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On Generalized PreSemi Closed Sets in Intuitionistic Fuzzy Topological Spaces

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ABSTRACT

In this paper, we introduce the notion of an intuitionistic fuzzy generalized presemi closed sets and intuitionistic fuzzy generalized presemi open sets in intuitionistic fuzzy topological spaces. Also we have provide some applications of intuitionistic fuzzy generalized presemi closed sets.

Keywords: *Intuitionistic fuzzy topology, intuitionistic fuzzy generalized presemi closed sets, intuitionistic fuzzy generalized presemi open sets, intuitionistic fuzzy points, IFPST$_{1/2}$ space and IFPST$^*_{1/2}$ space .*

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1. INTRODUCTION

In 1965, Zadeh [13] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of these notions. In 1986, the notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce intuitionistic fuzzy generalized presemi closed sets and intuitionistic fuzzy generalized presemi open sets in intuitionistic fuzzy topological space. We study some of its properties in intuitionistic fuzzy topological spaces.

2. PRELIMINARIES

Throughout this paper, (X, τ) or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1: [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form
$A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$ where the functions μ_A: X $\to[0,1]$ and ν_A: X $\to[0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{<x, \mu_A(x), \nu_A(x)>/ x\in X\}$ and $B = \{<x, \mu_B(x), \nu_B(x)>/ x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ <x, \nu_A(x), \mu_A(x)>/ x \in X \}$,
(iv) $A \cap B = \{ <x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) >/ x \in X \}$,
(v) $A \cup B = \{ <x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)>/ x\in X \}$.

For the sake of simplicity, we shall use the notation $A = <x, \mu_A, \nu_A>$ instead of $A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in\tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = <x, \mu_A, \nu_A>$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$$int(A) = \cup\{ G / G \text{ is an IFOS in X and } G \subseteq A \},$$
$$cl(A) = \cap\{ K / K \text{ is an IFCS in X and } A \subseteq K \}.$$

Definition 2.5: [4] An IFS A of an IFTS (X, τ) is an

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(i) intuitionistic fuzzy regular closed set (IFRCS in short) if A = cl(int(A)),
(ii) intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)),
(iii) intuitionistic fuzzy semiclosed set (IFSCS in short) if int(cl(A)) ⊆ A,
(iv) intuitionistic fuzzy semiopen set (IFSOS in short) if A ⊆ cl(int(A)),
(v) intuitionistic fuzzy preclosed set (IFPCS in short) if cl(int(A)) ⊆ A,
(vi) intuitionistic fuzzy preopen set (IFPOS in short) if A ⊆ int(cl(A)),
(vii) intuitionistic fuzzy α-closed set (IFαCS) if cl(int(cl(A))) ⊆ A,
(viii) intuitionistic fuzzy α-open set (IFαOS in short) if A ⊆ int(cl(int(A)))

Definition 2.6: [4] Let A = <x, μ_A, ν_A> be an IFS in X.

Then

pint(A) = ∪{G : G is an IFPOS in X and G ⊆ A},
pcl(A) = ∩{K : K is an IFPCS in X and A ⊆ K}.

Definition 2.7: [10] An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy w-closed set (IFWCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy w-open set (IFWOS in short) if A^c is an IFWCS in (X, τ).

Definition 2.8: [12] An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy semipre closed set (IFSPCS in short) if there exists an IFPCS B such that int(B) ⊆ A ⊆ B,
(ii) intuitionistic fuzzy semipre open set (IFSPOS in short) if there exists an IFPOS B such that B ⊆ A ⊆ cl(B).

Definition 2.9: [6] An IFS A is an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semipre closed set (IFGSPCS) if spcl(A) ⊆U whenever A ⊆ U and U is an IFSOS in (X, τ). An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy generalized semipre open set (IFGSPOS in short) if A^c is an IFGSPCS in X.

Definition 2.10: [6] Let A be an IFS in an IFTS (X, τ).

Then

(i) spint (A) = ∪{ G / G is an IFSPOS in X and G ⊆A },
(ii) spcl (A) = ∩{ K / K is an IFSPCS in X and A ⊆K }.

Note that for any IFS A in (X, τ), we have spcl(A^c) = $(\text{spint}(A))^c$ and spint(A^c) = $(\text{spcl}(A))^c$.

Definition 2.11: Let A be an IFS in an IFTS (X, τ). Then
(i) intuitionistic fuzzy generalized pre regular closed set (IFGPRCS for short) if pcl(A) ⊆ U whenever A ⊆ U and U is an intuitionistic fuzzy regular open in X [9],
(ii) intuitionistic fuzzy generalized pre closed set (IFGPCS for short) if pcl(A) ⊆ U whenever A ⊆ U and U is an intuitionistic fuzzy open in X [5].

An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy generalized pre regular open set and intuitionistic fuzzy generalized pre open set (IFGPROS and IFGPOS in short) if the complement A^c is an IFGPRCS and IFGPCS respectively.

Definition 2.12: [11] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semipre regular closed set (IFGSPRCS for short) if spcl(A) ⊆ U whenever A ⊆ U and U is an IFROS in (X, τ).

Definition 2.13: [11] The complement A^c of an IFGSPRCS A in an IFTS (X, τ) is called an intuitionistic fuzzy generalized semipre regular open set (IFGSPROS for short) in X.

Definition 2.14: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $T_{1/2}$ space (IF$T_{1/2}$ space for short) if every intuitionistic fuzzy generalized closed set in X is an intuitionistic fuzzy closed set in X.

Definition 2.15: [4] Two IFSs A and B are said to be q-coincident (A $_q$ B in short) if and only if there exists and element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.16: [4] Two IFSs are said to be not q-coincident (A $_q{}^c$ B in short) if and only if A ⊆ B^c.

Definition 2.17: [7] Let Let α, β ∈ [0, 1] and α + β ≤ 1.

An intuitionistic fuzzy point (IFP for short) $p_{(\alpha, \beta)}$ of X is an IFS of X defined by

$$p_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if} \quad y = p \\ (0, 1) & \text{if} \quad y \neq p \end{cases}$$

3. INTUITIONISTIC FUZZY GENERALIZED PRESEMI CLOSED SETS

In this section we have introduced intuitionistic fuzzy generalized presemi closed sets and have studied some of its properties.

Definition 3.1: An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized presemi closed set (IFGPSCS for short) if pcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ). The family of all IFGPSCSs of an IFTS(X, τ) is denoted by IFGPSC(X).
For the sake of simplicity, we shall use the notation A = <x, (μ, μ), (ν, ν)> instead of A = <x,(a/μ_a, b/μ_b), (a/ν_a, b/ν_b)> in all the examples used in this paper.

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Example 3.2: Let X= {a, b} and G = <x, (0.5, 0.4), (0.5, 0.6)>**.** Then τ = {$0_\sim$, G, $1_\sim$} is an IFT on X and the IFS A = <x, (0.4, 0.2), (0.6, 0.7)> is an IFGPSCS in (X, τ).

Theorem 3.3: Every IFCS in (X, τ) is an IFGPSCS in (X, τ) but not conversely.

Proof: Let A be an IFCS. Let A ⊆ U and U be an IFSOS in (X, τ). Then pcl(A) ⊆ cl(A) = A ⊆ U, by hypothesis. Hence A is an IFGPSCS in (X, τ).

Example 3.4: In Example 3.2., the IFS A = <x, (0.4, 0.2), (0.6, 0.7)> is an IFGPSCS but not an IFCS in (X, τ).

Theorem 3.5: Every IFRCS in (X, τ) is an IFGPSCS in (X, τ) but not conversely.

Proof: Let A be an IFRCS in (X, τ). Since every IFRCS is an IFCS, by Theorem 3.3., A is an IFGPSCS in (X, τ).

Example 3.6: In Example 3.2., the IFS A= <x, (0.4, 0.2), (0.6, 0.7)> is an IFGPSCS but not an IFRCS in (X, τ).

Theorem 3.7: Every IFαCS in (X, τ) is an IFGPSCS in (X, τ) but not conversely.

Proof: Let A be an IFαCS. Let A ⊆ U and U be an IFSOS in (X, τ). Then pcl(A) ⊆ αcl(A) = A ⊆ U, by hypothesis. Hence A is an IFGPSCS in (X, τ).

Example 3.8: In Example 3.2., the IFS A = <x, (0.4, 0.2), (0.6, 0.7)> is an IFGPSCS but not an IFαCS in (X, τ).

Theorem 3.9: Every IFPCS in (X, τ) is an IFGPSCS in (X, τ) but not conversely.

Proof: Let A be an IFPCS in (X, τ) and let A ⊆ U, U be an IFSOS in (X, τ). Then pcl(A) = A ⊆ U, by hypothesis. Hence A is an IFGPSCS in (X, τ).

Example 3.10: Let X= {a, b} and G_1= <x, (0.2, 0.1), (0.8, 0.9)>, G_2= <x, (0.5, 0.6), (0.5, 0.4)>. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} is an IFT on X and the IFS A = <x, (0.9, 0.6), (0.1, 0.4)> is an IFGPSCS in (X, τ) but not an IFPCS in (X, τ).

Theorem 3.11: Every IFWCS in (X, τ) is an IFGPSCS in (X, τ) but not conversely.

Proof: Let A be an IFWCS. Let A ⊆ U and U be an IFSOS in (X, τ). Then pcl(A) ⊆ cl(A) ⊆ U, by hypothesis. Hence A is an IFGPSCS in (X, τ).

Example 3.12: In Example 3.2., the IFS A = <x, (0.4, 0.2), (0.6, 0.7)> is an IFGPSCS but not an IFWCS in (X, τ).

Theorem 3.13: Every IFGPSCS in (X, τ) is an IFGPCS in (X, τ) but not conversely.

Proof: Let A be an IFGPSCS in (X, τ). Let A ⊆ U and U be an IFOS in (X, τ). Since every IFOS in (X, τ) is an IFSOS in (X, τ). Then pcl(A)⊆U. Hence A is an IFGPCS in (X, τ).

Example 3.14: Let X = {a, b} and G = <x, (0.3, 0.2), (0.7, 0.8)>. Then τ = {$0_\sim$, G, $1_\sim$} is an IFT on X and the IFS A = <x, (0.5, 0.4), (0.5, 0.6)> is an IFGPCS in (X, τ) but not an IFGPSCS in (X, τ).

Theorem 3.15: Every IFGPSCS in (X, τ) is an IFGPRCS in (X, τ) but not conversely.

Proof: Let A be an IFGPSCS and A ⊆ U, U be an IFROS in (X, τ). Since every IFROS in (X, τ) is an IFSOS in (X, τ). Then pcl(A) ⊆ U, by hypothesis. Hence A is an IFGPRCS in (X, τ).

Example 3.16: In Example 3.14., the IFS A = <x, (0.5, 0.4), (0.5, 0.6)> is an IFGPRCS but not an IFGPSCS in (X, τ).

Theorem 3.17: Every IFGPSCS in (X, τ) is an IFGSPCS in (X, τ) but not conversely.

Proof: Let A be an IFGPSCS and A ⊆ U, U be an IFSOS in (X, τ). Since spcl(A) ⊆ pcl(A) ⊆ U. Hence A is an IFGSPCS in (X, τ).

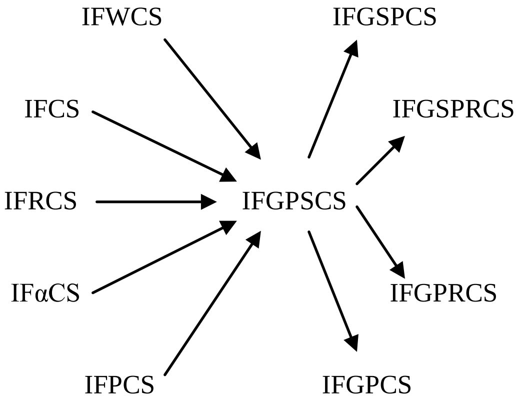
Example 3.18: In Example 3.14., the IFS A = <x, (0.5, 0.4), (0.5, 0.6)> is an IFGSPCS but not an IFGPSCS in (X, τ).

Theorem 3.19: Every IFGPSCS in (X, τ) is an IFGSPRCS in (X, τ) but not conversely.

Proof: Let A be an IFGPSCS and A ⊆ U, U be an IFROS in (X, τ). Since every IFROS is an IFSOS in (X, τ). Then spcl(A) ⊆ pcl(A)⊆ U, by hypothesis. Hence A is an IFGSPRCS in (X, τ).

Example 3.20: In Example 3.14., the IFS A = <x, (0.5, 0.4), (0.5, 0.6)> is an IFGSPRCS but not an IFGPSCS in (X, τ).

In the following diagram, we have provided the relation between various types of intuitionistic fuzzy closedness.



In this diagram by "A → B" we mean A implies B but not conversely.

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Remark 3.21: The union of any two IFGPSCSs in (X, τ) is not an IFGPSCS in (X, τ) in general as seen from the following example.

Example 3.22: Let X= {a, b} and τ = {0~, G_1, 1~} where G_1= <x, (0.3, 0.2), (0.7, 0.8)>**.** Then the IFSs A = <x, (0.2, 0.4), (0.8, 0.6)> and B = <x, (0.5, 0.1), (0.5, 0.9)> are IFGPSCSs in (X, τ) but A∪B = <x, (0.5, 0.4), (0.5, 0.6)> is not an IFGPSCS in (X, τ). Let U= <x, (0.5, 0.4), (0.5, 0.6)> be an IFSOS in (X, τ). Since A∪B ⊆U but pcl(A∪B) = <x, (0.7, 0.8), (0.3, 0.2)> ⊈ U.

Remark 3.23: The intersection of any two IFGPSCSs in (X, τ) is not an IFGPSCS in (X, τ) in general as seen from the following example.

Example 3.24: Let X= {a, b} and let τ = {0~, G_1, 1~} where G_1= <x, (0.3, 0.2), (0.7, 0.8)>**.** Then the IFSs A = <x, (0.5, 0.9), (0.5, 0.1)> and B= <x, (0.8, 0.4), (0.2, 0.6)> are IFGPSCSs in (X, τ) but A ∩ B = <x, (0.5, 0.4), (0.5, 0.6)> is not an IFGPSCS in (X, τ). Let U = <x, (0.5, 0.4), (0.5, 0.6)> be an IFSOS in (X, τ). Since A∩B ⊆ U but pcl(A∩B) = <x, (0.7, 0.8), (0.3, 0.2)> ⊈ U.

Theorem 3.25: Let (X, τ) be an IFTS. Then for every A ∈ IFGPSC(X) and for every IFS B ∈ IFS(X), A ⊆ B ⊆ pcl(A) implies B ∈ IFGPSC(X).

Proof: Let B ⊆ U and U is an IFSOS in (X, τ). Then since A ⊆ B, A ⊆ U. Since A is an IFGPSCS, it follows that pcl(A) ⊆ U. Now B ⊆ pcl(A) implies pcl(B) ⊆ pcl(pcl(A)) = pcl(A). Thus pcl(B) ⊆ U. This proves that B ∈ IFGPSC(X).

Theorem 3.26: If A is an IFSOS and an IFGPSCS in (X, τ), then A is an IFPCS in(X, τ).

Proof: Since A ⊆ A and A is an IFSOS in (X, τ), by hypothesis, pcl(A) ⊆ A. But since A ⊆ pcl(A). Therefore pcl(A) = A. Hence A is an IFPCS in (X, τ).

Theorem 3.27: Let A be an IFGPSCS in (X, τ) then pcl(A) −A does not contain any non empty IFSCS.

Proof: Let F be an IFSCS such that F ⊆ pcl(A) − A. Then F ⊆ X−A implies A⊆X−F. Since A is an IFGPSCS and X − F is an IFSOS. Therefore pcl(A) ⊆ X−F. That is F ⊆ X− pcl(A). Hence F ⊆ pcl(A) ∩ (X− pcl(A)) = φ. This shows F = φ, which is a contradiction.

Theorem 3.28: Let A be an IFGPSCS in (X, τ). Then A is an IFPCS if and only if pcl(A) − A is an IFSCS.

Proof: Necessity: Let A be an IFGPSCS. Assume that A is an IFPCS. Then pcl(A) = A and so pcl(A) − A= φ. Hence pcl(A) − A is an IFSCS.
Sufficiency: Assume that pcl(A) − A is an IFSCS. Then pcl(A) −A = φ since A is an IFGPSCS. That is pcl(A) = A. Hence A is an IFPCS.

Theorem 3.29: If A is an IFGPSCS then scl({x})∩ A ≠ φ, for each x ∈ pcl(A).

Poof: Let x ∈ pcl(A). If scl({x})∩A = φ, then A ⊆ $scl(\{x\})^c$. So pcl(A) ⊆ $scl(\{x\})^c$. Since x ∈ pcl(A), which implies x ∈ $scl(\{x\})^c$ which is a contradiction. Hence scl({x}) ∩A ≠ φ.

Theorem 3.30: Let A be a subset of X. If scl({x}) ∩ A ≠ φ, for each x ∈scl(A) then scl(A) – A contains no non empty IFSCS.

Poof: Let F be an IFSCS such that F ⊆ scl(A) – A. If there exists a x ∈ F, then φ ≠ scl({x}) ∩ A ⊆ F∩A ⊆ scl(A) – A, a contradiction. Hence F = φ.

Theorem 3.31: For every x ∈ X, X – {x} is an IFGPSCS or IFSOS.

Proof: Suppose X – {x} is not an IFSOS. Then X is the only IFSOS containing X – {x}. This implies pcl(X – {x}) ⊆X. Hence X – {x} is an IFGPSCS in X.

Theorem 3.32: Let (X, τ) be an IFTS and A is an IFS of X. Then A is an IFGPSCS if and only if $(A_q{}^cF)$ ⇒ $(pcl(A)_q{}^cF)$ for every IFSCS F of X.

Proof: Necessity: Let F be an IFSCS of X and $(A_q{}^cF)$. Then A ⊆ F^c and F^c is an IFSOS in X. Therefore pcl(A) ⊆ F^c because A is an IFGPSCS. Hence $(pcl(A)\,_q{}^cF)$.
Sufficiency: Let O be an IFSOS of X such that A ⊆ O i.e. A $\subseteq ((O)^c)^c$ then $(A_q{}^cO^c)$ and O^c is an IFSCS in X. Hence by hypothesis $(pcl(A)_q{}^cO^c)$. Therefore pcl(A) ⊆ $((O)^c)^c$. i .e. pcl (A) ⊆ O. Hence A is an IFGPSCS in X.

Theorem 3.33: Let A be an an IFGPSCS in an intuitionistic fuzzy topological space (X, τ) and $p_{(\alpha,\beta)}$ be an intuitionistic fuzzy point of X such that $p_{(\alpha,\beta)}\,_q$pcl(A) then $pcl(p_{(\alpha,\beta)})_q$A.

Proof: If $pcl(p_{(\alpha,\beta)})_q{}^cA$ then $pcl(p_{(\alpha,\beta)})$ ⊆A^c which implies that A ⊆ $(pcl(p_{(\alpha,\beta)}))^c$ and so pcl(A) ⊆ $(pcl(p_{(\alpha,\beta)}))^c$ ⊆ $(p_{(\alpha,\beta)})^c$, because A is an IFGPSCS in X. Hence $(p(\alpha,\beta)_q{}^c(pcl(A)))$, a contradiction.

4. INTUITIONISTIC FUZZY GENERALIZED PRESEMI OPEN SETS

In this section we have introduced intuitionistic fuzzy generalized presemi open sets and have studied some of its properties.
Definition 4.1: An IFS A is said to be an intuitionistic fuzzy generalized presemi open set (IFGPSOS for short) in (X, τ) if the complement A^c is an IFGPSCS in X.
The family of all IFGPSOSs of an IFTS (X, τ) is denoted by IFGPSO(X).

Theorem 4.2: Every IFOS, IFROS, IFαOS, IFPOS, IFWOS are an IFGPSOS and every IFGPSOS is an IFGPOS, IFGPROS, IFGSPOS, IFGSPROS but the converses are not true in general.

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Proof: Straight forward. The following examples show that the converses are not true in general.

Example 4.3: Let X= {a, b} and G = <x, (0.5, 0.4), (0.5, 0.6)>. Then $\tau = \{0_\sim, G, 1_\sim\}$ is an IFT on X and the IFS A = <x, (0.6, 0.7),(0.4, 0.2)> is an IFGPSOS in (X, τ) but not an IFOS, IFROS, IFαOS, IFWOS in (X, τ).

Example 4.4: Let X = {a, b} and G_1 = <x, (0.2, 0.1), (0.8, 0.9)>, G_2 = <x, (0.5, 0.6), (0.5, 0.4)>. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on X and the IFS A = <x, (0.1, 0.4),(0.9, 0.6)> is an IFGPSOS in (X, τ) but not an IFPOS in (X, τ).

Example 4.5: Let X = {a, b} and G = <x, (0.3, 0.2), (0.7, 0.8)>. Then $\tau = \{0_\sim, G, 1_\sim\}$ is an IFT on X and the IFS A = <x, (0.5, 0.6), (0.5, 0.4)> is an IFGPOS, IFGPROS, IFGSPOS, IFGSPROS in (X, τ) but not an IFGPSOS in (X, τ).

Theorem 4.6: An IFS A of an IFTS (X, τ) is an IFGPSOS if F ⊆ pint (A) whenever F is an IFSCS and F ⊆ A.

Proof: Follows from definition 3.1., and definition 2.15.

Theorem 4.7: Let A be an IFGPSOS of an IFTS (X, τ) and pint(A) ⊆ B ⊆ A. Then B is an IFGPSOS.

Proof: Suppose A is an IFGPSOS in X and pint(A) ⊆ B ⊆ A. $\Rightarrow A^c \subseteq B^c \subseteq (\text{pint}(A))^c \Rightarrow A^c \subseteq B^c \subseteq \text{pcl}(A^c)$ and A^c is an IFGPSCS it follows from theorem 3.25, that B^c is an IFGPSCS. Hence B is an IFGPSOS.

Theorem 4.8: An IFS A of an IFTS (X, τ) is an IFGPSOS in (X, τ) if and only if F ⊆ pint(A) whenever F is an IFSCS in (X, τ) and F ⊆ A.

Proof: Necessity: Suppose A is an IFGPSOS in (X, τ). Let F be an IFSCS in (X, τ) such that F ⊆ A. Then F^c is an IFSOS and $A^c \subseteq F^c$. By hypothesis A^c is an IFGPSCS in (X, τ), we have $\text{pcl}(A^c) \subseteq F^c$. Therefore F ⊆ pint (A).
Sufficiency: Let U be an IFSOS in (X, τ) such that $A^c \subseteq$ U. By hypothesis, $U^c \subseteq$ pint(A). Therefore $\text{pcl}(A^c) \subseteq$ U and A^c is an IFGPSCS in (X, τ). Hence A is an IFGPSOS in (X, τ).

5. APPLICATIONS OF INTUITIONISTIC FUZZY GENERALIZED PRESEMI CLOSED SETS

In this section we have provided some applications of intuitionistic fuzzy generalized presemi closed sets in intuitionistic fuzzy topological spaces.

Definition 5.1: If every IFGPSCS in (X, τ) is an IFPCS in (X, τ), then the space can be called as an intuitionistic fuzzy presemi $T_{1/2}$ ($IFPST_{1/2}$ space for short) space.

Theorem 5.2: An IFTS (X, τ) is an $IFPST_{1/2}$ space if and only if IFPOS(X) = IFGPSO(X).

Proof: Necessity: Let (X, τ) be an $IFPST_{1/2}$ space. Let A be an IFGPSOS in (X, τ). By hypothesis, A^c is an IFGPSCS in (X, τ) and therefore A is an IFPOS in (X, τ). Hence IFPO(X) = IFGPSO(X).
Sufficiency: Let IFPO(X, τ) = IFGPSO(X, τ). Let A be an IFGPSCS in (X, τ). Then A^c is an IFPOS in (X, τ). By hypothesis, A^c is an IFPOS in (X, τ) and therefore A is an IFPCS in (X, τ). Hence (X, τ) is an $IFPST_{1/2}$ space.

Theorem 5.3: For any IFS A in (X, τ) where X is an $IFPST_{1/2}$ space, A ∈ IFGPSO(X) if and only if for every IFP $p_{(\alpha, \beta)}$ ∈ A, there exists an IFGPSOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Proof: Necessity: If A ∈ IFGPSO(X), then we can take B = A so that $p_{(\alpha, \beta)} \in B \subseteq A$ for every IFP $p_{(\alpha, \beta)} \in A$.
Sufficiency: Let A be an IFS in (X, τ) and assume that there exists B ∈ IFGPSO(X) such that $p_{(\alpha,\beta)} \in B \subseteq A$. Since X is an $IFPST_{1/2}$ space, B is an IFPOS. Then $A = \bigcup_{p(\alpha,\beta) \in A} \{p(\alpha, \beta)\} \subseteq \bigcup_{p(\alpha,\beta) \in A} B \subseteq A$. Therefore $A = \bigcup_{p(\alpha,\beta) \in A} B$, which is an IFPOS. Hence by Theorem 5.2., A is an IFGPSOS in (X, τ).

Definition 5.4: An IFTS (X, τ) is said to be an intuitionistic fuzzy presemi $T^*_{1/2}$ space ($IFPST^*_{1/2}$ space for short) if every IFGPSCS is an IFCS in (X, τ).

Theorem 5.5: Let A ∈ $IFPST^*_{1/2}$ space then A ∈ $IFPST_{1/2}$ space.

Proof: Let A be an IFGPSCS in (X, τ). Assume that A ∈$IFPST^*_{1/2}$ space, which implies A is an IFCS in (X, τ). Since every IFCS is an IFPCS implies that A is an IFPCS in (X, τ). Hence A ∈ $IFPST_{1/2}$.

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