On Generalized PreSemi Closed Sets in Intuitionistic Fuzzy Topological Spaces

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ABSTRACT

In this paper, we introduce the notion of an intuitionistic fuzzy generalized presemi closed sets and intuitionistic fuzzy generalized presemi open sets in intuitionistic fuzzy topological spaces. Also we have provide some applications of intuitionistic fuzzy generalized presemi closed sets.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy generalized presemi closed sets, intuitionistic fuzzy generalized presemi open sets, intuitionistic fuzzy points, IPST₁/₀ space and IPFST₁/₀ space.

2010 Mathematics Subject Classification : 54A40, 03F55.

1. INTRODUCTION

In 1965, Zadeh [13] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of these notions. In 1986, the notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce intuitionistic fuzzy generalized presemi closed sets and intuitionistic fuzzy generalized presemi open sets in intuitionistic fuzzy topological space. We study some of its properties in intuitionistic fuzzy topological spaces.

2. PRELIMINARIES

Throughout this paper, (X, τ) or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A' respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1: [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form
A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\} where the functions \mu_A: X \rightarrow [0,1] and \nu_A: X \rightarrow [0,1] denote the degree of membership (namely \mu_A(x)) and the degree of non-membership (namely \nu_A(x)) of each element x \in X to the set A, respectively, and 0 \leq \mu_A(x) + \nu_A(x) \leq 1 for each x \in X. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\} and B = \{<x, \mu_B(x), \nu_B(x)>/ x \in X\}. Then

(i) A \subseteq B if and only if \mu_A(x) \leq \mu_B(x) and \nu_A(x) \geq \nu_B(x) for all x \in X,
(ii) A = B if and only if A \subseteq B and B \subseteq A,
(iii) A' = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\},
(iv) A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)>/ x \in X\},
(v) A \cup B = \{<x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)>/ x \in X\}.

For the sake of simplicity, we shall use the notation A = <x, \mu_A, \nu_A> instead of A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family \tau of IFSs in X satisfying the following axioms:

(i) 0, 1 \in \tau,
(ii) G_1 \cap G_2 \in \tau, for any G_1, G_2 \in \tau,
(iii) \bigcup G_i \in \tau for any family \{G_i/ i \in J\} \subseteq \tau.

In this case the pair (X, \tau) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \tau is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A' of an IFOS A in an IFTS (X, \tau) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, \tau) be an IFTS and A = <x, \mu_A, \nu_A> be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

int(A) = \bigcup \{ G / G is an IFOS in X and G \subseteq A \},
cl(A) = \bigcap \{ K / K is an IFCS in X and A \subseteq K \}.

Definition 2.5: [4] An IFS A of an IFTS (X, \tau) is an
generalized fuzzy regular closed set (IFRCS in short) if $A = \text{cl} (\text{int} (A))$,

(ii) intuitionistic fuzzy regular open set (IFROS in short) if $A = \text{int} (\text{cl} (A))$,

(iii) intuitionistic fuzzy semiclosed set (IFSCS in short) if $\text{int} (\text{cl} (A)) \subseteq A$,

(iv) intuitionistic fuzzy semipreopen set (IFSPOS in short) if $A \subseteq \text{cl} (\text{int} (A))$,

(v) intuitionistic fuzzy preclosed set (IFPCS in short) if $\text{cl} (\text{int} (A)) \subseteq A$,

(vi) intuitionistic fuzzy $\alpha$-closed set (IFaCS) if $\text{cl} (\text{int} (\text{cl} (A))) \subseteq A$,

(vii) intuitionistic fuzzy $\alpha$-open set (IFaOS in short) if $A \subseteq \text{int} (\text{int} (\text{int} (A)))$.

Definition 2.6: [4] Let $A = < x, \mu_A, \nu_A >$ be an IFS in $X$.

Then

$$\text{pint}(A) = \bigcup \{G : G \text{ is an IFPOS in } X \text{ and } G \subseteq A\},$$

$$\text{pcl}(A) = \bigcap \{K : K \text{ is an IFPCS in } X \text{ and } A \subseteq K\}.$$ 

Definition 2.7: [10] An IFS $A$ of an IFTS $(X, \tau)$ is called an intuitionistic fuzzy w-closed set (IFWCS in short) if $\text{cl} (A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFGS in $(X, \tau)$.

An IFS $A$ of an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy w-open set (IFWOS in short) if $\text{int}(A)$ is an IFWCS in $(X, \tau)$.

Definition 2.8: [12] An IFS $A$ of an IFTS $(X, \tau)$ is an intuitionistic fuzzy semipreopen set (IFaOS in short) if there exists an IFPOS $B$ such that $B \subseteq A \subseteq \text{cl}(B)$.

An intuitionistic fuzzy $\alpha$-closed set (IFaCS) is $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

An intuitionistic fuzzy $\alpha$-open set (IFaOS in short) if $A \subseteq \text{int} (\text{int} (\text{int} (A)))$.

Definition 2.9: [6] An IFS $A$ is an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized semipreopen set (IFGSPCS in short) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFGS in $(X, \tau)$. An IFS $A$ of an IFTS $(X, \tau)$ is called an intuitionistic fuzzy generalized semipreopen set (IFGSPCS in short) if $A^c$ is an IFGPSC in $X$.

Definition 2.10: [6] Let $A$ be an IFS in an IFTS $(X, \tau)$.

(i) intuitionistic fuzzy semipreopen set (IFaOS in short) if there exists an IFPOS $B$ such that $B \subseteq A \subseteq \text{cl}(B)$.

(ii) intuitionistic fuzzy $\alpha$-closed set (IFaCS) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

Note that for any IFS $A$ in $(X, \tau)$, we have $\text{spcl}(A^c) = (\text{spint}(A))^c$ and $\text{spint}(A^c) = (\text{spcl}(A))^c$.

Definition 2.11: Let $A$ be an IFS in an IFTS $(X, \tau)$. Then

(i) intuitionistic fuzzy generalized pre regular closed set (IFGPRCS for short) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an intuitionistic fuzzy regular open in $X$.

(ii) intuitionistic fuzzy generalized pre closed set (IFGPFCs for short) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an intuitionistic fuzzy open in $X$.

An IFS $A$ of an IFTS $(X, \tau)$ is called an intuitionistic fuzzy generalized pre regular open set and intuitionistic fuzzy generalized pre open set (IFGPFCs and IFGPPOS in short) if the complement $A^c$ is an IFGPRCS and IFGPFCs respectively.

Definition 2.12: [11] An IFS $A$ in an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized semipreopen set (IFGSPCS in short) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFROS in $(X, \tau)$.


Definition 2.14: [8] An IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $T_{1/2}$ space (IFT$_{1/2}$ space for short) if every intuitionistic fuzzy generalized closed set in $X$ is an intuitionistic fuzzy closed set in $X$.

Definition 2.15: [4] Two IFSs $A$ and $B$ are said to be $q$-coincident ($A \sim B$ in short) if and only if there exists and element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.16: [4] Two IFSs are said to be not $q$-coincident ($A \nsim B$ in short) if and only if $A \nsim B$.

Definition 2.17: [7] Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$.

An intuitionistic fuzzy point (IFP for short) $p_{\alpha, \beta}$ of $X$ is an IFS of $X$ defined by

$$p_{\alpha, \beta}(y) = \begin{cases} (\alpha, \beta) & \text{if } y = p \\ (0, 1) & \text{if } y \neq p \end{cases}$$

3. INTUITIONISTIC FUZZY GENERALIZED PRESEMI CLOSED SETS

In this section we have introduced intuitionistic fuzzy generalized presemi closed sets and have studied some of its properties.

Definition 3.1: An IFS $A$ in an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized presemi closed set (IFGPSCS for short) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFGS in $(X, \tau)$. The family of all IFGSCSs of an IFTS$(X, \tau)$ is denoted by IFGPSC(X).

For the sake of simplicity, we shall use the notation $A = <x, (\mu, \nu), (v, w)>$ instead of $A = <x, (\mu_a, b/\mu_b), (a/v_a, b/v_b)>$ in all the examples used in this paper.
Example 3.2: Let $X=\{a, b\}$ and $G=\langle x, (0.5, 0.4), (0.5, 0.6)\rangle$. Then $\tau = \{0., G, 1.\}$ is an IFT on $X$ and the IFS $A = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFGPSCS in $(X, \tau)$.

**Theorem 3.3:** Every IFCS in $(X, \tau)$ is an IFGPSCS in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFCS. Let $A \subseteq U$ and $U$ be an IFSOS in $(X, \tau)$. Then $pcl(A) \subseteq cl(A) = A \subseteq U$, by hypothesis. Hence $A$ is an IFGPSCS in $(X, \tau)$.

**Example 3.4:** In Example 3.2., the IFS $A = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFGPSCS but not an IFCS in $(X, \tau)$.

**Theorem 3.5:** Every IFRCS in $(X, \tau)$ is an IFGPSCS in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFRCS in $(X, \tau)$. Since every IFRCS is an IFCS, by Theorem 3.3., $A$ is an IFGPSCS in $(X, \tau)$.

**Example 3.6:** In Example 3.2., the IFS $A = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFGPSCS but not an IFRCS in $(X, \tau)$.

**Theorem 3.7:** Every IFaCS in $(X, \tau)$ is an IFGPSCS in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFaCS. Let $A \subseteq U$ and $U$ be an IFSOS in $(X, \tau)$. Then $pcl(A) \subseteq acl(A) = A \subseteq U$, by hypothesis. Hence $A$ is an IFGPSCS in $(X, \tau)$.

**Example 3.8:** In Example 3.2., the IFS $A = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFGPSCS but not an IFaCS in $(X, \tau)$.

**Theorem 3.9:** Every IFPCS in $(X, \tau)$ is an IFGPSCS in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFPCS in $(X, \tau)$ and let $A \subseteq U$, $U$ be an IFSOS in $(X, \tau)$. Then $pcl(A) = A \subseteq U$, by hypothesis. Hence $A$ is an IFGPSCS in $(X, \tau)$.

**Example 3.10:** Let $X=\{a, b\}$ and $G_1=\langle x, (0.2, 0.1), (0.8, 0.9)\rangle$, $G_2=\langle x, (0.5, 0.6), (0.5, 0.4)\rangle$. Then $\tau = \{0., G_1, G_2, 1.\}$ is an IFT on $X$ and the IFS $A = \langle x, (0.9, 0.6), (0.1, 0.4)\rangle$ is an IFGPSCS in $(X, \tau)$ but not an IFPCS in $(X, \tau)$.

**Theorem 3.11:** Every IFWCS in $(X, \tau)$ is an IFGPSCS in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFWCS. Let $A \subseteq U$ and $U$ be an IFSOS in $(X, \tau)$. Then $pcl(A) \subseteq cl(A) = A \subseteq U$, by hypothesis. Hence $A$ is an IFGPSCS in $(X, \tau)$.

**Example 3.12:** In Example 3.2., the IFS $A = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFGPSCS but not an IFWCS in $(X, \tau)$.

**Theorem 3.13:** Every IFGPSCS in $(X, \tau)$ is an IFGPCS in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFGPSCS in $(X, \tau)$. Let $A \subseteq U$ and $U$ be an IFSOS in $(X, \tau)$. Then $pcl(A) \subseteq U$, by hypothesis. Hence $A$ is an IFGPCS in $(X, \tau)$.

**Example 3.14:** Let $X=\{a, b\}$ and $G=\langle x, (0.3, 0.2), (0.7, 0.8)\rangle$. Then $\tau = \{0., G, 1.\}$ is an IFT on $X$ and the IFS $A = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ is an IFGPCS in $(X, \tau)$ but not an IFGPSCS in $(X, \tau)$.

**Theorem 3.15:** Every IFGPSCS in $(X, \tau)$ is an IFGPRCS in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFGPSCS and $A \subseteq U$, $U$ be an IFRCS in $(X, \tau)$. Since every IFRCS is an IFOS in $(X, \tau)$. Then $pcl(A) \subseteq U$, by hypothesis. Hence $A$ is an IFGPRCS in $(X, \tau)$.

**Example 3.16:** In Example 3.14., the IFS $A = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ is an IFGPRCS but not an IFGPSCS in $(X, \tau)$.

**Theorem 3.17:** Every IFGPSCS in $(X, \tau)$ is an IFGSPCS in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFGPSCS and $A \subseteq U$, $U$ be an IFOS in $(X, \tau)$. Since $spcl(A) \subseteq pcl(A) \subseteq U$, by hypothesis. Hence $A$ is an IFGSPCS in $(X, \tau)$.

**Example 3.18:** In Example 3.14., the IFS $A = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ is an IFGSPCS but not an IFGPSCS in $(X, \tau)$.

**Theorem 3.19:** Every IFGSPCS in $(X, \tau)$ is an IFGSPRC in $(X, \tau)$ but not conversely.

**Proof:** Let $A$ be an IFGSPCS and $A \subseteq U$, $U$ be an IFROS in $(X, \tau)$. Since every IFROS is an IFOS in $(X, \tau)$. Then $spcl(A) \subseteq pcl(A) \subseteq U$, by hypothesis. Hence $A$ is an IFGSPRC in $(X, \tau)$.

**Example 3.20:** In Example 3.14., the IFS $A = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ is an IFGSPRC but not an IFGSPCS in $(X, \tau)$.

In the following diagram, we have provided the relation between various types of intuitionistic fuzzy closedness.

![Diagram](https://example.com/diagram.png)

In this diagram by “A → B” we mean A implies B but not conversely.
Remark 3.23: The intersection of any two IFGSCSs in (X, τ) is not an IFGSCS in (X, τ) in general as seen from the following example.

Example 3.24: Let X= {a, b} and let τ = {0., G1, 1.} where G1= <x, (0.3, 0.2), (0.7, 0.8)> Then the IFSSs A = <x, (0.5, 0.9), (0.5, 0.1)> and B = <x, (0.5, 0.1), (0.5, 0.9)> are IFGSCSs in (X, τ) but A ∪ B = <x, (0.5, 0.4), (0.5, 0.6)> is not an IFGSCS in (X, τ). Let U = <x, (0.5, 0.4), (0.5, 0.6)> be an IFOS in (X, τ). Since A ∪ B ⊆ U but pcl(A ∪ B) = <x, (0.7, 0.8), (0.3, 0.2)> ⊄ U.

Theorem 3.25: Let (X, τ) be an IFTS. Then for every A ∈ IFGSCS(X) and for every IFS B ∈ IFSCS(X), A ⊆ B ⊆ pcl(A) implies B ⊆ IFGSCS(X).

Proof: Let B ⊆ U and U is an IFOS in (X, τ). Then since A ⊆ B, A ⊆ U. Since A is an IFGSCS, it follows that pcl(A) ⊆ U. Now B ⊆ pcl(A) implies pcl(B) ⊆ pcl(pcl(A)) = pcl(A). Thus pcl(B) ⊆ U. This proves that B ⊆ IFGSCS(X).

Theorem 3.26: If A is an IFOS and an IFGSCS in (X, τ), then A is an IFPCS in (X, τ).

Proof: Since A ⊆ A and A is an IFOS in (X, τ), by hypothesis, pcl(A) ⊆ A. But since A ⊆ pcl(A). Therefore pcl(A) = A. Hence A is an IFPCS in (X, τ).

Theorem 3.27: Let A be an IFGSCS in (X, τ) then pcl(A) − A does not contain any non empty IFSCS.

Proof: Let F be an IFSCS such that F ⊆ pcl(A) − A. Then F ⊆ X−A implies A ⊆ X−F. Since A is an IFGSCS and X − F is an IFOS, therefore pcl(A) ⊆ X−F. That is F ⊆ X− pcl(A). Hence F ⊆ pcl(A) ∩ (X− pcl(A)) = φ. This shows F = φ, which is a contradiction.

Theorem 3.28: Let A be an IFGSCS in (X, τ). Then A is an IFPCS if and only if pcl(A) − A is an IFSCS.

Proof: Necessity: Let A be an IFGSCS. Assume that A is an IFPCS. Then pcl(A) = A and so pcl(A) − A = φ. Hence pcl(A) − A is an IFSCS.

Sufficiency: Assume that pcl(A) − A is an IFSCS. Then pcl(A) − A = φ since A is an IFGSCS. That is pcl(A) = A. Hence A is an IFPCS.

Theorem 3.29: If A is an IFGSCS then scl({x}) ∩ A ≠ φ, for each x ∈ pcl(A).

Proof: Let x ∈ pcl(A). If scl({x})∩A = φ, then A ⊆ scl(x). So pcl(A) ⊆ scl(x). Since x ∈ pcl(A), which implies x ∈ scl({x}) which is a contradiction. Hence scl({x}) ∩ A ≠ φ.

Theorem 3.30: Let A be a subset of X. If scl({x}) ∩ A ≠ φ, for each x ∈ scl(A) then scl(A) − A contains no non empty IFSCS.

Proof: Let F be an IFSCS such that F ⊆ scl(A) − A. If there exists a x ∈ F, then φ ≠ scl({x}) ∩ A ⊆ F ∩ scl(A) − A, a contradiction. Hence F = φ.

Theorem 3.31: For every x ∈ X, X − {x} is an IFGSCS or IFOS.

Proof: Suppose X − {x} is not an IFSCS. Then X is the only IFSCS containing X − {x}. This implies pcl(X − {x}) ⊆ X. Hence X − {x} is an IFGSCS in X.

Theorem 3.32: Let (X, τ) be an IFTS and A is an IF of X. Then A is an IFGSCS if and only if (A ∩ F) ⇒ (pcl(A) − F) for every IFSCS F of X.

Proof: Necessity: Let F be an IFSCS of X and (A ∩ F). Then A ⊆ F and F is an IFSCS in X. Therefore pcl(A) ⊆ F because A is an IFGSCS. Hence (pcl(A) − F).

Sufficiency: Let O be an IFSCS of O such that A ⊆ O i.e. A ⊆ (O) − O then (A ∩ O) − O is an IFSCS in X. Hence by hypothesis pcl(A∩O) − O. Therefore pcl(A) ⊆ ((O) − O). i.e. pcl(A) ⊆ O. Hence A is an IFGSCS in X.

Theorem 3.33: Let A be an an IFGSCS in an intuitionistic fuzzy topological space (X, τ) and p_{α,β} be an intuitionistic fuzzy point of X such that p_{α,β} ⊆ pcl(A) then pcl(p_{α,β}) ⊆ A.

Proof: If pcl(p_{α,β}) ⊆ A then pcl(p_{α,β}) ⊆ A which implies that A ⊆ (pcl(p_{α,β})) and so pcl(A) ⊆ (pcl(p_{α,β})) which implies (pcl(p_{α,β})) ⊆ pcl(A).

4. INTUITIONISTIC FUZZY GENERALIZED PRESEMI OPEN SETS

In this section we have introduced intuitionistic fuzzy generalized presemi open sets and have studied some of its properties.

Definition 4.1: An IFS A is said to be an intuitionistic fuzzy generalized presemi open set (IFGPPOS) for short in (X, τ) if the complement A′ is an IFGPOS in X. The family of all IFGPOSs of an IFTS (X, τ) is denoted by IFGPOS(X).

Theorem 4.2: Every IFOS, IFROS, IFoOS, IFPOS, IFWOS are an IFGPOS and every IFGPOS is an IFGPOS. IFGPROS, IFGPOS, IFGSPROS but the converses are not true in general.
Proof: Straight forward. The following examples show that the converses are not true in general.

Example 4.3: Let X = [a, b] and G = \(<x, (0.5, 0.4), (0.5, 0.6)\>). Then \(\tau = \{0, 0.5, 1\}\) is an IFT on X and the IFS \(A = <x, (0.6, 0.7), (0.4, 0.2)\rangle\) is an IFGPSOS in (X, \(\tau\)) but not an IFOS, IFROS, IF\(\omega\)OS, IF\(\omega\)SOS in (X, \(\tau\)).

Example 4.4: Let X = [a, b] and \(G_1 = <x, (0.2, 0.1), (0.8, 0.9)\rangle, G_2 = <x, (0.5, 0.6), (0.5, 0.4)\rangle\). Then \(\tau = \{0, G_1, G_2, 1\}\) is an IFT on X and the IFS \(A = <x, (0.1, 0.4), (0.9, 0.6)\rangle\) is an IFGPSOS in (X, \(\tau\)) but not an IFPOS in (X, \(\tau\)).

Example 4.5: Let X = [a, b] and \(G = <x, (0.3, 0.2), (0.7, 0.8)\rangle\). Then \(\tau = \{0, G, 1\}\) is an IFT on X and the IFS \(A = <x, (0.5, 0.6), (0.5, 0.4)\rangle\) is an IFPOS, IFGPSOS, IFGPSROS, IFGSPROS in (X, \(\tau\)) but not an IFGPSOS in (X, \(\tau\)).

Theorem 4.6: An IFS A of an IFTS (X, \(\tau\)) is an IFGPSOS if \(F \subseteq \text{pint} (A)\) whenever \(F\) is an IFSCS and \(F \subseteq A\).

Proof: Follows from definition 3.1., and definition 2.15.

Theorem 4.7: Let A be an IFGPSOS of an IFTS (X, \(\tau\)) and \(\text{pint}(A) \subseteq B \subseteq A\). Then B is an IFGPSOS.

Proof: Suppose A is an IFGPSOS in X and \(\text{pint}(A) \subseteq B \subseteq A\). \(\Rightarrow A' \subseteq B' \subseteq (\text{pint}(A))' \Rightarrow A' \subseteq B' \subseteq \text{pcl}(A')\) and \(A'\) is an IFGPSOS it follows from theorem 3.25, that \(B'\) is an IFGPSOS. Hence B is an IFGPSOS.

Theorem 4.8: An IFS A of an IFTS (X, \(\tau\)) is an IFGPSOS in (X, \(\tau\)) if and only if \(F \subseteq \text{pint}(A)\) whenever \(F\) is an IFSCS in (X, \(\tau\)) and \(F \subseteq A\).

Proof: Necessity: Suppose A is an IFGPSOS in (X, \(\tau\)). Let \(F\) be an IFSCS in (X, \(\tau\)) such that \(F \subseteq A\). Then \(F'\) is an IFPOS and \(A' \subseteq F'\). By hypothesis \(A'\) is an IFGPSOS in (X, \(\tau\)), we have \(\text{pcl}(A') \subseteq F'\). Therefore \(F \subseteq \text{pint}(A)\).

Sufficiency: Let U be an IFPOS in (X, \(\tau\)) such that \(A' \subseteq U\). By hypothesis, \(U' \subseteq \text{pint}(A)\). Therefore \(\text{pcl}(A') \subseteq U\) and \(A'\) is an IFGPSCS in (X, \(\tau\)). Hence A is an IFGPSOS in (X, \(\tau\)).

5. APPLICATIONS OF INTUITIONISTIC FUZZY GENERALIZED PRESEMI CLOSED SETS

In this section we have provided some applications of intuitionistic fuzzy generalized presemi closed sets in intuitionistic fuzzy topological spaces.

Definition 5.1: If every IFGPSCS in (X, \(\tau\)) is an IFPCS in (X, \(\tau\)), then the space can be called as an intuitionistic fuzzy presemi \(T_{1/2}\) (IF\(PST_{1/2}\) space for short) space.

Theorem 5.2: An IFTS (X, \(\tau\)) is an IF\(PST_{1/2}\) space if and only if IFPOS(X) = IFGPSO(X).

Proof: Necessity: Let (X, \(\tau\)) be an IF\(PST_{1/2}\) space. Let A be an IFGPSOS in (X, \(\tau\)). By hypothesis, \(A'\) is an IFGPSCS in (X, \(\tau\)) and therefore A is an IFPOS in (X, \(\tau\)). Hence IFPO(X) = IFGPSO(X).

Sufficiency: Let IFPO(X, \(\tau\)) = IFGPSO(X, \(\tau\)). Let A be an IFGPSCS in (X, \(\tau\)). Then \(A'\) is an IFPOS in (X, \(\tau\)). By hypothesis, \(A'\) is an IFPOS in (X, \(\tau\)) and therefore A is an IFPCS in (X, \(\tau\)). Hence (X, \(\tau\)) is an IF\(PST_{1/2}\) space.

Theorem 5.3: For any IFS A in (X, \(\tau\)) where X is an IF\(PST_{1/2}\) space, A \(\subseteq\) IFGPSO(X) if and only if for every IFP \(p_{(\alpha, \beta)} \subseteq A\), there exists an IFGPSOS B in X such that \(p_{(\alpha, \beta)} \subseteq B \subseteq A\).

Proof: Necessity: If A \(\subseteq\) IFGPSO(X), then we can take B = A so that \(p_{(\alpha, \beta)} \subseteq B \subseteq A\) for every IFP \(p_{(\alpha, \beta)} \subseteq A\).

Sufficiency: Let A be an IF in (X, \(\tau\)) and assume that there exists B \(\subseteq\) IFGPSO(X) such that \(p_{(\alpha, \beta)} \subseteq B \subseteq A\). Since X is an IF\(PST_{1/2}\) space, B is an IFPOS. Then A = \(\bigcup p_{(\alpha, \beta)} \in A \bigcup p_{(\alpha, \beta)} \in B \subseteq A\). Therefore A = \(\bigcup p_{(\alpha, \beta)} \in A \bigcup p_{(\alpha, \beta)} \in B\), which is an IFPOS. Hence by Theorem 5.2, A is an IFGPSOS in (X, \(\tau\)).

Definition 5.4: An IFTS (X, \(\tau\)) is said to be an intuitionistic fuzzy presemi \(T^*_{1/2}\) space (IF\(PST^*_{1/2}\) space for short) if every IFGPSCS is an IFCS in (X, \(\tau\)).

Theorem 5.5: Let A \(\subseteq\) IF\(PST^*_{1/2}\) space then A \(\subseteq\) IF\(PST_{1/2}\) space.

Proof: Let A be an IFGPSCS in (X, \(\tau\)). Assume that A \(\subseteq\) IF\(PST^*_{1/2}\) space, which implies A is an IFCS in (X, \(\tau\)). Since every IFCS is an IFPCS implies that A is an IFPCS in (X, \(\tau\)). Hence A \(\subseteq\) IF\(PST_{1/2}\).

REFERENCES


