



Evaluating the Impact of Active Members on Church Growth

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ABSTRACT

Believe is an integral part of human nature. Man has always believed in something and if this believe is strong enough there is a passion to transmit it to others. A church is an organised group of people who believe in Christ. Church founders and members have a mandate to spread their faith (believe). The transmission dynamics of believe is similar to that of an epidemic disease. In this paper we modelled church growth using ideas from epidemiological modelling and evaluated the impact of active members on the growth of the church. We found that a Faith-free equilibrium state exists and that it is locally asymptotically stable. Furthermore we investigated the existence of a faithful endemic equilibrium state and numerically solved the model equations with MATLAB. From numerical simulations we established that maximum church growth is achieved in presence of effective active members who continue to reproduce themselves, that is, groom passive members into becoming active.

Keywords: Church growth, Faith, Faith-free equilibrium, Mathematical modelling, Numerical simulation, Stability.

1. INTRODUCTION

Although synagogues and similar places of worship exists before Jesus, what traditionally came to be known as church today was instituted by Jesus Christ as stated in the Bible “You are Peter, upon this rock I will build my church” (Mathew 16:18). His major goal was to preach His message of salvation and make converts (People that believe and follow Him). Growth therefore has been a concern of the church right from inception. Striving to make converts, He was able to pull a crowd of five thousand persons to a single meeting (Matthew 14:13-21; Mark 6:30-44; Luke 9:10-17; John 6:1-15) and later went on to tell His disciples “Go and make disciples of all nations” (Convert the whole world) (Mathew 28: 19-20). On the Pentecost day alone, Peter was able to convert three thousand persons (Act 2:14-36). Ever since then Churches have continue to spring up and grow.

1.1 Background to Church Growth

According to Brierley [1], The Christian church has grown substantially since its birth amounting up to 28% of the world’s population. Despite the predictions of secularism theory, numerous authors have noted that Christian churches have continued to grow [2].

Trying to answer the question: why churches grow? Kelly [3] asserts that strong religious movement make demands of their members in terms of both belief and behaviour. He concluded that conservative churches grow because they were able to attract and retain active and committed membership. Hoge [4] proposed that denominational growth rates correlate strongly with strictness and concomitants.

Iannaccone [5] argue that Kelly was correct, in showing how strictness overcomes “free rider” problems, He embedded Kelly’s thesis within a much broader rational choice approach to religion. He asserts that strictness makes organisation stronger and more attractive because it reduces free riding. It screens out members who lack commitment and stimulate

participation among those that remain. He concluded that members of stricter denominations devote more time and money to their religions and are more likely to describe themselves as strong members of their faith. They socialise more extensively with fellow members and are less involved in secular organisations.

Iannaccone *et al* [6] offer a model for the growth of religious organisations as a “Product” derived from inputs of time and money. They found that faster growing churches demand and receive larger commitments of time and money from their members. They also found that faster growing churches have members who contribute more in dollars and members who attend religious services several times a week. They argue that the decline of liberal/mainline denominations and the growth of their more strict/conservative counterparts is directly related to the different amount of time and money that they demand and receive from members, and that these two variables (time and money) are positively associated with church growth.

However, Bruce *et al* [7] found that participation in the congregation (enthusiasm) is not relevant to church growth. They argue that churches grow when larger percentages of members are growing spiritually and also when members see their leaders as empowering.

Medcalfe and Sharp [8] found that growing churches are associated with members leaving and hence asserts that churches can still grow even if they loose members. They found congregational growth being associated with internal factors specific to individual churches such as (1) Time commitment (2) Money commitment (3) Evangelism commitment. External factors such as income growth, location, competition and population growth are not associated with growing churches.

Most of the literature on church growth does not attempt to model the dynamics of church growth in terms of the underlying causes. The approaches are qualitative, using statistical analysis [3,6,8].

Hayward [2] was the earliest publication on the mathematics of church growth. He investigated the possibility of using mathematics to model church growth with ideas from population modelling. He applied epidemic models to church growth and affirms that these models prove useful because of the similarities between the spread of disease and the spread of beliefs (which ultimately leads to growth in the church), some of which are (1) There are at least two categories of people: those who have the disease (or belief in church growth), and those who do not. (2) Beliefs, like many diseases often spread by some sort of contact between the two categories of people. In the case of disease the contact may be physical or via some intermediary mechanisms such as air borne droplets. For belief the contact is via oral communication. He asserts that “enthusiasts” are the only church population that drive growth.

Using system dynamics to investigate church growth and assuming that enthusiasts retain their conversion potential throughout their life time; Hayward [9] modelled a situation where the whole population gets converted into the church. Building on this model of system dynamics, Hayward [10] included births, deaths and reversion into the model and found that the whole population does not get converted which is more realistic. The same result hold when in Hayward [11] the model was modified so that enthusiasts lose their enthusiasm after a length of time (what he called the limited enthusiasm).

Hayward [5] asserts that declining churches do so because the number of enthusiasts one enthusiast would make if the whole population were unbelievers is inadequate, that is, there is a failure to reach a threshold of reproduction potential. Therefore, churches die out not necessarily because they are losing members but because the enthusiasts have failed to reproduce themselves. He suggested that a church needs to make enthusiasts, not just converts, if it is to avoid extinction.

The model was built on the following assumptions:

- 1 Unbelievers are converted and recruited into the church through contact with a subset of believers called enthusiasts
- 2 After a period of time, the enthusiasts cease to be in active conversion, remaining in the church as inactive members
- 3 The enthusiastic period starts immediately after an unbeliever is converted

1.2 Why Model Church Growth

The mechanism for the transmission dynamics of beliefs is similar to that of epidemics. There are at least two categories of people: those who have the belief (believers) and those who do not (unbelievers). This categorisation is analogous to infectives and susceptible in mathematical epidemiology where the believer (those who transmit the belief) are likened to the infectives (those who transmit the disease) and the unbelievers are likened to the susceptible.

Applying mathematical modelling to epidemiology have proven to be a success and yielded a lot of meaningful results. For example, Daniel Bernoulli formulated a model for small pox in 1760. He solved the model and used it to evaluate the

effectiveness of inoculating healthy people against the small pox virus. Ever since then, mathematical modelling in epidemiology has recorded series of success. Mathematical model have become important tools in analysing the spread and control of infectious diseases. Models provide results such as reproduction numbers. Mathematical models have been used to find optimal control strategies in epidemics.

Noting these huge successes in mathematical epidemiology and considering the fact that the transmission dynamics of beliefs and diseases are similar, it is therefore hoped that applying mathematical modelling to the study of church growth (and decline) will provide useful result.

Hayward [2] reasoned that:

- 1 Models can help decide what sort of data should be gathered to best measure a church's effectiveness.
- 2 Model can help explain why there is such a wide variation in the speed and extent of church growth and decline. For example some growth is slow and steady, whereas some, often associated with revivals, are fast.

1.3 Mathematical Modelling

Mathematical modelling is an activity of translating a real life problem into mathematical equation(s) for subsequent analysis of the problem. Usually, a set of equations is used to describe the relationship between variables and the behaviour of the model is analysed using mathematical methods and computer simulations. From results obtained, the model can be used to make forecasts. The model formulation process clarifies assumptions, variables and parameters which lead to precision about relevant aspect of the problem. They are several types of models from which a modeller can chose. In population modelling, the modeller is usually faced with the choice of either a stochastic or a deterministic model.

Stochastic models are models that permit random fluctuations, they allow for probabilities. Upon simulations, usually with probabilities calculated using random numbers, the result of a stochastic model is different for different runs. Stochastic models provide information such as means and confidence intervals. A limitation is that solutions of stochastic models are more difficult to obtain than that of deterministic models.

On the other hand, for any given model structure with fixed parameter values and the same initial conditions, a deterministic model will produce the same output each time it is simulated. They are often appropriate for large populations. In a deterministic model every set of variable is uniquely determined by parameters in the model and by set of previous states of these variables.

In population modelling, the population is usually divided into distinct classes. Individuals flow from one class to another with respect to time. Population modellers capture this flow using compartments. The choice of which compartment to include in a model depend on the characteristics of the particular problem being modelled and the purpose of the model.

Our model is a non linear deterministic compartmental model. The total population N , is divided into three distinct classes or compartments: The susceptible church members, the passive church members and the active church members. The susceptible church members include all the non church members and members of other denominations.

The passive church members are those who are not actively involved in church activities. In Economics these types of organisational members are called “free – riders”. They shoulder less than their fair share of the cost of productions and consume more than their fair share of resources. In a labour setting, these types of members do not pay union dues but remain beneficiaries of union representation. In our model these are church members whose only contribution is to attend church service. They sing the praises, join in the dancing and listen to sermon but may end up not even giving offering. They have no passion for wining souls and therefore do not directly involve themselves in evangelism. However, they remain a numerical part of the church and may contribute to the numerical growth of the church. For example, a passive church member may mandate the members of his household (spouse and children) to attend church (It’s a common practice in Nigeria for all minors and spouse to attend the church of the bread winner). Also a passive church member may have a reason to invite friends and colleagues to the church (e.g. christening), these visitors may get converted through this encounter.

The active church members are those who actively and committed participate in church activities. Other authors referred to them as “enthusiasts” (see for example [2,8]). We proposed that they are responsible for “the majority” of conversion and recruitment into the church. Our argument agrees with the economics 80/20 principle which is basically an idea that in any given situation 80 percent of the work is done by 20 percent of those concerned. When it comes to church growth, the active members (which might be less than 20 percent) are responsible for most of the recruitment (which might be more than 80 percent) but certainly not all the recruitment.

Our definition of active members also includes those who contribute substantial amount of money to the church. They are also regarded as active members since the money they contribute can sponsor or enhance the effectiveness of evangelism. Medcalfe and Sharp [8] referred to them as “Identified givers”. They associated larger giving from “Identified givers” with church growth and assert that enthusiasm (what we call activeness) is multi faceted and manifests itself in time, money and evangelism commitment.

1.4 Assumptions of the Model

We shall build our church growth model based on the following assumptions.

- 1 Unbelievers are converted and recruited into the church through contact with either an active church member or a passive church member.
- 2 The probability that a contact results into a conversion (and hence recruitment) is higher with an active than a passive member

- 3 Removal (or reversion) does not confer a permanent nor temporal immunity against the church therefore those who revert enter into the susceptible compartment immediately and can be reconverted.
- 4 The active period does not start immediately upon conversion. A new convert enters into the passive population and stay for a period of time (no matter how short the time may be). Training (directly or indirectly) and listening to “The Word” transforms a passive member into an active member over time.
- 5 The passive class is the only gateway in and out of the church. Should an active member decides to leave the church, he first become passive for a period of time (no matter how short the time may be) before leaving the church.

1.5 Model Formulation

The total interacting population is divided into two classes, the believers and unbelievers. The unbelievers are the susceptible, $S(t)$. The believers are further divided into passive church members and active church members ($P(t)$ and $A(t)$ respectively). We assume that the local density of the total population is a constant though the total population size $N(t) = S(t) + P(t) + R(t)$ may vary with time. It is assumed that the susceptible individuals are recruited into the population at a rate Λ . The per capita rate β , which is the average number of effective contact (contact sufficient to result into conversion), is a constant. The parameter $m > 1$ captures the fact that individuals in the active population are more convincing, due to lifestyle, time commitment, money commitment and enthusiasm, than their passive counterparts and therefore more likely to convert a susceptible. We shall call this parameter the effectiveness of an Active member. We assumed that believers do not deliberately seek out unbelievers and has contacts uniformly mixed through the population, hence the probability of finding unbelievers at any time t , is given by:

$$\frac{S(t)}{N(t)}$$

Expressed in Hayward [9] as:

$$\frac{\text{Unbelievers}}{\text{Unbelievers+Believers}}$$

Which he referred to as homogenous mixing. It implies that church members have to be diffused into the society rather than confined to certain places. The total number of new conversion at any time t , is given by:

$$\frac{\beta(P(t)+mA(t))S(t)}{N(t)}$$

This form of mixing is called proportionate mixing by Busenberg and Van Driessche [13]. δ is the rate at which passive individuals “catches enthusiasm” and become active. α is the rate at which active individuals loses enthusiasm and become passive. ω is the rate at which passive individuals leaves the church (reversion). μ is the natural death rate of all individuals. The equations of the model are as presented below:

$$\frac{dS}{dt} = \Lambda N - \frac{\beta(P+mA)S}{N} - \mu S + \omega P, \tag{1.1}$$

$$\frac{dP}{dt} = \frac{\beta(P+mA)S}{N} - (\omega + \delta + \mu)P + \alpha A, \tag{1.2}$$

$$\frac{dA}{dt} = \delta P - (\alpha + \mu)A. \tag{1.3}$$

Adding equations (1.1) – (1.3) gives

$$\frac{dN}{dt} = (\Lambda - \mu)N. \tag{1.4}$$

We shall transform the model into proportions.

Let $x = \frac{S}{N}$, $y = \frac{P}{N}$, and $z = \frac{A}{N}$

Then

$$\begin{aligned} \frac{dx}{dt} &= \frac{1}{N} \left[\frac{dS}{dt} - x \frac{dN}{dt} \right], \\ \frac{dy}{dt} &= \frac{1}{N} \left[\frac{dP}{dt} - y \frac{dN}{dt} \right] \text{ and} \\ \frac{dz}{dt} &= \frac{1}{N} \left[\frac{dA}{dt} - z \frac{dN}{dt} \right]. \end{aligned}$$

From the derivatives above

$$\begin{aligned} \frac{dx}{dt} &= \frac{1}{N} \left[\Lambda N - \frac{\beta(P+mA)S}{N} - \mu S + \omega P - x(\Lambda - \mu)N \right] \\ &= \Lambda - \beta(y + mz)x - \mu x + \omega y - \Lambda x + \mu x \\ &= \Lambda(1 - x) - \beta(y + mz)x + \omega y \end{aligned} \tag{1.5}$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{1}{N} \left[\frac{\beta(P+mA)S}{N} - (\omega + \delta + \mu)P + \alpha A - y(\Lambda - \mu)N \right] \\ &= \beta(y + mz)x - (\omega + \delta + \mu)y + \alpha z - \Lambda y + \mu y \\ &= \beta(y + mz)x - (\omega + \delta + \Lambda)y + \alpha z \end{aligned} \tag{1.6}$$

$$\begin{aligned} \frac{dz}{dt} &= \frac{1}{N} [\delta p - (\alpha + \mu)A - z(\Lambda - \mu)N] \\ &= \delta y - (\alpha + \mu)z - z(\Lambda - \mu) \\ &= \delta y - (\alpha + \Lambda)z \end{aligned} \tag{1.7}$$

We shall carry out the analysis on the model equations transformed into proportions as follows:

$$\begin{aligned} \frac{dx}{dt} &= \Lambda(1 - x) - \beta(y + mz)x + \omega y, \tag{1.8} \\ \frac{dy}{dt} &= \beta(y + mz)x - (\omega + \delta + \Lambda)y + \alpha z, \tag{1.9} \\ \frac{dz}{dt} &= \delta y - (\alpha + \Lambda)z. \tag{2.0} \end{aligned}$$

2. MODEL ANALYSIS

Existence of Faith Free Equilibrium (FFE) state.

To establish this, we equate the left hand side of equations (1.8) – (2.0) to zero leading to the system

$$\Lambda(1 - x) - \beta(y + mz)x + \omega y = 0 \tag{2.1}$$

$$\beta(y + mz)x - (\omega + \delta + \Lambda)y + \alpha z = 0 \tag{2.2}$$

$$\delta y - (\alpha + \Lambda)z = 0 \tag{2.3}$$

At FLE state

$$\frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0 \text{ and } y = z = 0,$$

Then $(x, y, z) = (x^*, 0, 0)$.

Putting this condition in equation (2.1) – (2.3). we have

$$\Lambda(1 - x^*) = 0$$

Hence $x^* = 1$, since $\Lambda > 0$

We therefore have $(x^*, 0, 0) = (1, 0, 0)$ as the FLE state.

2.1 Faithful Equilibrium State

The faithful equilibrium state is denoted by $(x, y, z) = (x^*, y^*, z^*)$,

Solving x, y, z in equations (2.1) – (2.3) we have

$$x^* = \frac{\Lambda + \omega y}{\Lambda + \beta y + \beta m z}, \tag{2.4}$$

$$y^* = -\frac{(\alpha + \beta m x)z}{\beta x - \omega - \delta - \Lambda} \tag{2.5}$$

and

$$z^* = \frac{\delta y}{\alpha + \Lambda}. \tag{2.6}$$

Now, putting z^* in x^* and y^* we have

$$x^* = \frac{\alpha(\omega + 2\delta) - (\alpha + \omega + \delta)\Lambda - \Lambda^2}{\beta(\delta m - \alpha - \Lambda)}$$

Let $x^* = k$

Substituting $x^* = k$ in y^* and z^* we obtain

$$y^* = \frac{\Lambda(k-1)}{\omega - \beta k \Lambda}, \text{ where } \Lambda = 1 + \frac{\delta m}{\alpha + \Lambda}$$

Consequently,

$$z^* = \frac{\Lambda \delta (k-1)}{(\alpha + \Lambda)(\omega - \beta k \Lambda)}$$

Hence

$$(x^*, y^*, z^*) = \left(K, \frac{\Lambda(k-1)}{\omega - \beta k \Lambda}, \frac{\Lambda \delta (k-1)}{(\alpha + \Lambda)(\omega - \beta k \Lambda)} \right)$$

Becomes the faithful endemic state.

2.2 Stability

By linearization method

$$\begin{aligned} \text{Let } f_1 &= \Lambda(1 - x) - \beta(y + mz)x + \omega y \\ f_2 &= \beta(y + mz)x - (\omega + \delta + \alpha)y + \alpha z \end{aligned}$$

$$f_3 = \delta y - (\alpha + \Lambda)z$$

Then

$$\begin{aligned} \frac{\partial f_1}{\partial x} &= -\Lambda - \beta(y^* + mz^*), \frac{\partial f_1}{\partial y} = \omega - \beta x^*, \frac{\partial f_1}{\partial z} = -\beta m x^* \\ \frac{\partial f_2}{\partial x} &= \beta(y^* + mz^*), \frac{\partial f_2}{\partial y} = \beta x^* - (\omega + \delta + \Lambda), \frac{\partial f_2}{\partial z} \\ &= \alpha + \beta m x^* \\ \frac{\partial f_3}{\partial x} &= 0, \frac{\partial f_3}{\partial y} = \delta, \frac{\partial f_3}{\partial z} = -(\alpha + \Lambda) \end{aligned}$$

The jacobian matrix becomes

$$J^* = \begin{pmatrix} -\Lambda - \beta(y^* + mz^*) & \omega - \beta x^* & -\beta m x^* \\ \beta(y^* + mz^*) & \beta x^* - (\omega + \delta + \Lambda) & \alpha + \beta m x^* \\ 0 & \delta & -(\alpha + \Lambda) \end{pmatrix}$$

At FLE state $(x^*, 0, 0) = (1, 0, 0)$

$$J_0 = \begin{pmatrix} -\Lambda & \omega - \beta & -\beta m \\ 0 & \beta - (\omega + \delta + \Lambda) & \alpha + \beta \\ 0 & \delta & -(\alpha + \Lambda) \end{pmatrix}$$

To determine the eigenvalues, we employ the equation.

$|J_0 - \lambda I| = 0$, Where λ represents the eigenvalues

$$\begin{vmatrix} -\Lambda - \lambda & \omega - \beta & -\beta m \\ 0 & \beta - (\omega + \delta + \Lambda) - \lambda & \alpha + \beta m \\ 0 & \delta & -(\alpha + \Lambda) - \lambda \end{vmatrix} = 0$$

$$(-\Lambda - \lambda)[(\beta - (\omega + \delta + \Lambda)(-\Lambda - \lambda) - \delta\alpha + \beta m) = 0$$

Either $(-\Lambda - \lambda) = 0$ which implies $\lambda_1 = 0$

Or

$$\begin{aligned} \lambda^2 + (2\Lambda + \alpha + \omega + \delta - \beta)\lambda - \delta(\alpha + \beta m) \\ - (\alpha + \Lambda)(\beta - \omega - \delta - \Lambda) = 0 \end{aligned}$$

$$\lambda^2 + (2\Lambda + \alpha + \omega + \delta - \beta)\lambda - [(\alpha + \Lambda)(\beta - \omega - \Lambda) + \delta(\beta m - \Lambda)] = 0$$

And we have,

$$\lambda^2 + A_1 + A_2 = 0$$

Where,

$$A_1 = (2\Lambda + \alpha + \omega + \delta - \beta)$$

$$A_2 = [-(\alpha + \Lambda)(\beta - \omega - \Lambda) + \delta(\beta m - \Lambda)]$$

By Quadratic formula

$$\lambda_{2,3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda_{2,3}$$

$$= \frac{-(2\Lambda + \alpha + \omega + \delta - \beta) \pm \sqrt{(2\Lambda + \alpha + \omega + \delta - \beta)^2 - 4A_2}}{2}$$

Since $A_2 < 0$

$$\begin{aligned} \lambda_{2 \leq} &= -\frac{[(2\Lambda + \alpha + \omega + \delta - \beta) + \sqrt{(2\Lambda + \alpha + \omega + \delta - \beta)^2}]}{2} \\ \lambda_2 &= -\frac{(2\Lambda + \alpha + \omega + \delta - \beta) + (2\Lambda + \alpha + \omega + \delta - \beta)}{2} \end{aligned}$$

Similarly,

$$\begin{aligned} \lambda_{3 \leq} &= -\frac{[(2\Lambda + \alpha + \omega + \delta - \beta) - \sqrt{(2\Lambda + \alpha + \omega + \delta - \beta)^2}]}{2} \\ \lambda_3 &= -\frac{(2\Lambda + \alpha + \omega + \delta - \beta) - (2\Lambda + \alpha + \omega + \delta - \beta)}{2} \\ \lambda_3 &= -(2\Lambda + \alpha + \omega + \delta - \beta) < 0 \end{aligned}$$

Since

$\lambda_i < 0, i = 1, 2, 3$, We can hence conclude that the FLE state is locally asymptotically stable.

3. NUMERICAL EXPERIMENT

Having presented the governing equations for our church model, we will now solve the equations numerically using a fourth order Runge-Kutta method. The numerical experiments are carried out in order to study the impact of the Passive population on the general growth of the church. Parameter values are as presented in the table below.

Table 1: Parameter values for numerical experiments

Variables/ parameters	Experiment 1	Experiment 2	Experiment 3	Experiment 4	Experiment 5	Experiment 6	Experiment 7
x	0.6	0.6	0.6	0.6	0.6	0.6	0.6
y	0.1	0.1	0.1	0.1	0.1	0.1	0.1
z	0.07	0.1	0.07	0.07	0.07	0.07	0.07
Λ	0.2	0.2	0.2	0.2	0.2	0.2	0.2
β	0.35	0.35	0.35	0.35	0.35	0.3	0.3
m	2	2	4	5	2	5	5
ω	0.3	0.3	0.3	0.3	0.3	0.3	0.3
δ	0.1	0.1	0.1	0.1	0.3	0.3	0.5
α	0.06	0.06	0.06	0.06	0.06	0.06	0.06

Experiment 1

This experiment was carried out to study the prevalence of Active and Passive members when the effectiveness of the active members is low ($m = 2$). Parameter values are as

shown in Table 1. We see from Fig 1 that both populations continue to decrease with the Active population approaching zero.

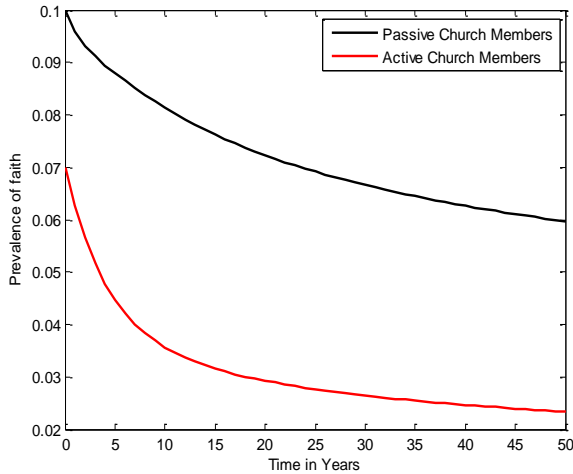


Fig. 1. Prevalence of faith with low effectiveness of Active members

Experiment 2

Here we kept all parameters constant as in Experiment 1 but increased the initial prevalence of Active members. Parameter values are as shown in Table 1. We see from fig 2 above that increasing the number of active members have no significant effect on the prevalence of faith. The Active population decreased and approaches zero as in the previous case.

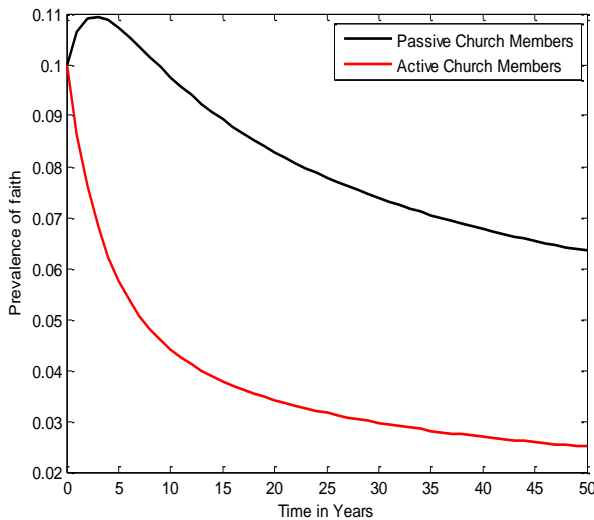


Fig 2. Prevalence of faith when active population was increased.

Experiment 3

In this experiment we simulated the prevalence of faith by increasing the effectiveness of the Active church members. Parameter values are as shown in Table 1. Parameters are kept constant as in experiment 1 except m which was increased from 2 to 4. We see from fig 3 above that this increment improved the prevalence of faith.

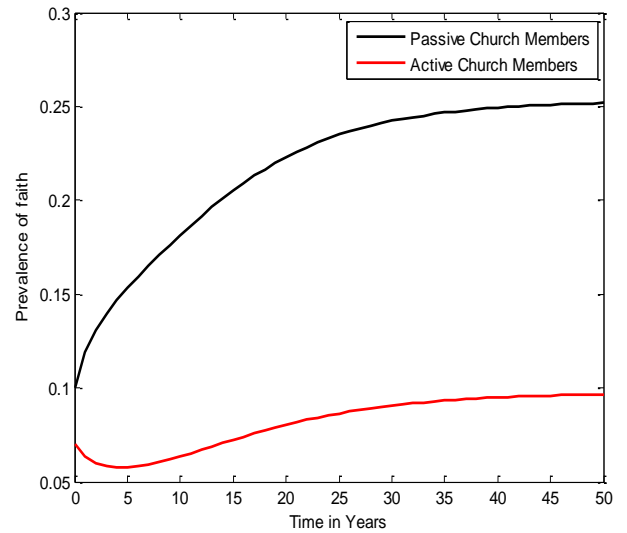


Fig 3. Prevalence of faith when $m = 4$

Experiment 4

We further increased m from 4 to 5, this further improved the prevalence of faith as seen in fig 4. We therefore establish that increasing the effectiveness of the active members is positively associated with church growth.

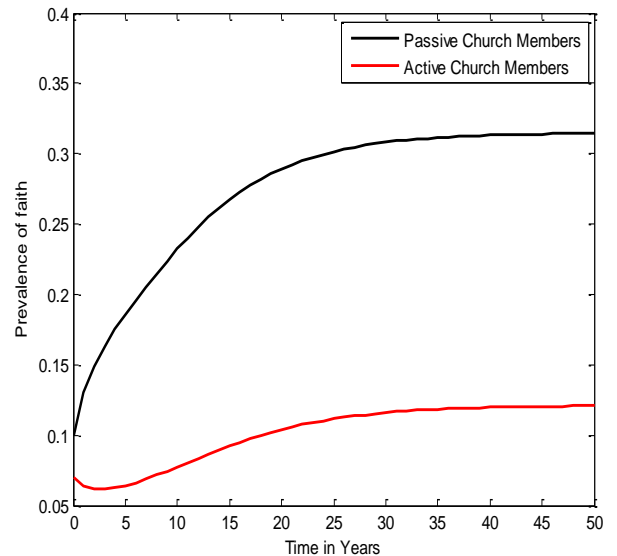


Fig 4. Prevalence of faith when $m = 5$

Experiment 5

We now investigated the effect of increasing the rate at which passive members become active (δ). We kept all parameters constant as in experiment 1 and increased δ from 0.1 to 0.3. The simulation revealed that increasing this rate alone has a very significant impact on the prevalence of faith. This is shown in Fig 5.

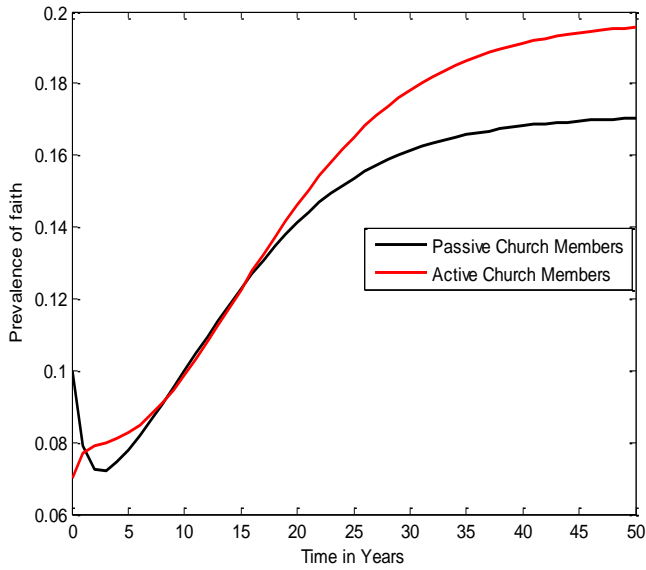


Fig 5. Effect of δ alone on the prevalence of faith

Experiment 6

In this experiment we investigated the combined effect of the rate at which passive members become active and the effectiveness of active members (δ and m respectively). We set $\delta = 0.3$ and $m = 5$. This gave a very good result. As seen in fig 6 the prevalence of faith was significantly improved.

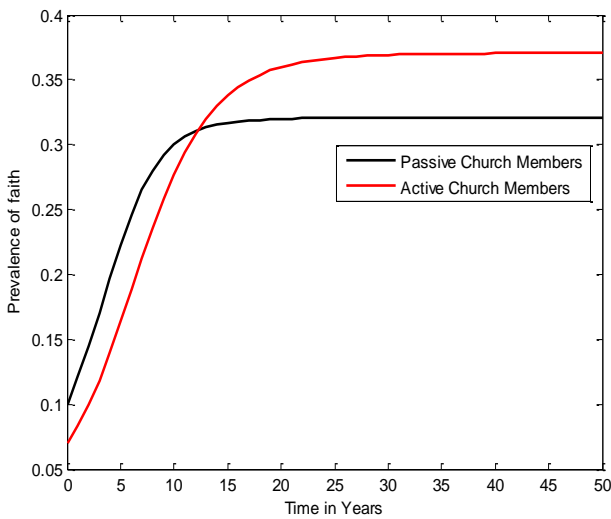


Fig 6. Combined effect of δ and m on the prevalence of faith when $\delta = 0.3$

Experiment 7

Here, we further increased the rate at which passive members become active. This increment as seen in fig 7 further improved the prevalence of faith. We therefore established that maximum church growth is achieved in presence of effective active members who continue to reproduce themselves, that is, groom passive members into becoming active.

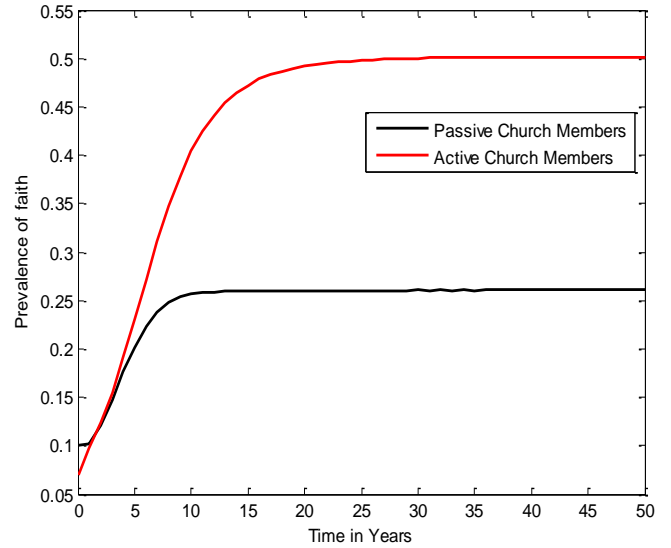


Fig 7. Combined effect of δ and m on the prevalence of faith when $\delta = 0.5$

4. CONCLUSION

A mathematical model for church growth was developed. The Faithless equilibrium state (FLE) was found to be locally asymptotically stable. Numerical simulations were used to evaluate the impact of the active population on church growth. Results showed that increasing the number of active population translates to no significant increase in church growth; however increasing the effectiveness of Active population and the rate at which passive members become active independently improved church growth significantly. Maximum church growth was achieved when effectiveness of Active members was combined with a greater rate at which Passive members become active members. We therefore suggest that church founders should, besides attracting members to their church, employs means of transforming their passive members into effective active members.

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