iJST EXPLORE International Journal of Science and Technology
CENTRE OF PROFESSIONAL RESEARCH PUBLICATIONS
ISSN 2049-7318

International Journal of Science and Technology Volume 2 No. 5, May, 2013

Some Algebraic Structures of Intuitionistic Fuzzy Multisets (IFMSs)

P. A. Ejegwa, J. A. Awolola
Department of Mathematics/Statistics/Computer Science,
University of Agriculture, P.M.B. 2373, Makurdi – Nigeria

ABSTRACT

Intuitionistic fuzzy multiset (IFMS) is an extension of intuitionistic fuzzy set (IFS). We showed some important properties of IFMS in this article. Since IFMS is a generalization of IFS, we extended the operations in IFS to IFMS and found that the algebraic laws in IFS are valid and sensible in IFMS. We authenticated the validity of the algebraic laws with numerical verification.

Keywords: *fuzzy sets, intuitionistic fuzzy sets, fuzzy multisets, intuitionistic fuzzy multisets, operations.*

1. INTRODUCTION

Intuitionistic fuzzy sets (IFSs) proposed by Atanassov [2,3] as a generalization of fuzzy sets introduced earlier by Zadeh [1] received much attentions in fuzzy community due to its flexibility and resourcefulness in tackling the issue of vagueness in Cantorian set theory. The main advantage of IFSs is their ability to cope with the hesitancy that may exist. This is achieved by incorporating a second function along with the membership function of the conventional fuzzy sets called *non-membership function*.

Subsequently, Shinoj and Sunil [9] proposed IFMSs from the combination of IFSs [3] and fuzzy multisets (FMSs) introduced by Yager[4] because there are times that each element has different membership values with a corresponding non-membership values. Nonetheless, in this paper we consider some important properties of IFMS and thereafter give some algebraic structures of IFMSs with respect to complement, union, intersection, addition and multiplication operations.

2. PRELIMINARIES

Definition 1[1]: Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x)$: $X \longrightarrow [0, 1]$ is the membership function of the fuzzy set A.

Definition 2[3]: Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \longrightarrow [0, 1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0, 1]$ i.e.,$\pi_A(x): X \longrightarrow [0, 1]$ for every $x \in X$. $\pi_A(x)$, expresses the lack of knowledge of whether x belongs to IFS A or not.

Definition 3[4]: Let X be a nonempty set. A fuzzy multiset (FMS) A drawn from X is characterized by a function, '*count membership*' of A denoted by CM_A *s.t.* CM_A: $X \longrightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval [0,1]. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from [0,1]. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x))$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq \ldots \geq \mu_A^n(x)$.

3. CONCEPT OF INTUITIONISTIC FUZZY MULTISETS (IFMSS)

Definition 4[9]: Let X be a nonempty set. An IFMS A drawn from X is characterized by two functions: "count membership" of A denoted as CM_A and "count non-membership" of A denoted as CN_A given respectively by CM_A: $X \longrightarrow Q$ and CN_A: $X \longrightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval [0,1] s.t. for each $x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CM_A(x)$ and it is denoted as $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x))$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq \ldots \geq \mu_A^n(x)$ whereas the corresponding non-membership sequence of elements in $CN_A(x)$ is denoted by $(\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^n(x))$ *s.t.* $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for every $x \in X$ and $i = 1, \ldots, n$.

We define IFMS alternatively. Let X be nonempty set. An IFMS A drawn from X is given as $A = \{\langle \mu_A^1(x), \ldots, \mu_A^n(x), \ldots, \nu_A^1(x), \ldots, \nu_A^n(x), \ldots \rangle x \in X\}$ where the functions $\mu_A^i(x), \nu_A^i(x): X \to [0,1]$ define the belongingness degrees and the non-belongingness degrees of A in X *s.t.* $0 \le \mu_A^i(x) + \nu_A^i(x) \le 1$ for $i = 1, \ldots$ If the sequence of the membership functions and non-membership (belongingness functions and non-belongingness functions) have only n-terms (i.e. finite), n is called the '*dimension*' of A. Consequently $A = \{\langle \mu_A^1(x), \ldots, \mu_A^n(x), \nu_A^1(x), \ldots, \nu_A^n(x)\rangle x \in X\}$ for $i = 1, \ldots, n$. when no ambiguity arises, we define $A = \{\langle \mu_A^i(x), \nu_A^i(x)\rangle x \in X\}$ for $i = 1, \ldots, n$.

For each IFMS A in X, $\pi_A^i(x) = 1 - \mu_A^i(x) - \nu_A^i(x)$ is the intuitionistic fuzzy multisets index or hesitation margin of x in A. The hesitation margin $\pi_A^i(x)$ *for each* $i = 1, \ldots, n$ is the degree of non-determinacy of $x \in X$, to the set A and $\pi_A^i(x) \in [0,1]$. The function $\pi_A^i(x)$ expresses lack of knowledge of whether x *in* A or not. In general, an IFMS A is given as $A = \{\langle x, \mu_A^i(x), \nu_A^i(x), \pi_A^i(x)\rangle : x \in X\}$.

Note: 1. $\mu_A^i(x) + \nu_A^i(x) + \pi_A^i(x) = 1$. 2. IFMS is an extension of IFS.

4. SOME PROPERTIES OF IFMS

1. Let X be nonempty set. The support of an IFMS A drawn from X is given as

supp $(A) = \{\mu_A^i(x) > 0$ or $\nu_A^i(x) > 0 : x \in X\}$, $i = 1, \ldots, n$.

2. Let X be nonempty set. The crossover point(s) of an IFMS A drawn from X is given as

cop $(A) = \{\mu_A^i(x) = 0.5$ or $\nu_A^i(x) = 0.5 : x \in X\}$, $i = 1, \ldots, n$.

3. Let X be nonempty set. The height of an IFMS A drawn from X is given as

hgt $(A) = \max\{supp(\mu_A^i(x), \nu_A^i(x))\}$ where $x \in X$ and $i = 1, \ldots, n$.

4. Let X be nonempty set. Two IFMSs A and B in X are similar or cognate if $\mu_A^i(x) = \mu_B^i(x)$ or $\nu_A^i(x) = \nu_B^i(x)$ for at least one i and $x \in X$ where $i = 1, \ldots, n$.

5. Let X be nonempty set. Two IFMSs A and B in X are comparable or equal if $\mu_A^i(x) = \mu_B^i(x)$

or $\nu_A^i(x) = \nu_B^i(x)$ for all $i = 1, \ldots, n$ *and* $x \in X$.

6. Let X be nonempty set. An IFMSs A in X is a sub-IFMS of an IFMS B in X (i.e. B is a super- IFMS) denoted as $A \subseteq B$ if $\mu_A^i(x) \le \mu_B^i(x)$ and $\nu_A^i(x) \ge \nu_B^i(x)$ for $x \in X$ and $i = 1, 2, \ldots, n$.

7. Let X be nonempty set. An IFMS A in X is a proper sub-IFMS of an IFMS B in X denoted as $A \subset B$ if $\mu_A^i(x) < \mu_B^i(x)$ and $\nu_A^i(x) > \nu_B^i(x)$ for all $i = 1, 2, \ldots, n$ and $x \in X$. Roughly speaking, $A \subset B$ if $A \subseteq B$ and $A \neq B$

8. Let X be nonempty set. The Cartesian product of two IFMSs A and B in X is defined as

$A \times B = \{\langle \mu_A^i(x)\mu_B^i(x), \mu_A^i(x)\nu_B^i(x), \nu_A^i(x)\mu_B^i(x), \nu_A^i(x)\nu_B^i(x)\rangle x \in X\}$ for $i = 1, \ldots, n$.

5. OPERATIONS IN INTUITIONISTIC FUZZY MULTISETS [9]

For any two IFMSs A and B drawn from X, the following operations hold. Let $A = \{\langle x, \mu_A^i(x), \nu_A^i(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B^i(x), \nu_B^i(x)\rangle : x \in X\}$, for each $i = 1, 2, \ldots, n$. Then:

[Complement]

$$A^c = \{\langle x, \nu_A^i(x), \mu_A^i(x)\rangle : x \in X\}$$

[Union]

$$A \cup B = \{\langle x, \max(\mu_A^i(x), \mu_B^i(x)), \min(\nu_A^i(x), \nu_B^i(x))\rangle : x \in X\}$$

[Intersection]

$$A \cap B = \{\langle x, \min(\mu_A^i(x), \mu_B^i(x)), \max(\nu_A^i(x), \nu_B^i(x))\rangle : x \in X\}$$

[Addition]

$$A \oplus B = \{\langle x, (\mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x)\mu_B^i(x)), (\nu_A^i(x)\, \nu_B^i(x))\rangle : x \in X\}$$

[Multiplication]

$$A \otimes B = \{\langle x, (\mu_A^i(x)\mu_B^i(x)), (\nu_A^i(x) + \nu_B^i(x) - \nu_A^i(x)\, \nu_B^i(x))\rangle : x \in X\}$$

6. SOME ALGEBRAIC STRUCTURES OF INTUITIONISTIC FUZZY MULTISETS

Let A, B and C be IFMSs in X, then the following algebraic laws are true:

1. $(A^c)^c = A$ i.e. complementary law.
2. (i) $A \cup A = A$ (ii) $A \cap A = A$ i.e. idempotent law.
3. (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$ i.e. commutative law.
4. (i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$ i.e. associative law.
5. (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ i.e. distributive law. It also holds for right distributive law.

6. $(i)\ (A \cup B)^c = A^c \cap B^c$ $(ii)\ (A \cap B)^c = A^c \cup B^c$ i.e. De Morgan's laws.
7. $(i)\ A \cap (A \cup B) = A$ $(ii)\ A \cup (A \cap B) = A$ i.e. absorption laws.
8. $(i) A \oplus B = B \oplus A$ $(ii) A \otimes B = B \otimes A$
9. $(i) A \oplus (B \oplus C) = (A \oplus B) \oplus C$ $(ii) A \otimes (B \otimes C) = (A \otimes B) \otimes C$
10. $(i)\ (A \oplus B)^c = A^c \otimes B^c$ $(ii) (A \otimes B)^c = A^c \oplus B^c$
11. $(i) A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$
$(ii)\ A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$
$(iii)\ A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$
$(iv)\ A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$.

It also holds for right distributive law.

7. NUMERICAL VERIFICATION OF THE ALGEBRAIC LAWS

Let $A = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$, $B = \{\langle(0.2, 0.1), (0.7, 0.8)\rangle\}$, $C = \{\langle(1.0, 0.5), (0.0, 0.5)\rangle\}$, then:

1. $A^c = \{\langle(0.4, 0.5), (0.3, 0.2)\rangle\}$
$(A^c)^c = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$\Rightarrow (A^c)^c = A$

2. $(ii)\ A \cup A = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$\Rightarrow A \cup A = A$
$(ii)\ A \cap A = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$\Rightarrow A \cap A = A$

3. $(i)\ A \cup B = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$B \cup A = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$\Rightarrow A \cup B = B \cup A$
$(ii)\ A \cap B = \{\langle(0.2, 0.1), (0.7, 0.8)\rangle\}$
$B \cap A = \{\langle(0.2, 0.1), (0.7, 0.8)\rangle\}$
$\Rightarrow A \cap B = B \cap A$

4. $(i)\ (A \cup B) \cup C = \{\langle(1.0, 0.5), (0.0, 0.5)\rangle\}$
$B \cup C = \{\langle(1.0, 0.5), (0.0, 0.5)\rangle\}$
$A \cup (B \cup C) = \{\langle(1.0, 0.5), (0.0, 0.5)\rangle\}$
$\Rightarrow (A \cup B) \cup C = A \cup (B \cup C)$
$(ii)\ (A \cap B) \cap C = \{\langle(0.2, 0.1), (0.7, 0.8)\rangle\}$
$B \cap C = \{\langle(0.2, 0.1), (0.7, 0.8)\rangle\}$
$A \cap (B \cap C) = \{\langle(0.2, 0.1), (0.7, 0.8)\rangle\}$
$\Rightarrow (A \cap B) \cap C = A \cap (B \cap C)$

5. $(i)\ A \cup (B \cap C) = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$A \cup C = \{\langle(1.0, 0.5), (0.0, 0.5)\rangle\}$
$(A \cup B) \cap (A \cup C) = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$\Rightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$(ii)\ A \cap (B \cup C) = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$A \cap C = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$(A \cap B) \cup (A \cap C) = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$\Rightarrow A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$

6. $(i)\ (A \cup B)^c = \{\langle(0.4, 0.5), (0.3, 0.2)\rangle\}$
$A^c \cap B^c = \{\langle(0.4, 0.5), (0.3, 0.2)\rangle\}$
$\Rightarrow (A \cup B)^c = A^c \cap B^c$
$(ii)\ (A \cap B)^c = \{\langle(0.7, 0.8), (0.2, 0.1)\rangle\}$
$A^c \cup B^c = \{\langle(0.7, 0.8), (0.2, 0.1)\rangle\}$
$\Rightarrow (A \cap B)^c = A^c \cup B^c$

7. (i) $A \cup B = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$A \cap (A \cup B) = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$\Rightarrow A \cap (A \cup B) =$ A
$(ii) A \cap B = \{\langle(0.2, 0.1), (0.7, 0.8)\rangle\}$
$A \cup (A \cap B) = \{\langle(0.3, 0.2), (0.4, 0.5)\rangle\}$
$\Rightarrow A \cup (A \cap B) =$ A

8. $(i)\ A \oplus B = \{\langle(0.44, 0.28), (0.28, 0.4)\rangle\}$
$B \oplus A = \{\langle(0.44, 0.28), (0.28, 0.4)\rangle\}$
$\Rightarrow A \oplus B = B \oplus A$
$(ii)\ A \otimes B = \{\langle(0.06, 0.02), (0.82, 0.9)\rangle\}$
$B \otimes A = \{\langle(0.06, 0.02), (0.82, 0.9)\rangle\}$
$\Rightarrow A \otimes B = B \otimes A$

9. $(i)\ B \oplus C = \{\langle(1.0, 0.55), (0.0, 0.4)\rangle\}$
$A \oplus (B \oplus C) = \{\langle(1.0, 0.64), (0.0, 0.2)\rangle\}$
$(A \oplus B) \oplus C = \{\langle(1.0, 0.64), (0.0, 0.2)\rangle\}$
$\Rightarrow A \oplus (B \oplus C) = (A \oplus B) \oplus C$
$(ii) B \otimes C = \{\langle(0.2, 0.05), (0.7, 0.9)\rangle\}$
$A \otimes (B \otimes C) = \{\langle(0.06, 0.01), (0.82, 0.95)\rangle\}$
$(A \otimes B) \otimes C = \{\langle(0.06, 0.01), (0.82, 0.95)\rangle\}$
$\Rightarrow A \otimes (B \otimes C) = (A \otimes B) \otimes C$

10. $(i)\ (A \oplus B)^c = \{\langle(0.28, 0.4), (0.44, 0.28)\rangle\}$
$A^c \otimes B^c = \{\langle(0.28, 0.4), (0.44, 0.28)\rangle\}$ s
$\Rightarrow (A \oplus B)^c = A^c \otimes B^c$
(ii) $(A \otimes B)^c = \{\langle(0.82, 0.9), (0.06, 0.02)\rangle\}$
$A^c \oplus B^c = \{\langle(0.82, 0.9), (0.06, 0.02)\rangle\}$
$\Rightarrow (A \otimes B)^c = A^c \oplus B^c$

11. $(i)\ A \oplus (B \cup C) = \{\langle(1.0, 0.6), (0.0, 0.25)\rangle\}$
$A \oplus B = \{\langle(0.44\ 0.28), (0.28, 0.4)\rangle\}$
$A \oplus C = \{\langle(1.0, 0.6), (0.0, 0.25)\rangle\}$
$(A \oplus B) \cup (A \oplus C) = \{\langle(1.0, 0.6), (0.0, 0.25)\rangle\}$
$\Rightarrow A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$
$(ii)\ A \oplus (B \cap C) = \{\langle(0.44, 0.28), (0.28, 0.4)\rangle\}$
$A \oplus B = \{\langle(0.44, 0.28), (0.28, 0.4)\rangle\}$
$A \oplus C = \{\langle(1.0, 0.6), (0.0, 0.25)\rangle\}$
$(A \oplus B) \cap (A \oplus C) = \{\langle(0.44, 0.28), (0.28, 0.4)\rangle\}$

$\Rightarrow A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$
$(iii)\ A \otimes (B \cup C) = \{\langle(0.3, 0.1), (0.4, 0.75)\rangle\}$
$A \otimes B = \{\langle(0.06, 0.02), (0.82, 0.9)\rangle\}$
$A \otimes C = \{\langle(0.3, 0.1), (0.4, 0.75)\rangle\}$
$(A \otimes B) \cup (A \otimes C) = \{\langle(0.3, 0.1), (0.4, 0.75)\rangle\}$
$\Rightarrow A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$
(iv) $A \otimes (B \cap C) = \{\langle(0.06, 0.02), (0.82, 0.9)\rangle\}$
$(A \otimes B) \cap (A \otimes C) = \{\langle(0.06, 0.02), (0.82, 0.9)\rangle\}$
$\Rightarrow A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$

8. CONCLUSION

We conclude that the algebraic laws that are valid in IFSs are also true in IFMSs since the latter is an extension of the former.

REFERENCES

[1] L.A. Zadeh, Fuzzy sets, Inform. and Control 8 (1965) 338-353.

[2] K. Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia, 1983.

[3] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87- 96.

[4] R. R. Yager, On the theory of bags, Int. J. of General Systems 13 (1986) 23-37.

[5] K. Atanassov, New operations defined over the intuitionistic fuzzy sets, Fuzzy Set and Systems 61 (1994) 137-142.

[6] K. Atanassov, Intuitionistic fuzzy sets, Springer, Heidelberg, 1999.

[7] K. De Supriya, R. Biswas, A. R. Roy, Some operations on intuitionistic fuzzy sets, Fuzzy Sets and Systems 114 (2000) 477-484.

[8] W. Zeng, H. Li, Note on "Some operations on intuitionistic fuzzy sets", Fuzzy Sets and Systems 157 (2006) 990-991.

[9] T. K. Shinoj, J. J. Sunil, Intuitionistic fuzzy multisets and its application in medical diagnosis, Int. J. of Mathematical and Computational Sciences 6 (2012).