International Journal of Science and Technology Volume 2 No. 10, October, 2013

Probabilistic Assessment and Optimization of the Second law efficiency of an Intercooled-Reheat- Regenerative Brayton Cycle Coupled to Variable Temperature Heat Reservoirs

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ABSTRACT

Thermodynamic analysis, pertaining to the Second law efficiency, of an intercooled, reheat, regenerative and irreversible Brayton cycle, coupled to variable temperature heat reservoirs, is presented in this paper. The Second law efficiency is found to first increase and then decrease, with increase in reheat pressure ratio. This means that there is an optimum value of Second law efficiency. This optimum Second law efficiency is found to be an increasing function of the components' efficiencies, the heat exchangers' effectiveness and heat capacitance rate of the working fluid, and a decreasing function of the heat capacitance rates of the external fluid. The thermal efficiency, at the optimized reheat pressure ratio, also follows a similar trend. The reasons behind these trends have been discussed in detail. A probabilistic assessment of the Second law efficiency is carried out, giving the relative impact of the various design parameters on the Second law efficiency.

Keywords: *Brayton Cycle; Second law analysis; Intercooling; reheating; regeneration; lost work; entropy analysis; probabilistic; variable temperature reservoirs.*

1. INTRODUCTION

The gas turbine engine works on the principles of Brayton cycle. One of the ways to improve the efficiency of the Brayton cycle is to increase the effective flame temperature [1]. This is the temperature which determines the total amount of exergy input to the cycle. The flame temperature, however, has a complex dependency on many design parameters like choice of fuel, fuel/air ratio, combustor design etc. and may not be amenable to upward increase. Another way to increase the efficiency is to improve the maximum temperature of the working fluid (air), which is the temperature at the inlet to the turbine. This approach has structural challenges and calls for research into new methods of turbine cooling [2, 3] and use of materials which can withstand the high temperatures [4-6]. The third approach is to improve the efficiencies of the individual components of the gas turbine engine. Finally, the efficiency of the gas turbine engine can be improved by the making changes in the Brayton cycle itself. These changes include, but are not limited to, intercooling, reheating, regeneration, isothermal heat addition etc. It may be noted that introduction of only intercooling or only reheat will always decrease the efficiency, because intercooling reduces the average temperature at which heat is added while reheat increases the average temperature at which heat is rejected. Therefore for improved efficiency, reheating and intercooling must be accompanied by regeneration.

Efficiency of a process or a cycle can be interpreted as a ratio of output of the real process to the output of an ideal (hypothetical) process. The way this ideal (hypothetical) process is defined depends on which definition of efficiency we are considering. For *thermal* efficiency of a cycle (which is the ratio of power output to heat input), the ideal cycle is one which converts all the heat input to power. The ideal cycle for thermal efficiency accordingly assumes no heat rejection and therefore violates the Second law of thermodynamics. In addition, while the maximum value of thermal efficiency of a heat engine between two constant temperature reservoirs is given by Carnot efficiency; the maximum possible thermal efficiency between two *variable-temperature* heat reservoirs cannot be found out in a straightforward manner and needs further research. Due to these reasons, it is difficult to gauge from thermal efficiency, how much more work is available to be extracted from the cycle/process.

To get a clearer picture of the irreversibility of a process/cycle, two efficiencies have been considered in literature: *isentropic* efficiency and *Second law* efficiency. In isentropic efficiency, the ideal (hypothetical) process, to which the real process is compared, is considered to be an isentropic process. The use of isentropic efficiency assumes the real process to be adiabatic. This means that we are neglecting the external irreversibility (the irreversibility associated with heat transfer across a finite temperature difference). So, while isentropic efficiency is a good measure of the performances of devices like compressors, turbines, nozzles etc., it is not suited for components which exchange heat, like heat exchangers or cycles which employ heat exchangers. Moreover, while using isentropic efficiency, the hypothetical isentropic process (to which the real process is compared) is not between the same initial and final states as the real process. Figure 1 shows the expansion process taking place in a turbine between two isobars P1=constant and P0 =constant. The real process in the turbine is shown by the dotted line AB, while the corresponding hypothetical isentropic process between the same two isobars is represented by the line AB'. Note that the final state of the real process B does not match with the final state of the ideal isentropic process, which is B'.

Second law efficiency, on the other hand suffers from no such inconsistencies. It takes into account, both the external as well as internal irreversibility, and thus it is favourable and in fact quite commonplace in literature, to use Second law efficiency for devices which exchange heat; e.g. heat exchangers or cycles which employ heat exchangers. Moreover, the ideal (hypothetical) process, to which the real process is compared while considering Second law efficiency, is a perfectly reversible process between the same initial and final states as the real process. In Figure 1, the reversible process between A and B is shown by the solid (curved) line AB. The real process (dotted AB) and the ideal, reversible process (solid AB) are between the same initial and final states. In addition, the theoretical maximum value which the Second law efficiency can take is 1, so the designer knows how much scope is there for improvement of the cycle. Optimization of Second law efficiency results in minimization of entropy generated, leading to maximization of work output [7]; which is the goal of thermodynamic optimization of devices [8]. For these reasons, in the present study, we have considered the probabilistic assessment and optimization of the Second law efficiency of an intercooled-reheat-regenerative Brayton cycle. To the best of our knowledge, this has not been considered in literature so far.

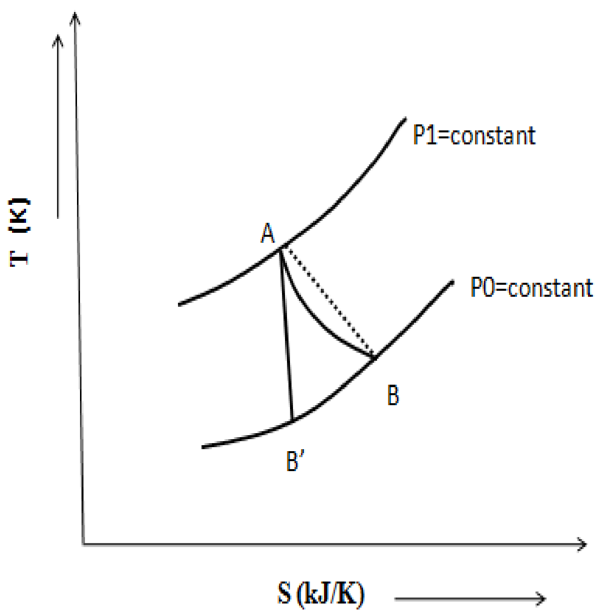

Figure 1: Difference between isentropic efficiency and Second law efficiency.

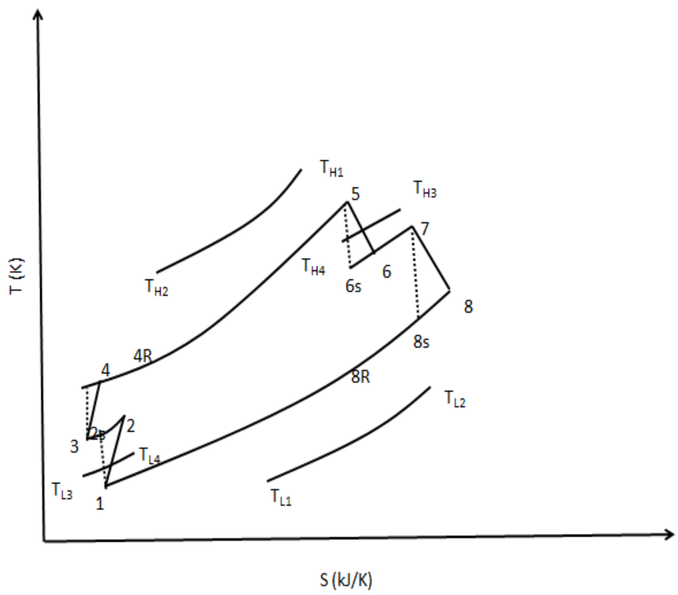

Figure 2: T-S diagram of an intercooled, reheat, regenerative Brayton Cycle.

Considerable work has been carried out to modify the Brayton cycle in order to improve its efficiency and/or power output. A brief survey of literature is presented. Bejan [7] proposed the heat leak model of irreversible power plants and proved that power output of a simple Brayton cycle is maximized when the entropy generation rate is minimized. Leff [9] considered a simple, reversible Brayton cycle and found out its efficiency at maximum work output. Ibrahim & others [10] considered an endoreversible Brayton cycle coupled to constant and variable temperature heat reservoirs and found out its optimal power output. Wang, Chen & co-workers [11] found out optimum thermal efficiency and optimum power of an intercooled, regenerative, irreversible Brayton cycle coupled to variable temperature heat reservoirs. Tyagi, Kaushik & co-workers [12] found out the optimal power and the optimum thermal efficiency of an intercooled, regenerated, reheat, irreversible Brayton cycle with variable temperature heat reservoirs. The present study carries forward the work of Tyagi et al and deals with the optimization and probabilistic assessment of the Second law Efficiency of the same cycle as theirs.

2. CYCLE DESCRIPTION

The T-S Diagram of the cycle is shown in Figure 2. The working fluid enters the engine at the state point 1. It is compressed in the first compressor to state point 2. The corresponding state point for isentropic compression is 2s. From 2, it gets cooled in the intercooler to state point 3. The external fluid in the intercooler, being of finite heat capacity, gets heated from T_{L3} to T_{L4} . The working fluid then gets compressed from state point 3 to 4 in the second compressor. The corresponding state point for isentropic compression is 4s. From 4, the fluid gets heated in the regenerator to T_{4R} and then again gets heated in the hot side HX to 5. The external fluid in the hot side HX cools from T_{H1} to T_{H2}. After 5, the working fluid expands in the first turbine to state point 6. The corresponding state point for isentropic expansion is 6s. From

6, the working fluid is reheated in the reheater to state point 7. The external fluid in the reheater gets cooled from T_{H3} to T_{H4}. The working fluid then expands in the second turbine to state point 8. The corresponding state point for isentropic expansion is 8s. From 8, the working fluid gets heated to state point T_{8R} in the regenerator and then back to 1 in the cold side HX. The external fluid in the cold side HX get heated from T_{L1} to T_{L2}.

3. THERMODYNAMIC ANALYSIS

3.1. Analysis for first law efficiency

The thermodynamic analysis of the cycle for the first law efficiency is presented in the Appendix.

3.2. Analysis for Second law efficiency

In this study, we have used a definition of Second law efficiency which is used for most work producing processes and is based on the concept of lost work .

$$\eta_{second} = \frac{\dot{W}_{actual}}{\dot{W}_{actual} + \dot{W}_{lost}} \quad (1)$$

where, from [13]

$$\dot{W}_{lost} = T_0 \dot{S}_{gen} \quad (2)$$

is the rate of work lost.

In the current case, $\dot{S}_{gen}$ is rate of total entropy generated inside the system and is given by:

$$\dot{S}_{gen} = \dot{S}_{gen,C1} + \dot{S}_{gen,L2} + \dot{S}_{gen,C2} + \dot{S}_{gen,H1} + \dot{S}_{gen,T1} + \dot{S}_{gen,H2} + \dot{S}_{gen,T2} + \dot{S}_{gen,L1} \quad (3)$$

For steady, one dimensional flow,

In the first compressor:

$$\dot{S}_{gen,C1} = \dot{m}(s_2 - s_1) \quad (4)$$

In the intercooler, neglecting pressure drop:

$$\dot{S}_{gen,L2} = \dot{m}(s_3 - s_2) + C_{L2} \ln \frac{T_{L4}}{T_{L3}} \quad (5)$$

In the second compressor:

$$\dot{S}_{gen,C2} = \dot{m}(s_4 - s_3) \quad (6)$$

In the hot side HX, neglecting pressure drop:

$$\dot{S}_{gen,H1} = \dot{m}(s_5 - s_{4R}) + C_{H1} \ln \frac{T_{H2}}{T_{H1}} \quad (7)$$

In the first turbine:

$$\dot{S}_{gen,T1} = \dot{m}(s_6 - s_5) \quad (8)$$

In the reheater, neglecting pressure drop:

$$\dot{S}_{gen,H2} = \dot{m}(s_7 - s_6) + C_{H2} \ln \frac{T_{H4}}{T_{H3}} \quad (9)$$

In the second turbine:

$$\dot{S}_{gen,T2} = \dot{m}(s_8 - s_7) \quad (10)$$

In the cold side HX, neglecting pressure drop:

$$\dot{S}_{gen,L1} = \dot{m}(s_1 - s_{8R}) + C_{L1} \ln \frac{T_{L2}}{T_{L1}} \quad (11)$$

In the regenerator:

$$\dot{S}_{gen,R} = \dot{m}(s_{8R} - s_8) + \dot{m}(s_{4R} - s_4) \quad (12)$$

Adding Eqs 4-12 ,we get

$$\dot{S}_{gen} = C_{L2} \ln \frac{T_{L4}}{T_{L3}} + C_{H1} \ln \frac{T_{H2}}{T_{H1}} + C_{H2} \ln \frac{T_{H4}}{T_{H3}} + C_{L1} \ln \frac{T_{L2}}{T_{L1}} \quad (13)$$

4. RESULTS AND DISCUSSION

To find out the effect of design parameters on the Second law efficiency, a typical set of values for the design parameters is chosen as follows: $T_{H1} = T_{H3} = 1500K$, $\varepsilon_{H1} = \varepsilon_{H2} = \varepsilon_{L1} = \varepsilon_{L2} = \eta_{C1} = \eta_{C2} = \eta_{T1} = \eta_{T2} = 0.90$, $\pi_i = 8, \pi = 15, C_{H1} = C_{H2} = C_{L1} = C_{L2} = 1.0kWK^{-1}$, $C_W = 0.95kWK^{-1}, T_{L1} = T_{L3} = T_o = 300K$. While choosing the parameters, care is taken to choose only such parameters for which the total entropy generation is positive and hence the Second law is not violated.

4.1. Total Pressure Fixed

i. Deterministic Analysis

Figure 3 is a characteristic curve which shows the Second law efficiency versus reheat pressure ratio for different values of efficiency of compressors. The reheat pressure ratio π_h is varied in the region $(1, \pi)$

As can be seen from the figure, the Second law efficiency increases with increase in efficiency of compressors. For $\pi_h = 1$ and $\pi_h = \pi$, the cycle reduces to an intercooled, regenerative, irreversible Brayton cycle (without any reheat), whose efficiency is less than the cycle with reheat, regeneration and intercooling considered here. Thus the Second law efficiency first increases with increase in reheat pressure ratio and then it decreases. This also means that there are values of reheat pressure ratio between $(1, \pi)$ where the Second law efficiency has local maxima.

ii. Probabilistic Assessment

What is shown in the previous section was the deterministic assessment of the Second law efficiency of the complex Brayton cycle. In the deterministic approach, an input parameter is given a particular value and the output, corresponding to that particular value of input, is calculated. Though deterministic analysis has its obvious uses, it has some short comings too. One of them is that it does not give the sensitivity of the output to the variability in various input parameters. This means that designer/decision maker does not know which input parameters, out of the many, are the most important for design. For example, through deterministic analysis, we know that increasing the effectiveness of both the reheater and the intercooler will increase the Second law efficiency, but we do not know which will have more impact.For this purpose, a Monte Carlo simulation was performed on the thermodynamic model. The noise parameters with their corresponding nominal values and their deviations are given in Table 1.

Table 1: Design Parameters with their mean values and variability

Design Parameter	Mean value	Standard Deviation
η_T	0.90	0.025
η_C	0.90	0.025
ε_{L2}	0.90	0.025
ε_R	0.90	0.025
ε_{H2}	0.90	0.025
ε_{H1}	0.90	0.025
ε_{L1}	0.90	0.025
C_{H1}, C_{H2}	1.15	0.12
C_{L1}, C_{L2}	1.15	0.12
C_W	0.8	0.05

To do the probabilistic assessment, 10,000 simulations were carried out for the thermodynamic model. The number 10,000 was chosen as a compromise between numerical accuracy and computational cost. The resulting data was analyzed (for mean, standard deviation etc.) and was plotted (See Fig 4 and Table 2).To gauge the sensitivity of the Second law efficiency to any input parameter variability, Spearman's rank order correlation test [14] between the ranks of the input variable and the corresponding output variable was used. The Spearman's rank order correlation coefficients (ρ) were calculated by the following formula;

$$\rho = 1 - \frac{6\sum d^2}{n(n^2 - 1)} \tag{1}$$

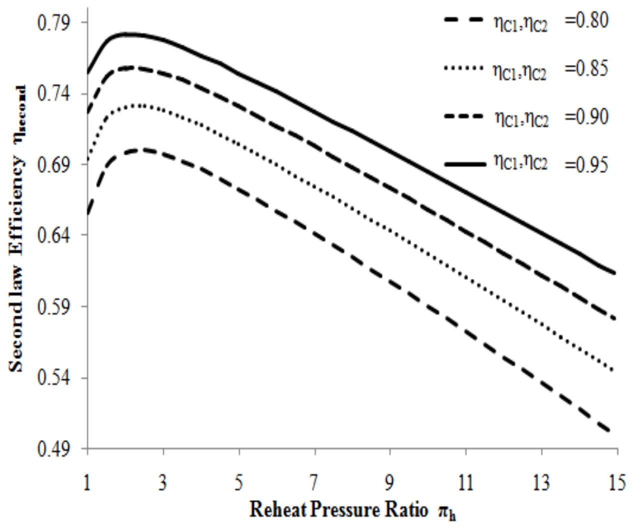

Figure 3: Second law efficiency versus reheat pressure ratio for different values of efficiency of the compressors

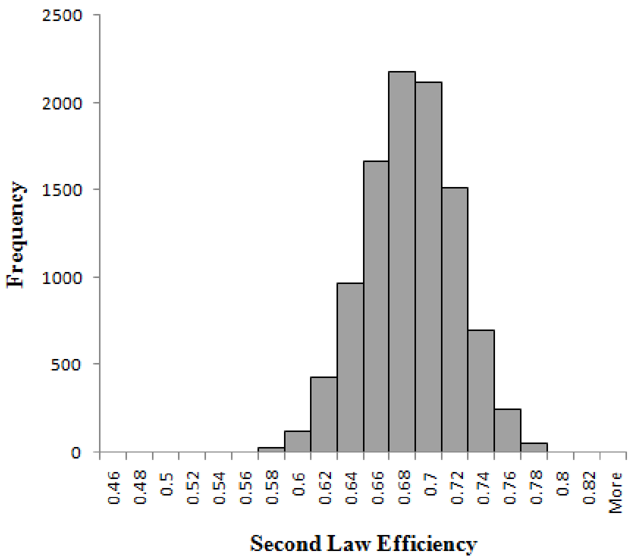

Figure 4: Frequency Distribution of Second law efficiency

Table 2: Mean and Standard Deviation of the Probabilistic distribution of Second law efficiency.

Mean	0.675869
Standard Deviation	0.034867

In Eq. (1) above, *n* is the number of data points and *d* is the difference in rank-order of any input variable and the corresponding output value. The value of the coefficient can range from -1 to 1. A value of 1 indicates strong positive correlation; a value of 0 indicates no correlation, while a value of -1 indicates strong negative correlation.

Table 3: Spearman's rank order correlation of different design parameters. efficiency.

Design parameter	Spearman's Rank order correlation coefficient
η_T	0.56331
η_C	0.379213
ε_{H1}	0.079735

ε_{H2}	0.028965
ε_{L1}	0.463745
ε_{L2}	0.27473
ε_{R}	0.136624
C_{H1}, C_{H2}	-0.08371
C_{L1}, C_{L2}	-0.28373
C_{W}	0.229148

The values of Spearman's rank order correlation, calculated for the different input parameters, are given in Table 3.
As can be seen from the Table 3, the efficiency of the turbines has a higher impact on the Second law efficiency than the efficiency of the compressors. Among the heat exchangers, the effectiveness of the cold side HX and the intercooler has a greater impact than the effectiveness of the hot side HX and the reheater, while the impact of the regenerator is between the two. The heat capacitance rates of the external fluid have a negative impact while the heat capacitance rate of the working fluid has a positive impact.

4.2. Total pressure variable

As pointed out in Sec 4.1.*i*, there are optimal values of the reheat pressure ratios for which the Second law efficiency attains local maxima. Optimization of the Second law efficiency will lead to minimization of entropy generation. According to Bejan [7] the minimization of entropy generation leads to maximization of work output. So, our goal in this paper is to optimize the Second law efficiency by varying the reheat pressure ratio and then to plot the optimum Second law efficiency for different total pressure ratio.

Figure 5- Figure 9 show, respectively, the effect of the efficiency of compressors, turbines, the effectiveness of the hot side HX, and the heat capacitance rates of the external fluid and the working fluid on the Optimum Second law efficiency vs Total pressure ratio plots. The thermal efficiency at the corresponding optimized reheat pressure ratio has also been plotted. The results are discussed as follows:

Effect of Efficiency of the compressors and the turbines

Figure 5 and Figure 6 show the optimum Second law efficiency versus total pressure ratio for different values of efficiency of the compressors and turbines respectively. The thermal efficiency at the optimized reheat pressure ratio is also plotted against total pressure ratio. As can be seen from the figures, the optimum Second law efficiency increases with increase in the components' efficiencies. The thermal efficiency at the optimized reheat pressure ratio also follows a similar trend. The result of these figures can be explained on the basis of both internal and external irreversibilities.

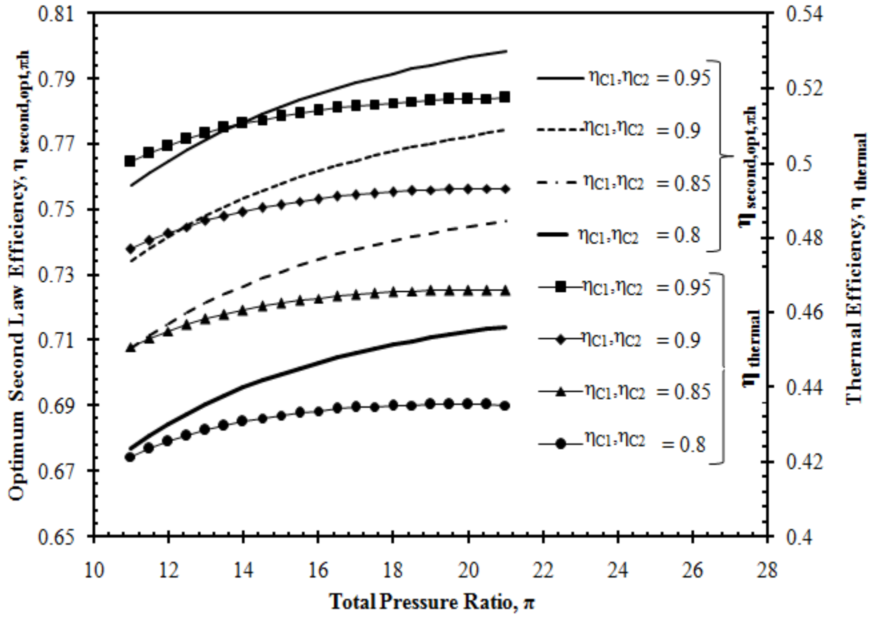

Figure 5: Optimum Second law efficiency and the thermal efficiency at optimized reheat pressure ratio, versus total pressure ratio for different values of efficiency of the compressors.

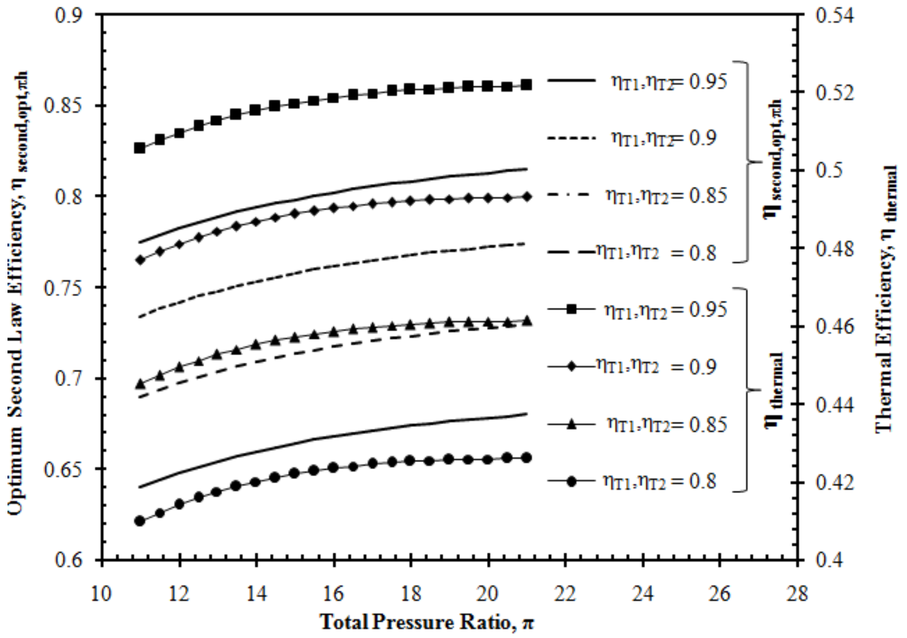

Figure 6: Optimum Second law efficiency and the thermal efficiency at optimized reheat pressure ratio, versus total pressure ratio for different values of efficiency of turbines.

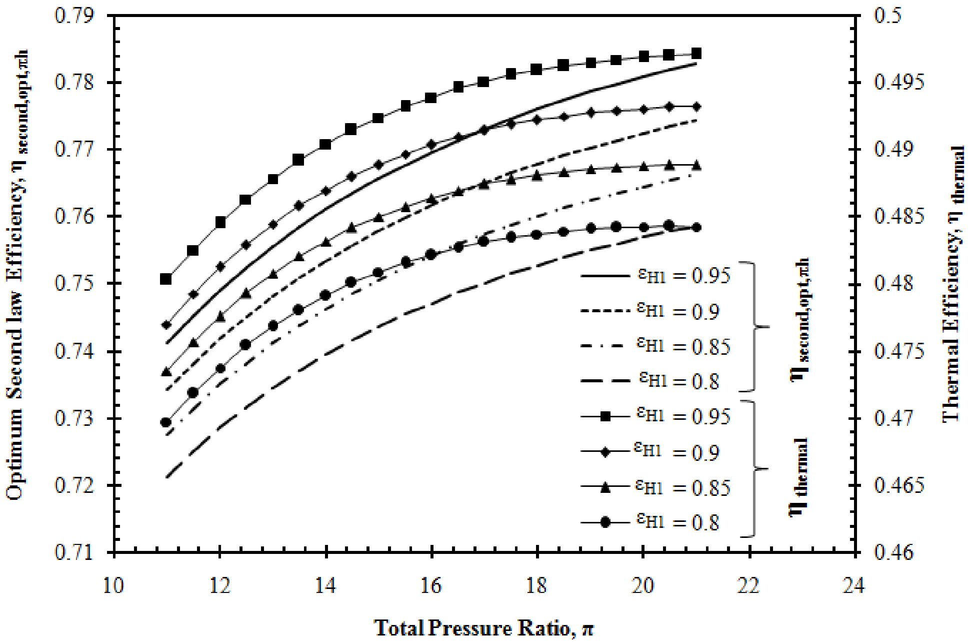

Figure 7: Optimum Second law efficiency and the thermal efficiency at optimized reheat pressure ratio, versus total pressure ratio for different values of effectiveness of the hot side HX.

In case of internal irreversibility, we know that as the efficiency of turbines/compressors increases, there is less entropy generated inside them and so there is less irreversibility in the cycle and the Second law efficiency increases. While accounting for external irreversibility (the irreversibility associated with heat transfer), the situation becomes a little more involved. As the efficiency of the second turbine increases, its exit temperature goes down, due to which there is less heat transfer in the regenerator and the cold side HX, which leads to decrease in external irreversibility. But in case of the first turbine, as its efficiency increases, the temperature at the exit of the first turbine goes down, which means more heat transfer in the reheater and hence more external irreversibility.

Similarly, when we increase the efficiency of the first compressor, its exit temperature goes down, due to which there is less heat transfer in the intercooler and hence less external irreversibility. On the other hand, when the efficiency of the second compressor is increased, its exit temperature goes down, which leads to more heat transfer in the regenerator and the hot side HX causing an increase in external irreversibility. Since the heat transfer in the regenerator plus the hot side HX is higher (compared to the intercooler), this increase in external irreversibility causes the compressors to have a lower impact on the Second law efficiency than turbines (see Table 3).

Effect of the Heat Exchangers' Effectiveness

Figure 7 shows the optimum Second law efficiency versus total pressure ratio for different values of the effectiveness of the hot side HX. The thermal efficiency at the optimized reheat pressure ratio has also been plotted versus total pressure ratio for different values of the effectiveness of the hot side HX. As can be seen from the plots, the Second law efficiency increases with increase in the effectiveness of the hot side HX. The thermal efficiency at the optimized reheat pressure ratio also follows the same trend. Qualitatively similar trends are also observed for the effectiveness of the other heat exchangers (the intercooler, the reheater, the cold side HX and the regenerator) but have not been reproduced here. All these trends can be explained on the basis of irreversibility.

As the effectiveness of the heat exchangers increase, the temperature difference between the external fluid and the working fluid decreases, due to which there is less external irreversibility (i.e. irreversibility due to heat transfer). Thus the Second law efficiency increases.

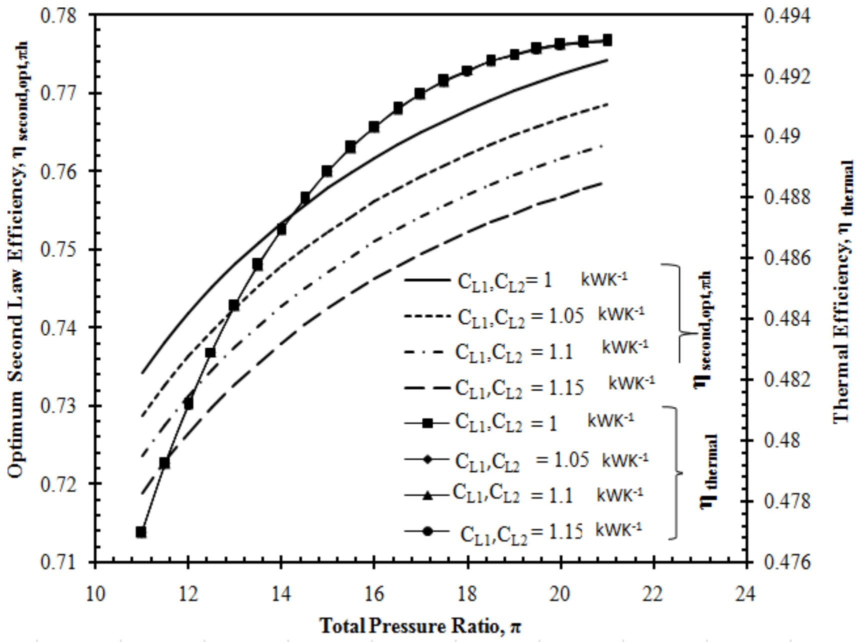

Figure 8: Optimum Second law efficiency and the thermal efficiency at optimized reheat pressure ratio, versus total pressure ratio for different values of heat capacitance rates of external fluid in the cold side HX and the intercooler.

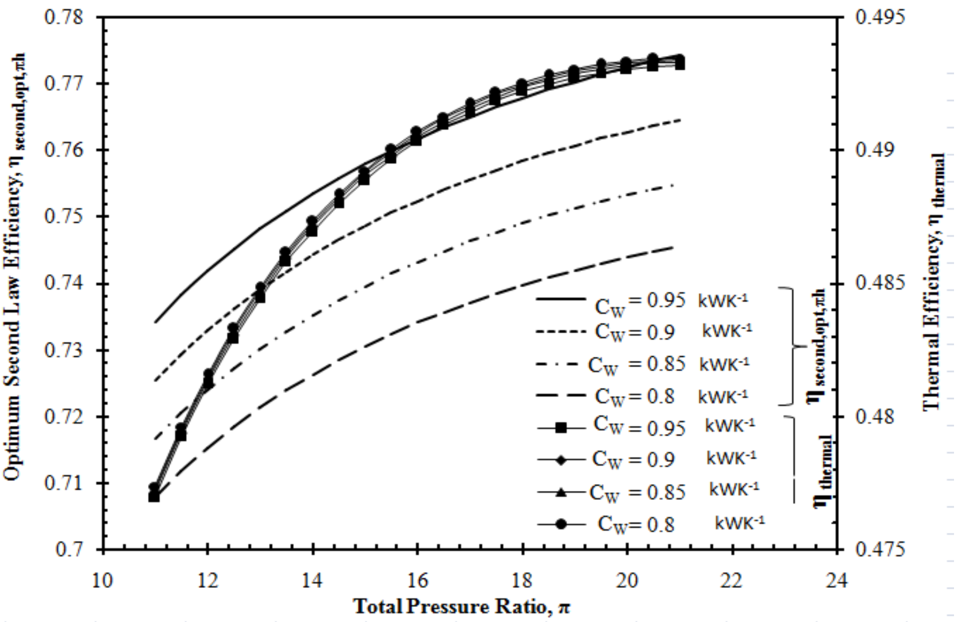

Figure 9: Optimum Second law efficiency and the thermal efficiency at optimized reheat pressure ratio, versus total pressure ratio for different values of heat capacitance rates of the working fluid.

Effect of heat capacitance rates of the External fluid in the Heat exchangers

Figure 8 shows the optimum Second law efficiency versus the total pressure ratio for different values of heat capacitance rates of the external fluid in cold side HX. The thermal efficiency at the optimized reheat pressure ratio has also been plotted versus total pressure ratio for different values of heat capacitance rates of the external fluid. As can be seen from the plots, the optimum Second law efficiency decreases with increase in heat capacitance rates of the external fluid, while the effect on the thermal efficiency is negligible. Similar trends are also observed for heat capacitance of external fluid in hot side HX and the re-heater, but are not reproduced here. The justification for these trends is as follows:

The parameters we have chosen are such that min (C_K, C_W) $= C_W$ where k= (H1, H2, L1, L2). If we substitute this in the Eq.A5-Eq.A8 (see Appendix), it is observed that as the heat capacitance rates of the external fluids increase, the temperature difference between the external fluid and the working fluid increases, due to which the irreversibility increases and the Second law efficiency decreases. Opposite behaviour should be expected if min (C_K, C_W) was equal to C_K.

The effect on the thermal efficiency (at the optimized reheat pressure ratio) is negligible, because, as can be seen from Eq. A5- Eq.A8 in Appendix, for the case where min (C_K, C_W) $= C_W$, changing C_K does not have any effect on the state points of the cycle, due to which the thermal efficiency does not change on changing the heat capacitance rates of the external fluid.

Effect of heat capacitance rates of the working fluid

Figure 9 shows the optimum Second law efficiency versus total pressure ratio for different values of the heat capacitance rates of the working fluid. The variation of thermal efficiency (at the optimized reheat pressure ratio) versus total pressure ratio for different values of heat capacitance rates of the working fluid has also been shown. From the plots it can be seen that the optimum Second law efficiency increases with increase in heat capacitance rates of the working fluid, while the thermal efficiency at the optimized reheat pressure ratio shows negligible variation. This can be explained as follows.
The parameters chosen for this cycle are such that min (C_K, C_W) = C_W (where k=H1, H2, L1, L2). Substituting this in Eq. A5-Eq.A8 of Appendix, it is found that as C_W increases, the temperature difference between the external fluid and the working fluid decreases, due to which the irreversibility associated with heat transfer (external irreversibility) will decrease.
This will lead to increase in the Second law efficiency. Opposite behaviour should be expected if min (C_K, C_W) was equal to C_K.

Also by looking at Eq. A5-Eq.A8, it is found that (for the case min (C_K, C_W) = C_W (where k=H1, H2, L1, L2)) changing C_W does not have any effect on the state points of the cycle. Since the state points remain unchanged, the thermal efficiency (at the optimized reheat pressure ratio) also remains, more or less, unchanged.

Figure 10-Figure 17 show the reheat pressure (at optimum Second law efficiency) versus total pressure ratio for different values of efficiencies of compressors and turbines, effectiveness of hot side HX, reheater, cold side HX , intercooler, heat capacitance rates of the external fluid and that of the working fluid respectively. It can be seen from the figures that the reheat pressure at optimum Second law efficiency increases with increase in total pressure ratio. Also, the reheat pressure at optimum Second law efficiency is an increasing function of the effectiveness of the reheater, cold side HX and the intercooler; and the heat capacitance rates of the external fluid. It is a decreasing function of efficiency of compressors and turbines, effectiveness of the hot side HX and the heat capacitance rates of the working fluid.

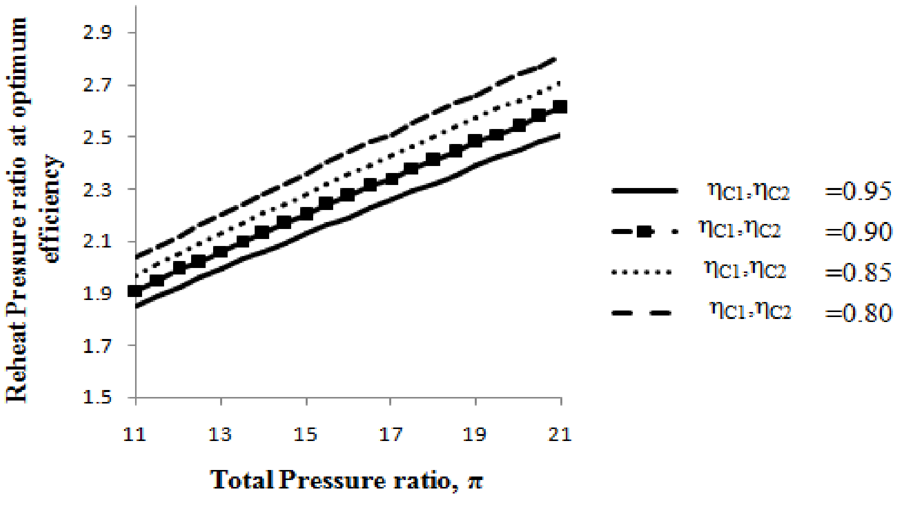

Figure 10: Reheat pressure at Optimum Second law efficiency, versus total pressure ratio, for different values of efficiency of the compressors.

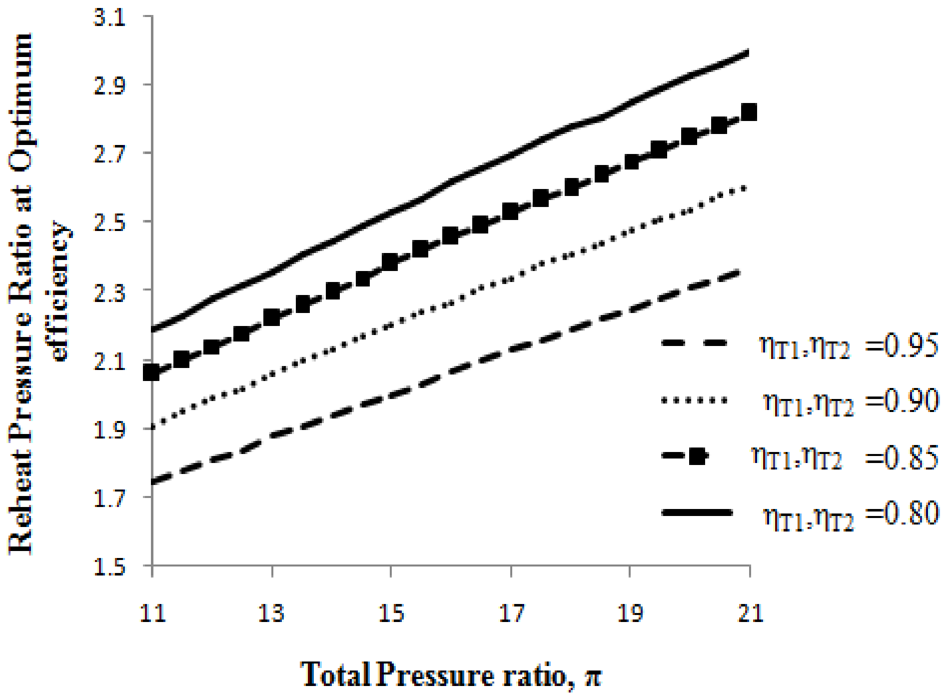

Figure 11: Reheat pressure at Optimum Second law efficiency versus total pressure ratio, for different values of efficiency of turbines.

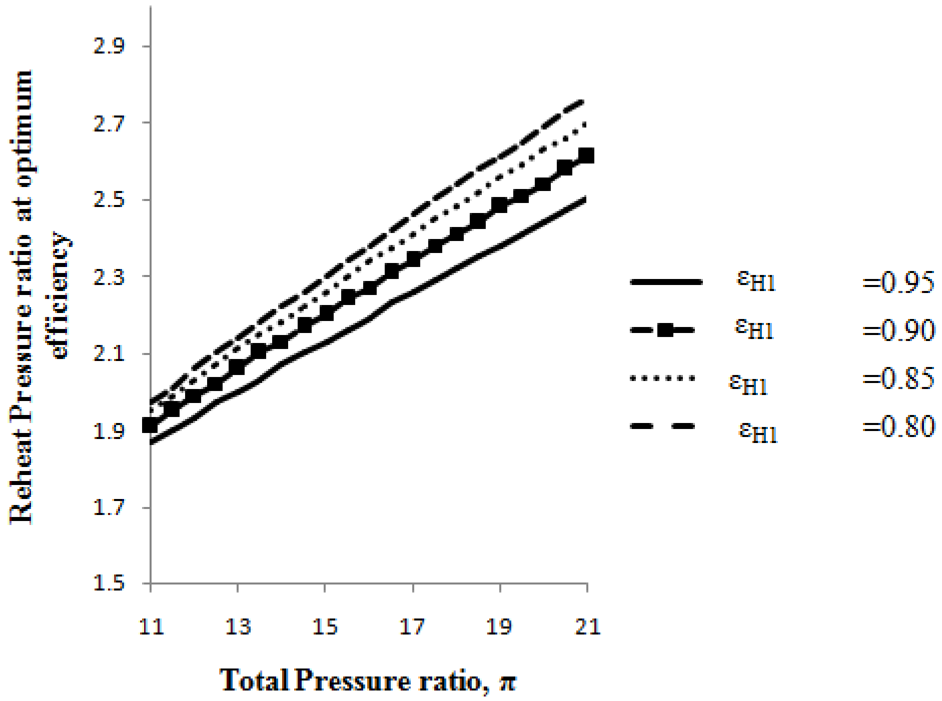

Figure 12: Reheat pressure at Optimum Second law efficiency, versus total pressure ratio, for different values of effectiveness of hot side HX.

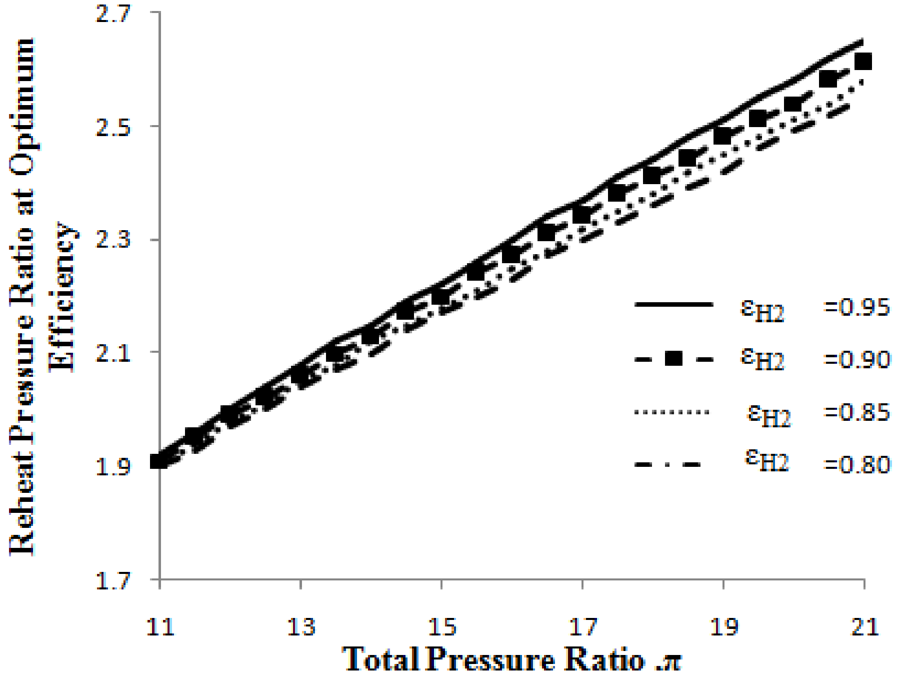

Figure 13: Reheat pressure at Optimum Second law efficiency, versus total pressure ratio, for different values of effectiveness of reheater.

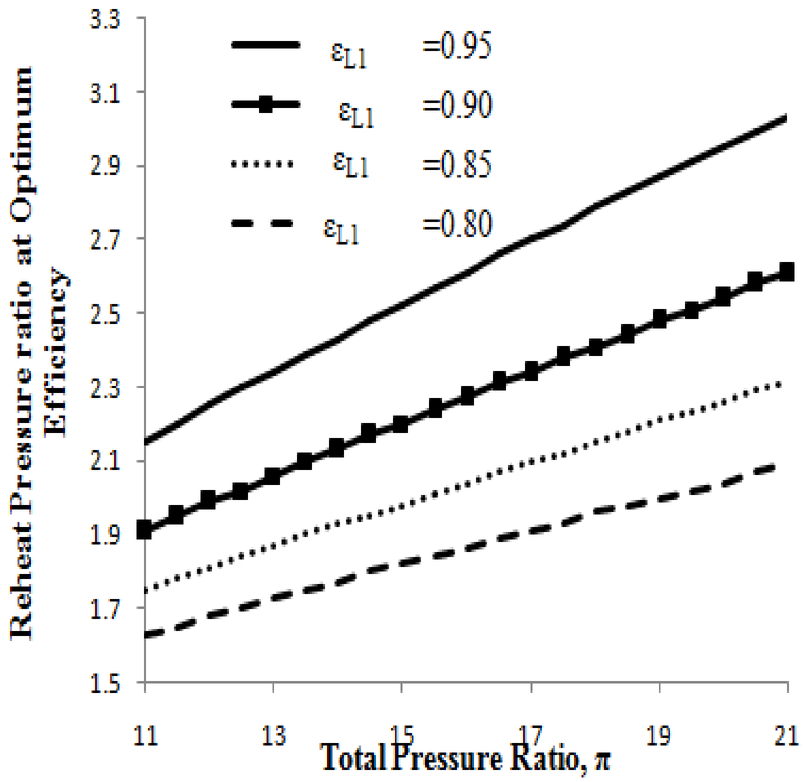

Figure 14: Reheat pressure at Optimum Second law efficiency versus total pressure ratio, for different values of effectiveness of cold side HX.

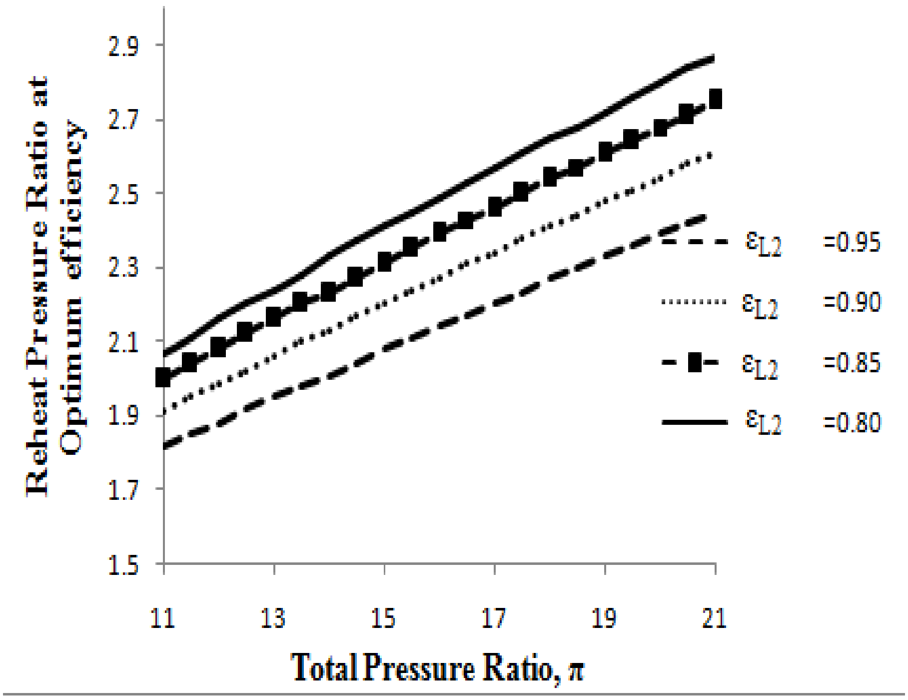

Figure 15: Reheat pressure at Optimum Second law efficiency, versus total pressure ratio, for different values of effectiveness of intercooler.

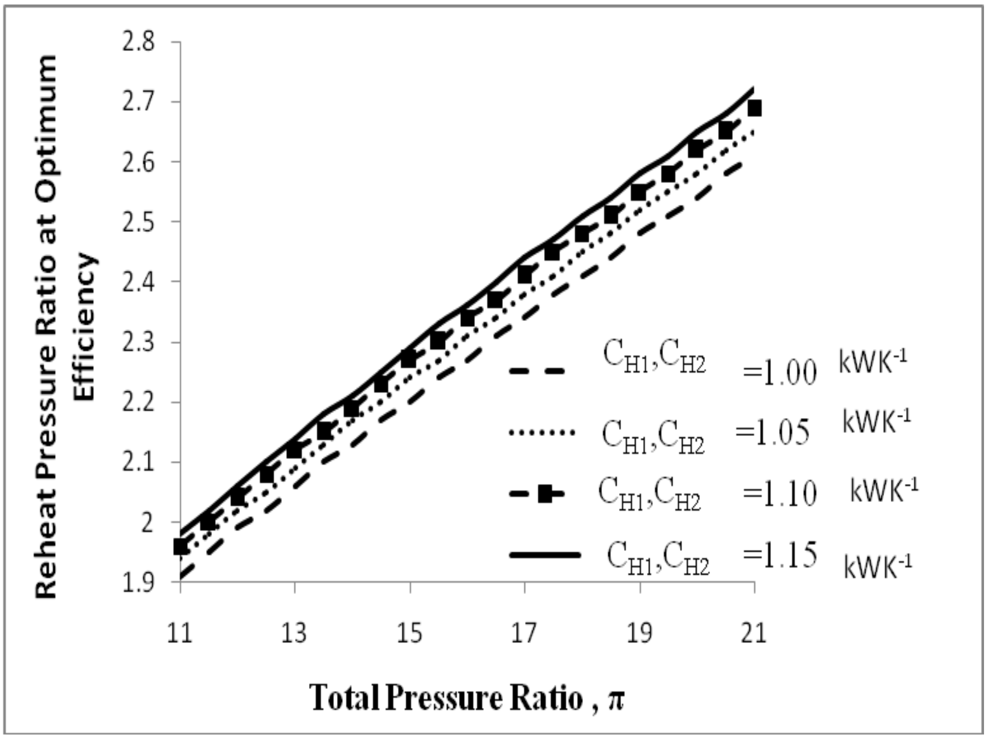

Figure 16: Reheat pressure at Optimum Second law efficiency, versus total pressure ratio, for different values of heat capacitance rates of the external fluid in the hot side HX and in the reheater.

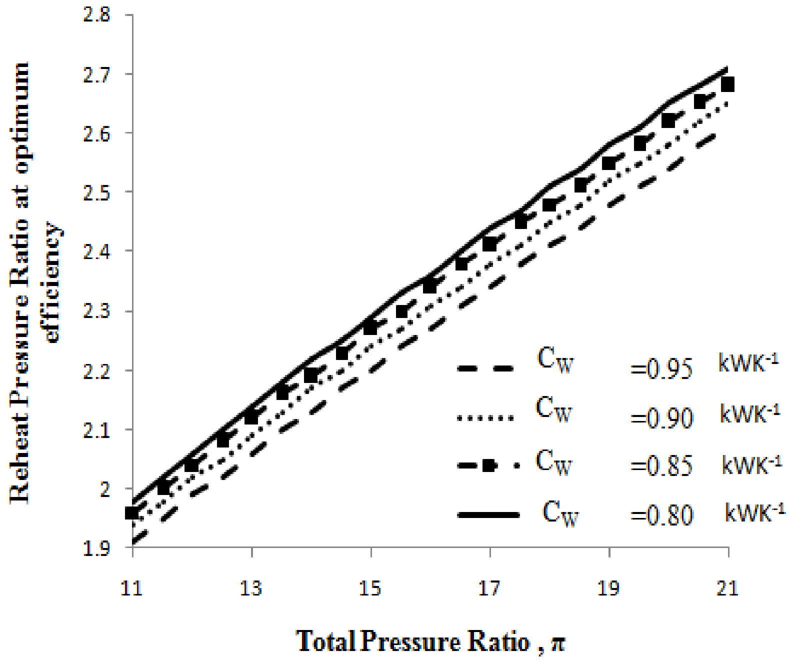

Figure 17: Reheat pressure at Optimum Second law efficiency, versus total pressure ratio, for different values of heat capacitance rates of the working fluid.

5. CONCLUSIONS

Second law efficiency of an irreversible, intercooled, reheat, regenerative Brayton cycle with variable temperature heat reservoirs has been studied in this research paper. Both deterministic and probabilistic assessment has been carried out for a given set of design parameters and the following conclusions have been deduced.

1. For a fixed total pressure ratio, the Second law efficiency first increases and then decreases with the increase in reheat pressure ratio. Thus there is a value of reheat pressure ratio for which the Second law efficiency reaches its optimum value.
2. The optimum Second law efficiency is an increasing function of efficiency of the compressors and the turbines, effectiveness of the various heat exchangers and the heat capacitance rates of the working fluid. It is a decreasing function of heat capacitance rates of the external fluid, for the case where the heat capacitance rates of the external fluid is more than that of the working fluid.
3. For the same case, the thermal efficiency at the optimized reheat pressure ratio, follows the same trend as the optimum Second law efficiency, except that it is unaffected by the changes in heat capacitance rates of the working fluid and the external fluid.
4. The reheat pressure ratio, at which the Second law efficiency attains its optimum value, is an increasing function of the effectiveness of the reheater, cold side HX and the intercooler and the heat capacitance rates of the external fluid, while it is a decreasing function of efficiency of compressors and turbines, effectiveness of the hot side HX and the heat capacitance rates of the working fluid.
5. A probabilistic assessment of the Second law efficiency shows that efficiency of the turbines has the highest effect on the Second law efficiency, followed by the effectiveness of the cold side HX, efficiency of the compressors, effectiveness of the intercooler, heat capacitance rate of the working fluid and the effectiveness of the regenerator, hot side HX and the re-heater in that order. The heat capacitance rate of the external fluid has a negative impact.

It is hoped that the results presented in this paper will help the designers/decision makers to make smarter decisions while carrying out the preliminary design of the Brayton cycle power plant.

Acknowledgements

The author would like to thank Dr T.K. Vashist of Infotech Enterprises Limited for his guidance and review of the manuscript. Thanks are also due to Mr. Harikrishnan L. for his help in plotting some of the graphs.

NOMENCLATURE

A: Area *(m^2)*

C: Mass flow rate times specific heat *(kWK^{-1})*

$\dot{Q}$: Heat transfer rates *(kW)*

$\dot{S}$: Entropy rate *(kWK^{-1})*

T: Temperature (*K*)

$\dot{W}$: Rate of work (*kW)*

1, 2, 3, 4, 4R, 4s, 5, 6, 6s, 7, 8, 8R thermodynamic state points

Greek Symbols

η_{second} : Second law efficiency

$\eta_{thermal}$: Thermal efficiency

$\eta_{second,opt,\pi h}$: Second law efficiency optimized with respect to reheat pressure ratio

ε : Effectiveness of heat exchanger

π_i, π_h : Intercooling and reheat pressure ratio respectively
π : Total pressure ratio
γ : Ratio of specific heats

Subscripts
actual: Actual
C1, C2: First and second compressor respectively
gen: Generated
H1, H2: Hot side HX and reheater respectively
L1, L2: Cold side HX and intercooler respectively
lost: Lost
R: Regenerator
system: System
T1, T2: First and second turbine respectively
W: Working fluid

Appendix

With reference to the model of the cycle drawn on the T-S diagram of Figure 2, the isentropic efficiencies of the compressors and the turbines can be written as [13, 15-17]:

$$\eta_{C1} = (T_{2s} - T_1)/(T_2 - T_1) \quad \text{(A1)}$$

$$\eta_{C2} = (T_{4s} - T_3)/(T_4 - T_3) \quad \text{(A2)}$$

$$\eta_{T1} = (T_5 - T_6)/(T_5 - T_{6s}) \quad \text{(A3)}$$

$$\eta_{T2} = (T_7 - T_8)/(T_7 - T_{8s}) \quad \text{(A4)}$$

The various heat transfer rates to and from the cycle can be given as follows [18-19]:

$$\dot{Q}_{H1} = (UA)_{H1}[(T_{H2} - T_4) - (T_{H1} - T_5)] \times (\ln \frac{(T_{H2} - T_4)}{(T_{H1} - T_5)})^{-1}$$
$$= C_W(T_5 - T_{4R}) = C_{H1}(T_{H1} - T_{H2})$$
$$= C_{H1,\min}\varepsilon_{H1}(T_{H1} - T_{4R}) \quad \text{(A5)}$$

$$\dot{Q}_{H2} = (UA)_{H2}[(T_{H4} - T_6) - (T_{H3} - T_7)] \times (\ln \frac{(T_{H4} - T_6)}{(T_{H3} - T_7)})^{-1}$$
$$= C_W(T_7 - T_6) = C_{H2}(T_{H3} - T_{H4})$$
$$= C_{H2,\min}\varepsilon_{H2}(T_{H3} - T_6) \quad \text{(A6)}$$

$$\dot{Q}_{L1} = (UA)_{L1}[(T_{8R} - T_{L2}) - (T_1 - T_{L1})] \times (\ln \frac{(T_{8R} - T_{L2})}{(T_1 - T_{L1})})^{-1}$$
$$= C_{L1}(T_{L2} - T_{L1}) = C_W(T_{8R} - T_1)$$
$$= C_{L1,\min}\varepsilon_{L1}(T_{8R} - T_{L1}) \quad \text{(A7)}$$

$$\dot{Q}_{L2} = (UA)_{L2}[(T_2 - T_{L4}) - (T_3 - T_{L3})] \times (\ln \frac{(T_2 - T_{L4})}{(T_3 - T_{L3})})^{-1}$$
$$= C_{L2}(T_{L4} - T_{L3}) = C_W(T_2 - T_3)$$
$$= C_{L2,\min}\varepsilon_{L2}(T_2 - T_{L3}) \quad \text{(A8)}$$

$$\dot{Q}_R = (UA)_R[(T_8 - T_{4R}) - (T_{8R} - T_4)] \times (\ln \frac{(T_8 - T_{4R})}{(T_{8R} - T_4)})^{-1}$$
$$= C_W(T_8 - T_{8R}) = C_W(T_{4R} - T_4)$$
$$= C_W\varepsilon_R(T_8 - T_4) \quad \text{(A9)}$$

Here, $(UA)_j$'s (j=H1, H2, L1, L2, R) are the overall heat transfer coefficient – area products, while ε_j's are the effectiveness of the hot side HX, reheater, cold side HX, intercooler and the regenerator. C_k's and C_W are the heat capacitance rates (specific heat multiplied by the mass flow rates) of the external fluids and the working fluids respectively. $C_{K,\min}$ =Minimum (C_K, C_W) where k=(H1, H2, L1, L2, R).

Solving for various state points temperatures in terms of T_4 and T_8, we get [12]

$$T_1 = a_1T_4 + b_1T_8 + c_1 \quad \text{(A10)}$$

$$T_2 = aT_1 \quad \text{(A11)}$$
$$T_3 = a_2T_4 + b_2T_8 + c_2 \quad \text{(A12)}$$
$$T_{4s} = a_3T_4 + b_3T_8 + c_3 \quad \text{(A13)}$$
$$T_5 = a_4T_4 + b_4T_8 + c_4 \quad \text{(A14)}$$
$$T_6 = bT_5 \quad \text{(A15)}$$
$$T_7 = a_5T_4 + b_5T_8 + c_5 \quad \text{(A16)}$$
$$T_{8s} = a_6T_4 + b_6T_8 + c_6 \quad \text{(A17)}$$
$$T_{4R} = (1-\varepsilon_R)T_4 + \varepsilon_R T_8$$
$$T_{8R} = (1-\varepsilon_R)T_8 + \varepsilon_R T_4$$

Where the different parameters are given as,

$$x_1 = \frac{C_{H1,\min}\varepsilon_{H1}}{C_W}$$
$$y_1 = \frac{C_{L1,\min}\varepsilon_{L1}}{C_W}$$
$$x_2 = \frac{C_{H2,\min}\varepsilon_{H2}}{C_W}$$
$$y_2 = \frac{C_{L2,\min}\varepsilon_{L2}}{C_W}$$
$$a_1 = (1-\varepsilon_R)(1-y_1)$$
$$b_1 = \varepsilon_R(1-y_1)$$
$$c_1 = y_1T_{L1}$$
$$a_2 = ab_1(1-y_2)$$
$$b_2 = aa_1(1-y_2)$$
$$c_2 = \{a(1-y_2)y_1 + y_1\}T_{L1}$$
$$a_3 = \eta_{C2} + (1-\eta_{C2})a_2$$
$$b_3 = (1-\eta_{C2})b_2$$
$$c_3 = (1-\eta_{C2})b_2$$
$$a_4 = (1-\varepsilon_R)(1-x_1)$$
$$b_4 = \varepsilon_R(1-x_1)$$
$$c_4 = x_1T_{H1}$$
$$a_5 = bb_4(1-x_2)$$
$$b_5 = ba_4(1-x_2)$$
$$c_5 = bc_4(1-x_2) + x_2T_{H3}$$
$$a_6 = (1-\eta_{T2}^{-1})a_5$$
$$b_6 = (1-\eta_{T2}^{-1})b_5 + \eta_{T2}^{-1}$$
$$c_6 = (1-\eta_{T2}^{-1})c_5$$
$$a = (\pi_i^{1-\frac{1}{\gamma}} + \eta_{C1} - 1)\eta_{C1}^{-1}$$

$$b = (\pi_h^{1-\frac{1}{\gamma}} + \eta_{T1}^{-1} - 1)\eta_{T1}$$

To solve for T_4 and T_8 in terms of the input parameters, we need to use the Second law of Thermodynamics for endoreversible cycle 1-2s-3-4s-5-6s-7-8s-1, which gives us

$$T_{2s}T_{4s}T_{6s}T_{8s} = T_1T_3T_5T_7 \qquad \text{(A20)}$$

Substituting the values of the various pressure ratios, viz. $\pi^m = (T_5T_2)/(T_{6s}T_{8s})$, $\pi_i^m = T_{2s}/T_1$, $\pi_h^m = T_5/T_{6s}$ (where $m = (\gamma-1)/\gamma$),we get two equations in T_4 and T_8 :

$$(a_6(\pi/\pi_h)^m - a_5)T_4 + (b_6(\pi/\pi_4)^m - b_5)T_8 + (\pi/\pi_h)^m c_6 - c_5 = 0 \qquad \text{(A21)}$$

$$(a_3(\pi_i/\pi)^m - a_2)T_4 + (b_3(\pi_i/\pi)^m - b2)T_8 + (\pi_i/\pi)^m c_3 - c_2 = 0 \qquad \text{(A22)}$$

Solving Eq (A19) and (A20) for T_4 and T_8 , we get:

$$T_4 = Nr1/Dr \quad ,T_8 = Nr2/Dr \qquad \text{(A23)}$$

where;

$$Nr1 = (\pi_i/\pi_h)^m(c_6b_3 - c_3b_6) + (\pi_i/\pi)^m(c_3b_5 - c_5b_3) + (\pi/\pi_h)^m(c_2b_6 - c_6b_2) + c_5b_2 - c_2b_5$$
$$Nr2 = (\pi_i/\pi_h)^m(a_6c_3 - a_3c_6) + (\pi_i/\pi)^m(a_3c_5 - a_5c_3) + (\pi/\pi_h)^m(a_2c_6 - a_6c_2) + a_5c_2 - a_2c_5$$
$$Dr = (\pi_i/\pi_h)^m(a_3b_6 - a_6b_3) + (\pi_i/\pi)^m(a_5b_3 - b_5a_3) + (\pi/\pi_h)^m(a_6b_2 - a_2b_6) + a_2b_5 - a_5b_2$$

The thermal efficiency of the cycle is given by [13,15-17]

$$\eta_{thermal} = 1 - (\dot{Q}_{L1} + \dot{Q}_{L2})/(\dot{Q}_{H1} + \dot{Q}_{H2})$$

$$= 1 - \frac{((T_{8R} - T_1) + (T_2 - T_3))}{((T_5 - T_{4R}) + (T_7 - T_6))} \qquad \text{(A24)}$$

Substituting the values of $T_1, T_2, T_3, T_{4R}, T_5, T_6, T_7, T_{8R}$

from equations (A10) to (A23) we get

$$\eta_{thermal} = 1 - Nre/Dre \qquad \text{(A25)}$$

Where;

$$Nre = ((1 - \varepsilon_R - b_1 + ab_1 - b_2)Nr2 + (\varepsilon_R - a_1 + aa_1 - a_2)Nr1 + (ac_1 - c_1 - c_2)Dr)/Dr$$
$$Dre = ((b_4 - \varepsilon_R + b_5 - bb_4)Nr2 + (a_4 - (1 - \varepsilon_R) + a_5 - ba_4)Nr1 + (c4 - bc4 + c5)Dr)/Dr$$

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