A is strictly dominated by B (i.e. $A \prec B$), if (i) $A \preccurlyeq B$ and (ii) A is not equinumerous with B.

Definition 9 (Relations): Let A, B and C be IFSs. Then;

i. $A \preccurlyeq A$ i.e. A is reflexive relation,
ii. $A \preccurlyeq B$ and $B \preccurlyeq A$ i.e. symmetric relation,
iii. $A \preccurlyeq B$ and $B \preccurlyeq C \Rightarrow A \preccurlyeq C$ i.e. transitive relation.

Corollary1: For any IFSs A and B, if $A \preccurlyeq B$ and $B \preccurlyeq A \Rightarrow A \sim B$.

Corollary 2: For any IFSs A and B, if $A \preccurlyeq A$, $A \preccurlyeq B$ and $B \preccurlyeq A \Rightarrow$ A and B are compatible to each other.

Note: 1. If a relation is reflexive, symmetric and transitive, such a relation is called an *"equivalence relation"*.
2. The proofs of Cor.1 and Cor.2 are obvious.

Application in Electoral System

Presently, we are in an era of democracy where the electorates exercise their franchise in the poll. Due to the existence of the fundamental human right, every electorate has the right to vote a preferable aspirant, and as such, decision on whom to elect preoccupies the electorates. In this scenario, some voters must of necessity vote a candidate, some against and some of course, will remain undecided or cast invalid vote. Interpreting into an intuitionistic fuzzy set, the electorates that voted for a candidate stand for the membership function μ, those that voted against stand for the non-membership function ν, and those that remain undecided or cast invalid ballot paper stand for the hesitation margin π.

For example, let X be the set of all countries with elective governments. Assume that we know for every country $x \in X$ the percentage of the electorates that voted for a particular party in each country, denote that by $M(x)$ and let $\mu_A(x) = \frac{M(x)}{100}$ (i.e. degree of membership). Also, assume that we know for every country $x \in X$ the percentage of the electorates that voted against in each country, denote by $N(x)$ and let

$\nu_A(x) = \frac{N(x)}{100}$ (i.e. degree of non-membership). Then, the part of the electorates who refuse to vote or cast invalid ballot paper is given as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. Thus, we construct the set as $\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and obviously, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and $\pi_A(x) \in [0,1]$.

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