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Plane Strain Deformation of a Poroelastic Half-Space in Welded Contact with an Isotropic Elastic Half Space

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ABSTRACT

The Biot linearized theory for fluid saturated porous materials is used to study the plane strain deformation of an isotropic, homogeneous, poroelastic half space in welded contact with an isotropic, homogeneous, perfectly elastic half space caused by an inclined line-load in elastic half space. The integral expressions for the displacements and stresses in the two half spaces in welded contact are obtained from the corresponding expressions for an unbounded elastic and poroelastic medium by applying boundary conditions at the interface. The integrals for inclined line-load are solved analytically for the limiting case i.e. undrained conditions in high frequency limit. The undrained displacements, stresses and pore pressure for poroelastic half space are shown graphically.

Keywords: *Inclined line-load, plane strain, poroelastic, welded half-spaces.*

1. INTRODUCTION

Poroelasticity is the mechanics of poroelastic solids with fluid filled pores. Its mathematical theory deals with the mechanical behaviour of an elastic porous medium which is either completely filled or partially filled with pore fluid and study the time dependent coupling between the deformation of the rock and fluid flow within the rock. The study of deformation by buried sources of a fluid saturated porous medium is very important because of its applications in earthquake engineering, soil mechanics, seismology, hydrology, geomechanics, geophysics etc. Biot (1941, 1956) developed linearized constitutive and field equations for poroelastic medium which has been used by many researchers (see e.g. Wang (2000) and the references listed there in).

When the source surface is very long in one direction in comparison with the others, the use of two dimensional approximation is justified and consequently calculations are simplified to a great extent and one gets a closed form analytical solution. A very long strip-source and a very long line-source are examples of two dimensional sources. Love (1944) obtained expressions for the displacements due to a line-source in an isotropic elastic medium. Maruyama (1966) obtained the displacements and stress fields corresponding to long strike-slip faults in a homogeneous isotropic half-space. The two dimensional problem has also been discussed by Rudnicki (1987), Rudnicki and Roeloffs (1990),Singh and Rani (2006) Rani and Singh(2007),Singh et al. (2007).

Different approaches and methods like boundary value method, displacement discontinuity method, Galerkin vector approach, displacement function approach and eigen value approach, Biot stress function approach etc. have been made to study the plane strain (two dimensional) problem of poroelasticity. The use of eigen value approach has the advantage of finding the solutions of the governing equations in the matrix form notations that avoids the complicated nature of the problem. Kumar et al. (2000, 2002), Garg et al. (2003), Kumar and Ailwalia (2005), Selim and Ahmed (2006) , Selim (2007,2008), Chugh et al.(2011) etc. have used this approach for solving plane strain problem of elasticity and poroelasticity.

In the present paper we study the plane strain deformation of a two phase medium consisting of an isotropic, homogeneous, poroelastic half space in welded contact with an isotropic, homogeneous, perfectly elastic half space caused by an inclined line-load in elastic half space. Using Biot stress function(Biot 1956d,Roeloffs 1988) and Fourier transform ,we find stresses ,displacement and pore pressure for poroelastic unbounded medium in integral form and using eigen value approach following Fourier transform ,we find stresses and displacement for unbounded elastic medium in integral form. Then we obtain the integral expressions for the displacements and stresses in the two half spaces in welded contact from the corresponding expressions for an unbounded elastic and poroelastic medium by applying suitable boundary conditions at the interface. These integrals cannot be solved analytically for arbitrary values of the frequency We evaluate these integrals for the limiting case i.e. undrained conditions in high frequency limit. The undrained displacements, stresses and pore pressure for poroelastic half space are shown graphically.

Geomechanics problems, such as loading by a reservoir lake or seabed structure that is very extensive in one direction on the earth's surface, can be solved as two dimensional plane strain problem. Bell and Nur (1978) used two dimensional half space models with surface loading to study the change in strength produced by reservoir-induced pore pressure and stresses for thrust, normal and strike-slip faulting.

2. FORMULATION OF THE PROBLEM

Consider a homogeneous, isotropic, elastic half space over a homogeneous, isotropic, poroelastic half space which is in welded contact at the interface. A rectangular Cartesian coordinate system oxyz is taken in such a way that plane z=0 coincides with the intersecting surface of the two half spaces. We take z-axis vertically downwards in the poroelastic half space so that homogeneous, isotropic, poroelastic half space becomes the medium-I (z≥0) and the elastic half space becomes the medium- II (z≤0). Further an inclined line-load of magnitude F_0, per unit length is acting on the y-axis and its inclination with z-direction is δ. The geometry of the problem is as shown in figure 1 and it conforms to the two dimensional approximation. Considering the cartesian coordinates (x, y, z) as (x_1, x_2, x_3). We have $\frac{\partial}{\partial x_2} \equiv 0$ and the displacement components (U_1, U_2, U_3) are independent of the Cartesian coordinate x_2 for the present two dimensional problem. Under this assumption the plane strain problem $(U_2 = 0)$ and the antiplane strain problem $(U_1 = U_3 = 0)$ get decoupled, and can therefore be solved independently. Here, we consider only the plane strain problem.

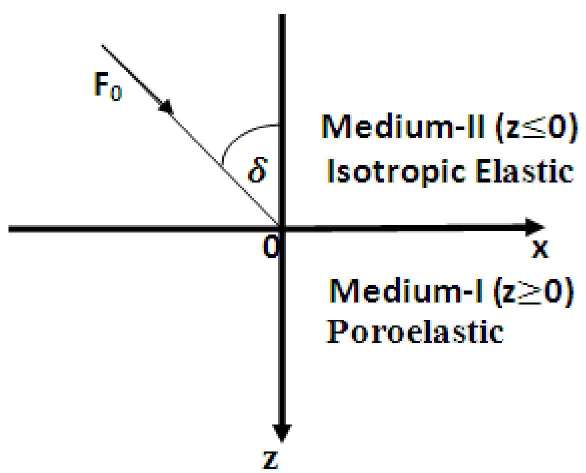

Fig. 1 An Inclined Line-Load F_0 Acting Along y-axis

3. SOLUTION FOR POROELASTIC HALF SPACE MEDIUM-I (Z≥0)

A homogeneous, isotropic, poroelastic medium can be described by five poroelastic parameters: drained Poisson's ratio (ν), undrained Poisson's ratio (ν_μ), shear modulus (G), hydraulic diffusivity (c) and Skempton's coefficient (B). Darcy conductivity (χ) and Biot-willis coefficient α can be expressed in terms of these five parameters:

$$\chi = \frac{9c(1-\nu_\mu)(\nu_\mu - \nu)}{2GB^2(1-\nu)(1+\nu_\mu)^2}, \tag{1}$$

$$\alpha = \frac{3(\nu_\mu - \nu)}{B(1-2\nu)(1+\nu_\mu)}, \tag{2}$$

The two dimensional plane strain problem for an isotropic poroelastic medium can be solved in terms of Biot's stress function F [Wang (2000)] as

$$\sigma_{11} = \frac{\partial^2 F}{\partial z^2}, \sigma_{33} = \frac{\partial^2 F}{\partial x^2}, \sigma_{13} = -\frac{\partial^2 F}{\partial x \partial z}, \tag{3}$$

$$\nabla^2(\nabla^2 F + 2\eta p) = 0, \tag{4}$$

$$(c\nabla^2 - \frac{\partial}{\partial t})[\nabla^2 F + \frac{3}{(1+\nu_\mu)B} p] = 0, \tag{5}$$

where σ_{ij} denotes the total stress in the fluid saturated porous elastic material, p is the excess fluid pore pressure (compression negative) and

$$\eta = \frac{(1-2\nu)\alpha}{2(1-\nu)}, \tag{6}$$

is the poroelastic stress coefficient.

From equations (4) and (5), we get the following decoupled equations

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2 p = 0, \tag{7}$$

and

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^4 F = 0. \tag{8}$$

The general solution of equation (7) may be expressed as

$$p = p_1 + p_2 \tag{9}$$

where p_1 and p_2 satisfies the following equations:

$$c\nabla^2 p_1 = \frac{\partial p_1}{\partial t}, \tag{10}$$

and

$$\nabla^2 p_2 = 0. \tag{11}$$

Similarly, the general solution of equation (8) may be expressed as

$$F = F_1 + F_2, \tag{12}$$

Where

$$c\nabla^2 F_1 = \frac{\partial F_1}{\partial t}, \quad (13)$$

$$\nabla^4 F_2 = 0. \quad (14)$$

Separation of time and space variables can be made for each of the four functions p_1, p_2, F_1 and F_2. Assuming the time dependence as $\exp(-i\omega t)$, equations (10), (11), (13) and (14) become

$$\nabla^2 p_1 + \frac{i\omega}{c} p_1 = 0, \quad (15)$$

$$\nabla^2 p_2 = 0, \quad (16)$$

$$\nabla^2 F_1 + \frac{i\omega}{c} F_1 = 0, \quad (17)$$

$$\nabla^4 F_2 = 0, \quad (18)$$

where p_1, p_2, F_1 and F_2 are now functions of x and z only.

Fourier transforms are now used to get suitable solutions of equations (15)-(18), which on using equations (9) and (12), can be written as

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{-mz} + A_2 e^{-|k|z} + A_3 e^{mz} + A_4 e^{|k|z}\right] e^{-ikx} dk, \quad (19)$$

$$F = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mz} + B_4 e^{mz} + (B_2 + B_3|k|z)e^{-|k|z} + (B_5 + B_6|k|z)e^{|k|z}\right] e^{-ikx} dk, \quad (20)$$

For medium I (z≥0), using the relation conditions, we have

$$p = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{-mz} + A_2 e^{-|k|z}\right] e^{-ikx} dk, \quad (21)$$

$$F = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mz} + (B_2 + B_3|k|z)e^{-|k|z}\right] e^{-ikx} dk, \quad (22)$$

where B_1, B_2, B_3, A_1 and A_2 are functions of k. From (4), (5), (21) and (22). We find

$$A_1 = \frac{i\omega}{2\eta\, c} B_1, \quad A_2 = \frac{2}{3}(1 + v_\mu) B k^2 B_3, \quad m = \left(\left(\frac{ck^2 - i\omega}{c}\right)\right)^{\frac{1}{2}}, (\mathrm{Re}\, m > 0), \quad (23)$$

Using (22) in (3), the stresses in medium 1 (z≥0) are obtained as

$$\sigma_{11} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m^2 e^{-mz} + (B_2 - 2B_3 + B_3|k|z)k^2 e^{-|k|z}\right] e^{-ikx} dk, \quad (24)$$

$$\sigma_{33} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mz} + (B_2 + B_3|k|z)e^{-|k|z}\right] k^2 e^{-ikx} dk, \quad (25)$$

$$\sigma_{13} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m e^{-mz} + (B_2 - B_3 + B_3|k|z)|k| e^{-|k|z}\right] (-ik) e^{-ikx} dk. \quad (26)$$

Corresponding to these stresses, the displacements are obtained as (Singh and Rani 2006)

$$2GU_1{}^1 = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mz} + \{B_2 + B_3(2v_\mu - 2 + |k|z)\}e^{-|k|z}\right] (-ik) e^{-ikx} dk, \quad (27)$$

$$2GU_3{}^1 = \frac{1}{2\pi}\int_{-\infty}^{\infty}[B_1 m e^{-mz} + \{B_2 + B_3(1 - 2v_\mu + |k|z)\}|k| e^{-|k|z}] e^{-ikx} dk. \quad (28)$$

Also from equation (21), we have

$$\frac{\partial p}{\partial z} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-mA_1 e^{-mz} - |k|A_2 e^{-|k|z}\right] e^{-ikx} dk, \quad (29)$$

4. SOLUTION FOR ELASTIC SOLID HALF SPACE, MEDIUM-II (Z≤0)

The equilibrium equations in Cartesian coordinate system (x_1, x_2, x_3) in absence of body forces are

$$\frac{\partial \tau_{11}}{\partial x_1}+\frac{\partial \tau_{12}}{\partial x_2}+\frac{\partial \tau_{13}}{\partial x_3}=0, \quad (30)$$

$$\frac{\partial \tau_{21}}{\partial x_1}+\frac{\partial \tau_{22}}{\partial x_2}+\frac{\partial \tau_{23}}{\partial x_3}=0, \quad (31)$$

$$\frac{\partial \tau_{31}}{\partial x_1}+\frac{\partial \tau_{32}}{\partial x_2}+\frac{\partial \tau_{33}}{\partial x_3}=0, \quad (32)$$

where $\tau_{ij}(i,j=1,2,3)$ are components of stress tensor.

The stress-strain relations for an isotropic elastic medium are

$$\begin{bmatrix}\tau_{11}\\ \tau_{22}\\ \tau_{33}\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0\\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0\\ 0 & 0 & 0 & \mu & 0 & 0\\ 0 & 0 & 0 & 0 & \mu & 0\\ 0 & 0 & 0 & 0 & 0 & \mu\end{bmatrix}\begin{bmatrix}e_{11}\\ e_{22}\\ e_{33}\\ 2e_{23}\\ 2e_{13}\\ 2e_{12}\end{bmatrix}, \quad (33)$$

where λ and μ are known as Lame's elastic moduli, e_{ij} are the components of the strain tensor and are related with displacement components (u_1, u_2, u_3) through the relations

$$e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), 1\le i,j\le 3, \quad (34)$$

we shall write (x_1, x_2, x_3)=(x,y,z), (u_1, u_2, u_3)=(u,v,w)

The equilibrium equations in terms of the displacement components are to be found from equations (30)-(32) by using (33) and (34) and for the present two dimensional problem

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial z^2}+(\lambda+\mu)\frac{\partial^2 w}{\partial x\,\partial z}=0, \quad (35)$$

$$(\lambda+\mu)\frac{\partial^2 u}{\partial x\,\partial z}+\mu\frac{\partial^2 w}{\partial x^2}+(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2}=0. \quad (36)$$

Now, we define Fourier transform $\bar{f}(x, k)$ of $f(x, y)$ [Debnath, (1995)] as

$$\bar{f}(x,k)=F[f(x,y)]=\int_{-\infty}^{\infty} f(x,y)e^{iky}\,dy, \quad (37)$$

So that

$$f(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(x,k)e^{-iky}\,dk, \quad (38)$$

where k is the transformed Fourier parameter. We know that (Sneddon, 1951)

$$F\left(\frac{\partial}{\partial y}f(x,y)\right)=(-ik)\bar{f}(x,k),$$

$$F\left(\frac{\partial^2}{\partial y^2}f(x,y)\right)=(-ik)^2\bar{f}(x,k), \quad (39)$$

Applying Fourier Transform as defined above in equations (35) and (36), we get

$$-(\lambda+2\mu)k^2\bar{u}+\mu\frac{d^2\bar{u}}{dz^2}-ik(\lambda+\mu)\frac{d\bar{w}}{dz}=0, \quad (40)$$

$$-ik(\lambda+\mu)\frac{d\bar{u}}{dz}-\mu k^2\bar{w}+(\lambda+2\mu)\frac{d^2\bar{w}}{dz^2}=0, \quad (41)$$

Where $\bar{u}$ and $\bar{w}$ are Fourier transform of u(x, z) and w(x, z) w.r.t. x

The above equations (40)-(41) can be written in vector matrix equation form as

$$A\frac{d^2N}{dz^2}+B\frac{dN}{dz}+CN=0, \quad (42)$$

where

$$A=\begin{bmatrix}(\lambda+2\mu) & 0\\ 0 & \mu\end{bmatrix}, B=\begin{bmatrix}0 & -ik(\lambda+\mu)\\ -ik(\lambda+\mu) & 0\end{bmatrix}, C=\begin{bmatrix}-k^2\mu & 0\\ 0 & -k^2(\lambda+2\mu)\end{bmatrix}, N=\begin{bmatrix}\bar{w}\\ \bar{u}\end{bmatrix} \quad (43)$$

We note that the matrices A, B, C are all symmetric. Matrices A, B, C depends upon elastic moduli only. Applying eigen value method to solve equation (42), we try a solution of the matrix equation (42) of the form

$$N(z,k)=E(k)e^{sz}, \quad (44)$$

where s is a parameter and E(k) is a matrix of the type 2×1.Substituting the value of N from equation (44) into equation (42),we get the following characteristic equation

$$s^4-2k^2s^2+k^4=0, \quad (45)$$

The solution of the characteristic equation (45) gives repeated eigenvalues as s=±k, ±k, or eigenvalues may be written as

$$s_1=s_2=-s_3=-s_4=|k|. \quad (46)$$

Equation (42) can be rewritten as

$$\frac{dN_2}{dz}=A_2N_2, \quad (47)$$

where

$$N_2 = \begin{bmatrix} \bar{w} \\ \bar{u} \\ \frac{d\bar{w}}{dz} \\ \frac{d\bar{u}}{dz} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{\mu}{\lambda + 2\mu}\right)k^2 & 0 & 0 & i\left(\frac{\lambda+\mu}{\lambda + 2\mu}\right)k \\ 0 & \left(\frac{\lambda+2\mu}{\mu}\right)k^2 & i\left(\frac{\lambda+\mu}{\mu}\right)k & 0 \end{bmatrix}, \quad (48)$$

The differential equation (47) is of first order. We use the eigenvalue method to solve it .Ross (1984), has given a procedure to solve the problem with repeated eigenvalues provided the governing vector differential equation is of the first order. We follow the procedure as given by Ross (1984) and get the eigenvectors as

$$X_{1,} = \begin{bmatrix} i|k| \\ k \\ ik^2 \\ k|k| \end{bmatrix}, X_2 = \begin{bmatrix} i\left\{|k|z - 2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\} \\ k\left(z - \frac{1}{|k|}\right) \\ i|k|\left\{|k|z - \left(\frac{\lambda+3\mu}{\lambda+\mu}\right)\right\} \\ k|k|z \end{bmatrix}, X_3 = \begin{bmatrix} -i|k| \\ k \\ ik^2 \\ -k|k| \end{bmatrix} \text{and } X_4 = \begin{bmatrix} -i\left\{|k|z + 2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\} \\ k\left(z + \frac{1}{|k|}\right) \\ i|k|\left\{|k|z + \left(\frac{\lambda+3\mu}{\lambda+\mu}\right)\right\} \\ k|k|z \end{bmatrix} \quad (49)$$

Thus, the solution of the vector-matrix differential equation (47) is

$$N_2 = (E_1X_1 + E_2X_2)e^{|k|z} + (E_3X_3 + E_4X_4)e^{-|k|z}, \quad (50)$$

where E_1, E_2, E_3 and E_4 are coefficients which may depend upon k and two elastic constants λ and μ of an isotropic elastic medium. We write

$$N_2 = \begin{bmatrix} N_{21} \\ N_{22} \\ N_{23} \\ N_{24} \end{bmatrix}, \quad (51)$$

Where

$$N_{21} = \left(\frac{i}{\lambda+\mu}\right)\begin{bmatrix} \{(\lambda+\mu)E_1|k| + E_2\{(\lambda+\mu)|k|z - 2(\lambda+2\mu)\}\}e^{|k|z} \\ -\{(\lambda+\mu)E_3|k| + E_4\{(\lambda+\mu)|k|z + 2(\lambda+2\mu)\}\}e^{-|k|z} \end{bmatrix}, \quad (52)$$

$$N_{22} = \left(\frac{k}{|k|}\right)\left[\{E_1|k| + E_2(|k|z-1)\}e^{|k|z} + \{E_3|k| + E_4(|k|z+1)\}e^{-|k|z}\right], \quad (53)$$

$$N_{23} = \left(\frac{i}{\lambda+\mu}\right)\begin{bmatrix} \{(\lambda+\mu)E_1k^2 + E_2|k|\{(\lambda+\mu)|k|z - (\lambda+3\mu)\}\}e^{|k|z} \\ +\{(\lambda+\mu)E_3k^2 + E_4|k|\{(\lambda+\mu)|k|z + (\lambda+3\mu)\}\}e^{-|k|z} \end{bmatrix}, \quad (54)$$

$$N_{24} = k|k|\left[(E_1 + E_2z)e^{|k|z} - (E_3 + E_4z)e^{-|k|z}\right]. \quad (55)$$

From equations (48) and (51)-(53), the displacements in the transformed domain are found to be

$$\bar{w} = \left(\frac{i}{\lambda+\mu}\right)\begin{bmatrix} \{(\lambda+\mu)E_1|k| + E_2\{(\lambda+\mu)|k|z - 2(\lambda+2\mu)\}\}e^{|k|z} \\ -\{(\lambda+\mu)E_3|k| + E_4\{(\lambda+\mu)|k|z + 2(\lambda+2\mu)\}\}e^{-|k|z} \end{bmatrix}, \quad (56)$$

$$\bar{u} = \left(\frac{k}{|k|}\right)\left[\{E_1|k| + E_2(|k|z-1)\}e^{|k|z} + \{E_3|k| + E_4(|k|z+1)\}e^{-|k|z}\right], \quad (57)$$

Inversion of equations (56) and (57) gives the following displacements for an isotropic elastic medium.

$$w(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{i}{\lambda+\mu}\right)\begin{bmatrix} \{(\lambda+\mu)E_1|k| + E_2\{(\lambda+\mu)|k|z - 2(\lambda+2\mu)\}\}e^{|k|z} \\ -\{(\lambda+\mu)E_3|k| + E_4\{(\lambda+\mu)|k|z + 2(\lambda+2\mu)\}\}e^{-|k|z} \end{bmatrix} e^{-ikx}dk, \quad (58)$$

$$u(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{k}{|k|}\right)\begin{bmatrix} \{E_1|k| + E_2(|k|z-1)\}e^{|k|z} \\ +\{E_3|k| + E_4(|k|z+1)\}e^{-|k|z} \end{bmatrix} e^{-ikx}dk, \quad (59)$$

To calculate the transformed stresses for plane strain deformation of an isotropic elastic medium, we make use of equation (33)-(34) and (37)-(39), we get

$$\bar{\tau}_{11} = (-ik)(\lambda + 2\mu)\bar{u} + \lambda\frac{d\bar{w}}{dz}, \quad (60)$$

$$\bar{\tau}_{13} = (-ik)\mu\bar{w} + \mu\frac{d\bar{u}}{dz}, \quad (61)$$

$$\bar{\tau}_{33} = (-ik)\lambda\bar{u} + (\lambda + 2\mu)\frac{d\bar{w}}{dz}, \quad (62)$$

where $\bar{u}$, $\bar{w}$ used in equations (60)-(62) are Fourier Transform of u(x, z) and w(x, z).

Using equations (48), (51)-(55) and (60) - (62) stresses $\bar{\tau}_{11}$, $\bar{\tau}_{13}$, $\bar{\tau}_{33}$ are given as

$$\bar{\tau}_{11} = \begin{bmatrix} \left\{-2\mu ik^2E_1 - 2i\mu|k|E_2\left(-\frac{\mu}{\lambda + \mu} + |k|z\right)\right\}e^{|k|z} \\ -\left\{2\mu ik^2E_3 + 2i\mu|k|E_4\left(\frac{\mu}{\lambda + \mu} + |k|z\right)\right\}e^{-|k|z} \end{bmatrix}, \quad (63)$$

$$\bar{\tau}_{13} = 2\mu k|k|\left[\left\{E_1 + E_2\left(z - \frac{(\lambda + 2\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{|k|z} - \left\{E_3 + E_4\left(z + \frac{(\lambda + 2\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{-|k|z}\right], \quad (64)$$

$$\bar{\tau}_{33} = 2\mu ik^2\left[\left\{E_1 + E_2\left(z - \frac{(2\lambda + 3\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{|k|z} + \left\{E_3 + E_4\left(z + \frac{(2\lambda + 3\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{-|k|z}\right], \quad (65)$$

Inversion of equations (63)-(65) gives the stresses in the following integral forms

$$\tau_{11} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\begin{bmatrix} \left\{-2\mu ik^2E_1 - 2i\mu|k|E_2\left(-\frac{\mu}{\lambda + \mu} + |k|z\right)\right\}e^{|k|z} \\ -\left\{2\mu ik^2E_3 + 2i\mu|k|E_4\left(\frac{\mu}{\lambda + \mu} + |k|z\right)\right\}e^{-|k|z} \end{bmatrix}e^{-ikx}dk, \quad (66)$$

$$\tau_{13} = \frac{1}{2\pi}\int_{-\infty}^{\infty}2\mu k|k|\left[\left\{E_1 + E_2\left(z - \frac{(\lambda + 2\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{|k|z} - \left\{E_3 + E_4\left(z + \frac{(\lambda + 2\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{-|k|z}\right]e^{-ikx}dk, \quad (67)$$

$$\tau_{33} = \frac{1}{2\pi}\int_{-\infty}^{\infty}2\mu ik^2\left[\left\{E_1 + E_2\left(z - \frac{(2\lambda + 3\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{|k|z} + \left\{E_3 + E_4\left(z + \frac{(2\lambda + 3\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{-|k|z}\right]e^{-ikx}dk. \quad (68)$$

The displacements and stress components for elastic half space medium II (z≤0) are now obtained as from equations (58)-(59) and (66)-(68)

$$w(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{i}{\lambda + \mu}\right)\left[\{(\lambda + \mu)E_1|k| + E_2\{(\lambda + \mu)|k|z - 2(\lambda + 2\mu)\}\}e^{|k|z}\right]e^{-ikx}dk, \quad (69)$$

$$u(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{k}{|k|}\right)\left[\{E_1|k| + E_2(|k|z - 1)\}e^{|k|z}\right]e^{-ikx}dk, \quad (70)$$

$$\tau_{11} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-2\mu ik^2E_1 - 2i\mu|k|E_2\left(-\frac{\mu}{\lambda + \mu} + |k|z\right)\right]e^{|k|z}e^{-ikx}dk, \quad (71)$$

$$\tau_{13} = \frac{1}{2\pi}\int_{-\infty}^{\infty}2\mu k|k|\left\{E_1 + E_2\left(z - \frac{(\lambda + 2\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{|k|z}e^{-ikx}dk, \quad (72)$$

$$\tau_{33} = \frac{1}{2\pi}\int_{-\infty}^{\infty}2\mu ik^2\left[\left\{E_1 + E_2\left(z - \frac{(2\lambda + 3\mu)}{(\lambda + \mu)|k|}\right)\right\}e^{|k|z}\right]e^{-ikx}dk. \quad (73)$$

5. CASE (a) NORMAL LINE-LOAD

Consider a normal line-load F_1 , per unit length, acting in the positive z-direction on the interface z=0 along y-axis(see figure 2). Since the half spaces are assumed to be in welded contact along the plane z=0, the continuity of the stresses and the displacements give the following boundary conditions at z=0:

$$\sigma_{13} = \tau_{13}, \tag{74}$$

$$\sigma_{33} - \tau_{33} = -F_1\delta(x), \tag{75}$$

$$U_1{}^1(x,z) = u(x,z), \tag{76}$$

$$U_3{}^1(x,z) = w(x,z), \tag{77}$$

where $\delta(x)$ in equation (75) is the Dirac delta function and it satisfies the following properties

$$\int_{-\infty}^{\infty} \delta(x)dx = 1\,, \delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikx}dk, \tag{78}$$

Also, we assume that the interface is impermeable, the hydraulic boundary condition at z=0 is

$$\frac{\partial p}{\partial z} = 0. \tag{79}$$

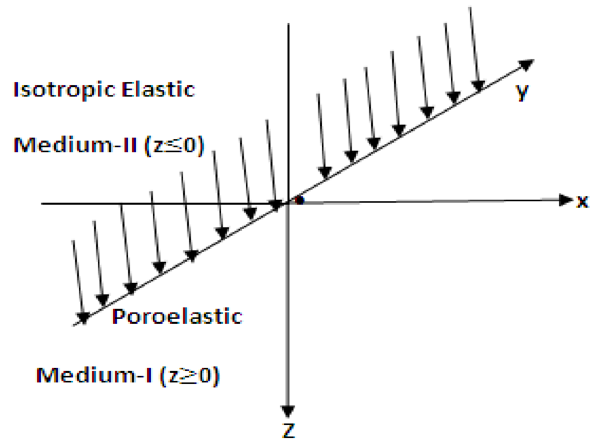

Figure 2: A Normal Line-Load F_1, per unit length, acting in the positive z-direction on the interface z=0 along y-axis

Now using equations (25)-(29), (69)-(70) and (72)-(73) in the boundary conditions (74)-(77) and (79), we get the following system of equations

$$mB_1 + |k|B_2 - |k|B_3 - 2\mu i|k|E_1 + 2\mu i\frac{(\lambda+\ 2\mu)}{(\lambda+\ \mu)}E_2 = 0, \tag{80}$$

$$B_1 + B_2 + 2\mu iE_1 - \frac{2\mu i}{|k|}\frac{(2\lambda+\ 3\mu)}{(\lambda+\mu)}E_2 = \frac{F_1}{K^2}, \tag{81}$$

$$B_1 + B_2 + \left(2v_\mu - 2\right)B_3 + 2GiE_1 - \frac{1}{|k|}2GiE_2 = 0, \tag{82}$$

$$B_1m + B_2|k| + |k|\left(1 - 2v_\mu\right)B_3 - 2Gi|k|E_1 + 4Gi\frac{(\lambda+\ 2\mu)}{(\lambda+\ \mu)}E_2 = 0, \tag{83}$$

$$ma^1B_1 + |k|b^1B_3 = 0, \tag{84}$$

where

$$a^1 = \frac{i\omega}{2\eta c}, b^1 = \frac{2}{3}\left(1 + v_\mu\right)Bk^2, \tag{85}$$

Solving the system of equations (80)-(84) using (85) we obtain

$$B_1 = \frac{-|k|b^1}{ma^1}B_3, \tag{86}$$

$$B_2 = \frac{-|k|(\lambda+\ \mu)pB_3 + 2|k|(\lambda+\ \mu)\mu iE_1 - 2\mu i(\lambda+\ 2\mu)E_2}{|k|(\lambda+\ \mu)}, \tag{87}$$

$$B_3 = \frac{-F_1(L(\mu-G)+1)}{k^2(E-M)(\mu-G)}, \tag{88}$$

$$E_1 = \frac{(p-\alpha)B_3 + \frac{2\mu i}{|k|}\left(\frac{3\lambda+5\mu}{\lambda+\mu}\right)E_2 + \frac{F_1}{k^2}}{4\mu i}, \tag{89}$$

$$E_2 = \frac{-(F_1L + EB_3k^2)}{2iF|K|}, \tag{90}$$

Where

$$\alpha = \frac{-|k|b^1}{ma^1},$$

$$p = \frac{m\alpha}{|k|} - 1,$$

$$r = \alpha + 2v_\mu - 2,$$

$$q = \frac{m\alpha}{|k|} + 1 - 2v_\mu, \tag{91}$$

$$E = \frac{(p-\alpha)(\mu-G) + 2\mu(\alpha-r)}{2\mu(\mu-G)}, \tag{92}$$

$$F = \frac{(3\lambda+5\mu)}{2(\lambda+\ \mu)} + \frac{(G(\lambda+\ \mu) - \mu(2\lambda+3\mu))}{(\lambda+\ \mu)(\mu-G)}, \tag{93}$$

$$L = \frac{-(\mu+G)}{2\mu(\mu-G)}, \tag{94}$$

$$M = \frac{2G(\alpha-r) - (r-q)(\mu-G)}{2G(\mu-G)}, \tag{95}$$

5.1 Undrained State, $\omega \to \infty$

Putting the values of B_1, B_2 and B_3 from equations (86)-(88) into equations(24)-(29) that corresponds to the stresses, displacements and pore pressure for poroelastic half space medium I ($z \geq 0$) and then taking limit $\omega \to \infty$ and then integrate, we get stresses ,displacements and pore pressure for normal line-load as

$$\sigma_{11} = \frac{F_1}{\pi}(P_4 + 2P_1)\frac{z}{x^2+z^2} - \frac{F_1P_1}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}, \tag{96}$$

$$\sigma_{33} = -\frac{F_1P_4}{\pi}\frac{z}{x^2+z^2} + \frac{F_1P_1}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}, \tag{97}$$

$$\sigma_{13} = -\frac{F_1}{\pi}(P_4 + P_1)\frac{x}{x^2+z^2} + \frac{2F_1P_1}{\pi}\frac{xz^2}{(x^2+z^2)^2}, \tag{98}$$

$$2GU_1{}^1 = \frac{F_1}{\pi}\Big((P_4 + P_1(2v_\mu - 2)\Big)\tan^{-1}\frac{x}{z} - \frac{F_1P_1}{\pi}\frac{xz}{x^2+z^2}, \tag{99}$$

$$2GU_3{}^1 = -\frac{F_1}{2\pi}\big(P_4 - P_1(1-2v_\mu)\big)\log(x^2+z^2) - \frac{F_1P_1}{\pi}\frac{z^2}{x^2+z^2}, \tag{100}$$

$$p = -\frac{2F_1P_1(1+v_\mu)B}{3\pi}\frac{z}{x^2+z^2}, \tag{101}$$

Where P_1, P_4 are as follow

$$P_1 = \frac{G}{-G+\mu(4v_\mu - 3)},\quad P_2 = \frac{L-EP_1}{F},\quad P_3 = \frac{P_1(\lambda+\mu)-\mu(3\lambda+5\mu)P_2+(\lambda+\mu)}{4\mu(\lambda+\mu)}, \text{ And } \quad P_4 = \frac{-P_1(\lambda+\mu)+2\mu(\lambda+\mu)P_3+\mu(\lambda+2\mu)P_2}{(\lambda+\mu)}, \tag{102}$$

Putting the values of E_1, and E_2 from equations (89)-(90) into equations (69)-(73) that corresponds to the stresses, and displacements for elastic half space medium II ($z \leq 0$) for normal line-load and then taking limit $\omega \to \infty$ and then integrate, we get

$$\tau_{11} = \frac{F_1\mu}{\pi}\left\{2P_3 + P_2\left(\frac{\mu}{\lambda+\mu}\right)\right\}\frac{z}{x^2+z^2} + \frac{\mu F_1P_2}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}, \tag{103}$$

$$\tau_{13} = \frac{-F_1\mu}{\pi}\left\{2P_3 + P_2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\frac{x}{x^2+z^2} - \frac{2\mu F_1P_2}{\pi}\frac{xz^2}{(x^2+z^2)^2}, \tag{104}$$

$$\tau_{33} = \frac{-F_1\mu}{\pi}\left\{2P_3 + P_2\left(\frac{2\lambda+3\mu}{\lambda+\mu}\right)\right\}\frac{z}{x^2+z^2} - \frac{\mu F_1P_2}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}, \tag{105}$$

$$u(x,z) = \frac{F_1}{\pi}\left(\frac{P_2+2P_3}{2}\right)\tan^{-1}\left(\frac{x}{z}\right) + \frac{F_1P_2}{2\pi}\frac{xz}{x^2+z^2}, \tag{106}$$

$$w(x,z) = \frac{-F_1}{2\pi}\left\{P_3 + P_2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\log(x^2+z^2) + \frac{F_1P_2}{2\pi}\frac{z^2}{x^2+z^2}, \tag{107}$$

5.2 Case (b) Tangential Line-Load

Consider a tangential line-load F_2 , per unit length, is acting at the origin in the positive x-direction(as shown in figure (3). Since the half spaces are assumed to be in welded contact along the plane z=0, the continuity of the stresses and the displacements give the following boundary conditions at z=0:

$$\sigma_{13} - \tau_{13} = -F_2\delta(x), \tag{2.1}$$

$$\sigma_{33} = \tau_{33}, \tag{2.2}$$

$U_1{}^1(x,z) = u(x,z),$ (2.3)

$U_3{}^1(x,z) = w(x,z),$ (2.4)

where $\delta(x)$ in equation (2.1) is the Dirac delta function

Also, if we assume that the interface is impermeable, the hydraulic boundary condition at z=0 is

$$\frac{\partial p}{\partial z} = 0. \quad (2.5)$$

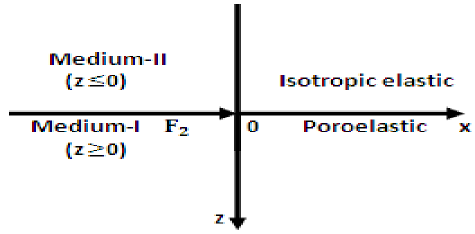

Figure 3: A tangential line-load F_2, per unit length, acting at the origin in the positive x-direction.

Now using equations (25)-(29), (69)-(70) and (72)-(73) in the boundary conditions (2.1)-(2.5), we get the following system of equations

$$mB_1 + |k|B_2 - |k|B_3 - 2\mu i|k|E_1 + 2\mu i\frac{(\lambda+2\mu)}{(\lambda+\mu)}E_2 = \frac{F_2}{ik}, \quad (2.6)$$

$$B_1 + B_2 + 2\mu iE_1 - \frac{2\mu i}{|k|}\frac{(2\lambda+3\mu)}{(\lambda+\mu)}E_2 = 0, \quad (2.7)$$

$$B_1 + B_2 + (2v_\mu - 2)B_3 + 2GiE_1 - \frac{1}{|k|}2GiE_2 = 0, \quad (2.8)$$

$$B_1 m + B_2|k| + |k|(1-2v_\mu)B_3 - 2Gi|k|E_1 + 4Gi\frac{(\lambda+2\mu)}{(\lambda+\mu)}E_2 = 0, \quad (2.9)$$

$$ma^1B_1 + |k|b^1B_3 = 0, \quad (2.10)$$

where

a^1, b^1 are same as defined in (85)

Solving the system of equations (2.6)-(2.10) using (85) we obtain

$$B_1 = \frac{-|k|b^1}{ma^1}B_3, \quad (2.11)$$

$$B_2 = \frac{-ik|k|(\lambda+\mu)pB_3 - 2k|k|(\lambda+\mu)\mu E_1 + 2\mu k(\lambda+2\mu)E_2 + F_2(\lambda+\mu)}{ik|k|(\lambda+\mu)}, \quad (2.12)$$

$$B_3 = \frac{-F_2 i}{2k|k|\mu(E-M)}, \quad (2.13)$$

$$E_1 = \frac{(p-\alpha)B_3 + \frac{2\mu i}{|k|}\left(\frac{3\lambda+5\mu}{\lambda+\mu}\right)E_2 - \frac{F_2}{ik|k|}}{4\mu i}, \quad (2.14)$$

$$E_2 = \frac{-(2iF_2 + 4EB_3\mu k|K|)}{8iF\mu k}, \quad (2.15)$$

Where α ,p ,r, q, E, F, M, are same as defined in (91)-(93) &(95)

Putting the values of B_1, B_2 and B_3 from equations (2.11)-(2.13) into equations(24)-(29) that corresponds to the expressions for the stresses ,displacements and pore pressure for poroelastic half space medium-I ($z \geq 0$) and then taking limit $\omega \to \infty$ and then integrate, we get stresses ,displacements and pore pressure for tangential line-load as under

$$\sigma_{11} = \frac{-F_2}{\pi}(q_4 - 2q_1)\frac{x}{x^2+z^2} - \frac{2F_2q_1}{\pi}\frac{xz^2}{(x^2+z^2)^2}, \quad (2.16)$$

$$\sigma_{33} = \frac{F_2}{\pi}\frac{q_4 x}{x^2+z^2} + \frac{2F_2q_1}{\pi}\frac{xz^2}{(x^2+z^2)^2}, \quad (2.17)$$

$$\sigma_{13} = -\frac{F_2}{\pi}(q_4 - q_1)\frac{z}{x^2+z^2} - \frac{F_2q_1}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}, \quad (2.18)$$

$$2GU_1{}^1 = \frac{-F_2}{2\pi}\left(\left(q_4 + q_1(2v_\mu - 2)\right)\right)\log(x^2+z^2) + \frac{F_2 q_1}{\pi}\frac{z^2}{x^2+z^2}, \tag{2.19}$$

$$2GU_3{}^1 = -\frac{F_2}{\pi}\left(q_4 + q_1(1-2v_\mu)\right)\tan^{-1}\frac{x}{z} - \frac{F_2 q_1}{\pi}\frac{xz}{x^2+z^2}, \tag{2.20}$$

$$p = -\frac{2F_2 q_1(1+v_\mu)B}{3\pi}\frac{x}{x^2+z^2}, \tag{2.21}$$

Where q_1, q_4 are as follow

$$q_1 = \frac{G}{-G+\mu(4v_\mu - 3)},$$

$$q_2 = \frac{(\mu - G) + q_1\left(\mu - G + 2\mu(2v_\mu - 2)\right)}{4F\mu(\mu - G)},$$

$$q_3 = \frac{q_1(\lambda+\mu) - 2\mu(3\lambda+5\mu)q_2 + (\lambda+\mu)}{4\mu(\lambda+\mu)},$$

$$q_4 = \frac{q_1(\lambda+\mu) - 2\mu(\lambda+\mu)q_3 - 2\mu(\lambda+2\mu)q_2 + (\lambda+\mu)}{(\lambda+\mu)}, \tag{2.22}$$

Putting the values of E_1, and E_2 from equations (2.14)-(2.15) into equations (69)-(73) that corresponds to the stresses and displacements for elastic half space medium II ($z \le 0$) for tangential line-load and then taking limit $\omega \to \infty$ and then integrate, we get

$$w(x,z) = \frac{-F_2}{\pi}\left\{q_3 + 2q_2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\tan^{-1}\left(\frac{x}{z}\right) - \frac{F_2 q_2}{\pi}\frac{xz}{x^2+z^2}, \tag{2.23}$$

$$u(x,z) = \frac{-F_2}{2\pi}(q_2+q_3)\log(x^2+z^2) + \frac{F_2 q_2}{\pi}\frac{z^2}{x^2+z^2}, \tag{2.24}$$

$$\tau_{11} = \frac{-2F_2\mu}{\pi}\left\{q_3 + q_2\left(\frac{\mu}{\lambda+\mu}\right)\right\}\frac{x}{x^2+z^2} - \frac{4\mu F_2 q_2}{\pi}\frac{xz^2}{(x^2+z^2)^2}, \tag{2.25}$$

$$\tau_{13} = \frac{-2F_2\mu}{\pi}\left\{q_3 + q_2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\frac{z}{x^2+z^2} - \frac{2\mu F_2 q_2}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}, \tag{2.26}$$

$$\tau_{33} = \frac{2F_2\mu}{\pi}\left\{q_3 + q_2\left(\frac{2\lambda+3\mu}{\lambda+\mu}\right)\right\}\frac{x}{x^2+z^2} + \frac{4\mu F_2 q_2}{\pi}\frac{xz^2}{(x^2+z^2)^2}, \tag{2.27}$$

5.3 Case(c) Inclined Line-Load

For an inclined line-load F_0,per unit length, (as shown in figure 1) is acting on the y-axis and its inclination with z-direction is δ. We get (Saada, 1974)

$$F_1 = F_0\cos\delta, \quad F_2 = F_0\sin\delta, \tag{3.1}$$

The stresses and displacements subjected to inclined line-load can be obtained by superposition of the normal and tangential cases. The final deformation of the formulated problem is given by

$$U_1{}^{1(IN)}(x,z) = U_1{}^{1(N)}(x,z) + U_1{}^{1(T)}(x,z), \tag{3.2}$$

$$U_3{}^{1(IN)}(x,z) = U_3{}^{1(N)}(x,z) + U_3{}^{1(T)}(x,z), \tag{3.3}$$

$$\sigma_{13}{}^{(IN)}(x,z) = \sigma_{13}{}^{(N)}(x,z) + \sigma_{13}{}^{(T)}(x,z), \tag{3.4}$$

$$\sigma_{33}{}^{(IN)}(x,z) = \sigma_{33}{}^{(N)}(x,z) + \sigma_{33}{}^{(T)}(x,z), \tag{3.5}$$

$$\sigma_{11}{}^{(IN)}(x,z) = \sigma_{11}{}^{(N)}(x,z) + \sigma_{11}{}^{(T)}(x,z), \tag{3.6}$$

$$p^{(IN)}(x,z) = p^{(N)}(x,z) + p^{(T)}(x,z), \tag{3.7}$$

Where deformation due to a normal line-load F_1 and a tangential line-load F_2 have been obtained in case (a) and case (b) respectively. The superscript (IN) indicates results due to inclined line-load F_0 and N indicates results due to Normal line-load F_1 and T indicates results due to Tangential line-load F_2.

Stresses, displacements and pore pressure due to inclined line-load F_0 per unit length in medium I ($z \ge 0$) are given by

$$\sigma_{11}{}^{(IN)} = \cos\delta\left(\frac{F_0}{\pi}(P_4+2P_1)\frac{z}{x^2+z^2} - \frac{F_0P_1}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}\right) + \sin\delta\left(\frac{-F_0}{\pi}(q_4-2q_1)\frac{x}{x^2+z^2} - \frac{2F_0q_1}{\pi}\frac{xz^2}{(x^2+z^2)^2}\right), \tag{3.8}$$

$$\sigma_{33}{}^{(IN)} = \cos\delta\left(-\frac{F_0P_4}{\pi}\frac{z}{x^2+z^2} + \frac{F_0P_1}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}\right) + \sin\delta\left(\frac{F_0}{\pi}\frac{q_4x}{x^2+z^2} + \frac{2F_0q_1}{\pi}\frac{xz^2}{(x^2+z^2)^2}\right), \tag{3.9}$$

$$\sigma_{13}{}^{(IN)} = \cos\delta\left(-\frac{F_0}{\pi}(P_4+P_1)\frac{x}{x^2+z^2} + \frac{2F_0P_1}{\pi}\frac{xz^2}{(x^2+z^2)^2}\right) + \sin\delta\left(-\frac{F_2}{\pi}(q_4-q_1)\frac{z}{x^2+z^2} - \frac{F_2q_1}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}\right), \tag{3.10}$$

$$U_1{}^{1(IN)}(x,z) = \frac{1}{2G}\left\{\cos\delta\left(\frac{F_0}{\pi}\Big(\big(P_4+P_1(2v_\mu-2)\big)\tan^{-1}\frac{x}{z} - \frac{F_0P_1}{\pi}\frac{xz}{x^2+z^2}\right) + \sin\delta\left(\frac{-F_0}{2\pi}\Big(\big(q_4+q_1(2v_\mu-2)\big)\log(x^2+z^2) + \frac{F_0q_1}{\pi}\frac{z^2}{x^2+z^2}\right)\right\},$$

$$U_3{}^{1(IN)}(x,z) = \frac{1}{2G}\left\{\cos\delta\left(-\frac{F_0}{2\pi}\big(P_4-P_1(1-2v_\mu)\big)\log(x^2+z^2) - \frac{F_0P_1}{\pi}\frac{z^2}{x^2+z^2}\right) + \sin\delta\left(-\frac{F_0}{\pi}\big(q_4+q_1(1-2v_\mu)\big)\tan^{-1}\frac{x}{z} - \frac{F_0q_1}{\pi}\frac{xz}{x^2+z^2}\right)\right\}, \tag{3.11}$$

$$p^{(IN)}(x,z) = \cos\delta\left\{-\frac{2F_1P_1(1+v_\mu)B}{3\pi}\frac{z}{x^2+z^2}\right\} + \sin\delta\left\{-\frac{2F_2q_1(1+v_\mu)B}{3\pi}\frac{x}{x^2+z^2}\right\}, \tag{3.12}$$

Stresses and displacements due to inclined line-load F_0 , per unit length, in medium II ($z\leq 0$) are given by

$$u^{(IN)}(x,z) = \cos\delta\left\{\frac{F_0}{\pi}\left(\frac{P_2+2P_3}{2}\right)\tan^{-1}\left(\frac{x}{z}\right) + \frac{F_0P_2}{2\pi}\frac{xz}{x^2+z^2}\right\} + \sin\delta\left\{\frac{-F_0}{2\pi}(q_2+q_3)\log(x^2+z^2) + \frac{F_0q_2}{\pi}\frac{z^2}{x^2+z^2}\right\}, \tag{3.13}$$

$$w^{(IN)}(x,z) = \cos\delta\left\{\frac{-F_0}{2\pi}\left\{P_3+P_2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\log(x^2+z^2) + \frac{F_0P_2}{2\pi}\frac{z^2}{x^2+z^2}\right\} + \sin\delta\left\{\frac{-F_0}{\pi}\left\{q_3+2q_2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\tan^{-1}\left(\frac{x}{z}\right) - \frac{F_0q_2}{\pi}\frac{xz}{x^2+z^2}\right\}, \tag{3.14}$$

$$\tau_{11}{}^{(IN)}(x,z) = \cos\delta\left\{\frac{F_0\mu}{\pi}\left\{2P_3+P_2\left(\frac{\mu}{\lambda+\mu}\right)\right\}\frac{z}{x^2+z^2} + \frac{\mu F_0P_2}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}\right\} + \sin\delta\left\{\frac{-2F_0\mu}{\pi}\left\{q_3+q_2\left(\frac{\mu}{\lambda+\mu}\right)\right\}\frac{x}{x^2+z^2} - \frac{4\mu F_0q_2}{\pi}\frac{xz^2}{(x^2+z^2)^2}\right\}, \tag{3.15}$$

$$\tau_{13}{}^{(IN)}(x,z) = \cos\delta\left\{\frac{-F_0\mu}{\pi}\left\{2P_3+P_2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\frac{x}{x^2+z^2} - \frac{2\mu F_0P_2}{\pi}\frac{xz^2}{(x^2+z^2)^2}\right\} + \sin\delta\left\{\frac{-2F_0\mu}{\pi}\left\{q_3+q_2\left(\frac{\lambda+2\mu}{\lambda+\mu}\right)\right\}\frac{z}{x^2+z^2} - \frac{2\mu F_0q_2}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}\right\}, \tag{3.16}$$

$$\tau_{33}{}^{(IN)}(x,z) = \cos\delta\left\{\frac{-F_0\mu}{\pi}\left\{2P_3+P_2\left(\frac{2\lambda+3\mu}{\lambda+\mu}\right)\right\}\frac{z}{x^2+z^2} - \frac{\mu F_0P_2}{\pi}\frac{z(z^2-x^2)}{(x^2+z^2)^2}\right\} + \sin\delta\left\{\frac{2F_0\mu}{\pi}\left\{q_3+q_2\left(\frac{2\lambda+3\mu}{\lambda+\mu}\right)\right\}\frac{x}{x^2+z^2} + \frac{4\mu F_0q_2}{\pi}\frac{xz^2}{(x^2+z^2)^2}\right\}, \tag{3.17}$$

6. NUMERICAL RESULTS AND DISCUSSION

We define the following dimensionless quantities

$X = \frac{x}{h}$, $Z = \frac{z}{h}$, $U_i = \frac{U_i{}^1}{h}$,i=1,3, And $\Sigma_{ij} = \frac{\sigma_{ij}}{G}$, $1\leq i,j \leq 3$, Considering elastic half space as Poissonion half space and poroelastic constants correspond to Ruhr Sandstone (Wang 2000).We have plotted graphs in figures (4-8) for the variation of the dimensionless displacements and stresses against the dimensionless horizontal distance X for a fixed value of Z=1/4. Each figure has four curves corresponding to four different values of δ,namely,$\delta = 0^0, 45^0, 60^0, 90^0$. The case $\delta = 0^0$ corresponds to a normal line-load and $\delta = 90^0$ for a tangential line load. Figures (4-5) show the variation of the normal displacement (U_1) and the variation of tangential displacement (U_3) respectively. These figures show that the displacements for $\delta = 45^0, 60^0$ lie between the corresponding displacements for a normal line-load and tangential line-load. Figure (6-8) correspond to the variation of stresses. In figure (7) the curves for different values of δ changes steadily. Figure (8) points frequent inter crossing of various curves for different values of δ. Figure (9) correspond to the variation of pore pressure against the dimensionless horizontal distance X.

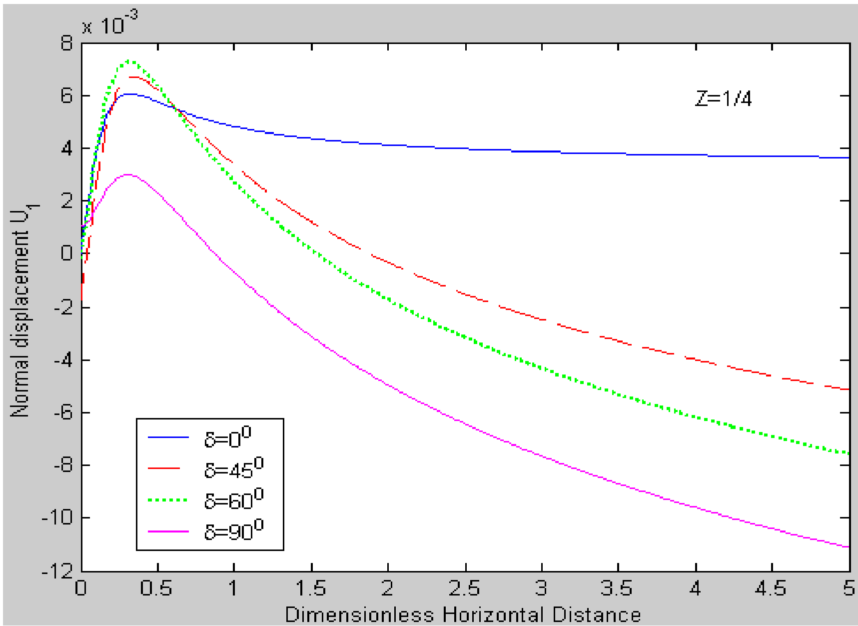

Figure 4

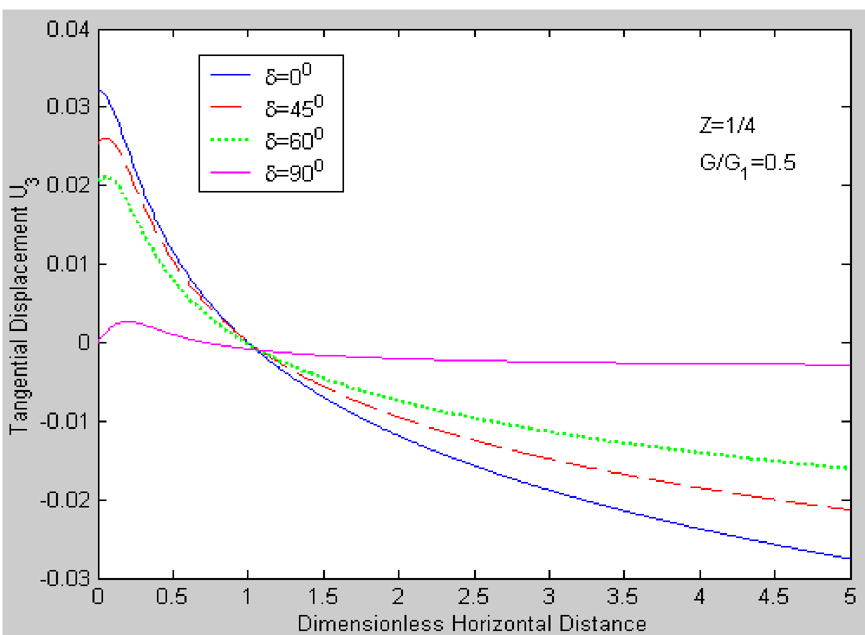

Figure 5

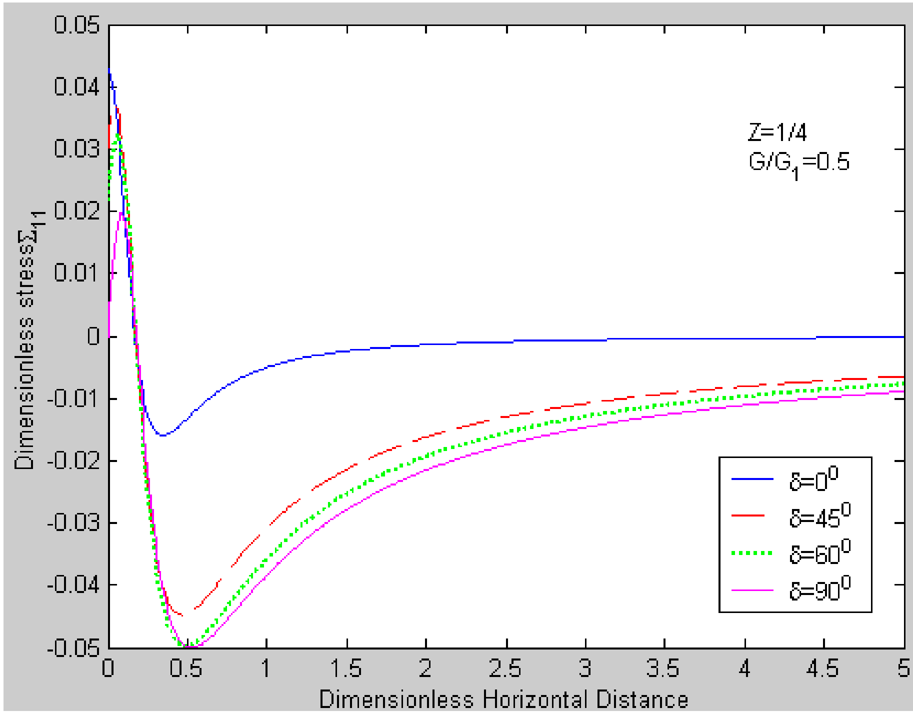

Figure 6

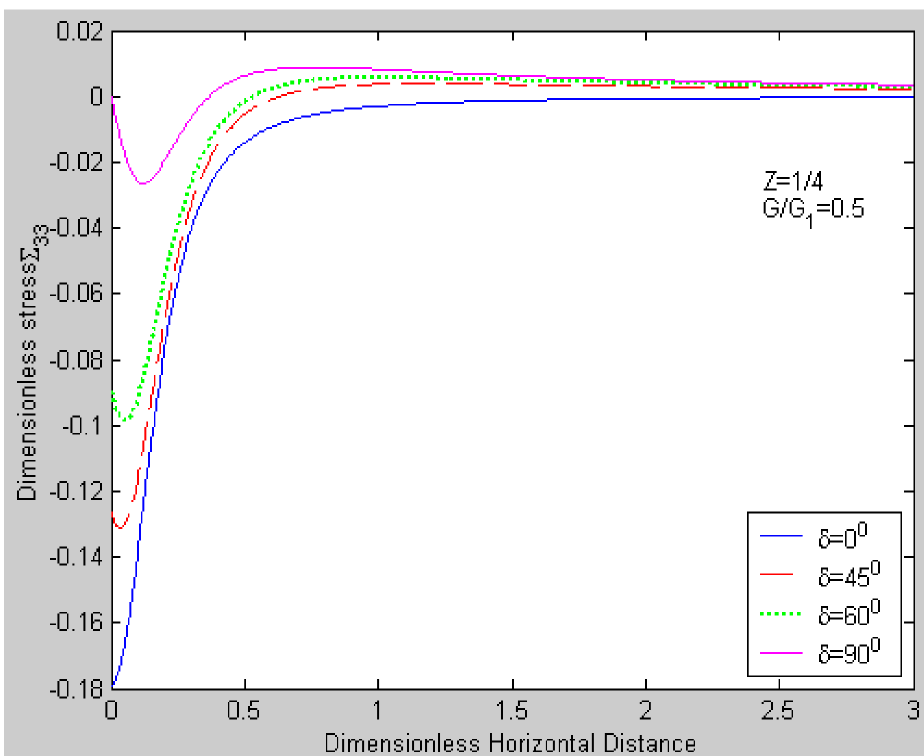

Figure 7

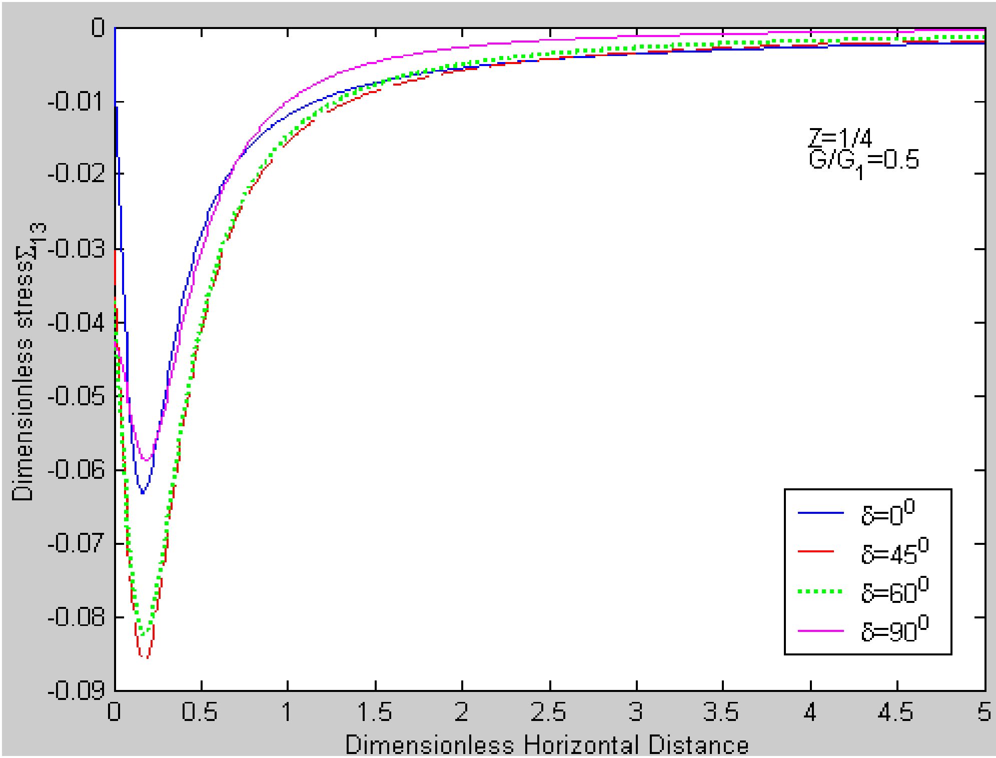

Figure 8

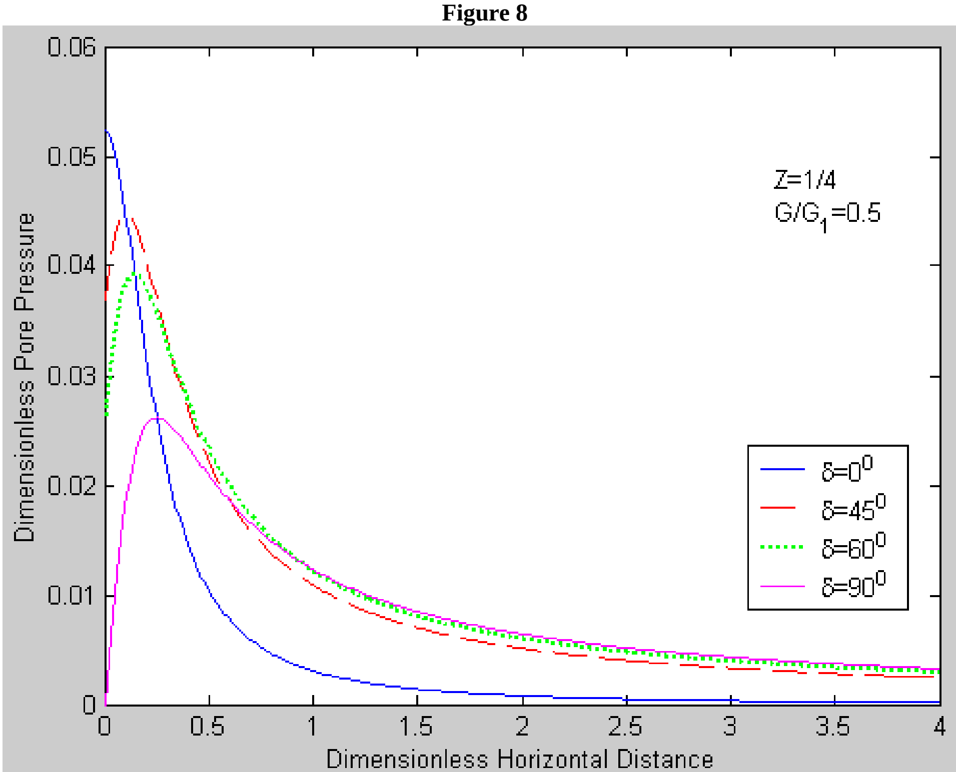

Figure 9

APPENDIX $(\boldsymbol{x} > 0)$

$$\int_{-\infty}^{\infty} e^{-|k|x} e^{-\iota ky}\, dk = \frac{2x}{y^2 + x^2},$$

$$\int_{-\infty}^{\infty} |k| e^{-|k|x} e^{-\iota ky}\, dk = \frac{2(x^2 - y^2)}{(y^2 + x^2)^2},$$

$$\int_{-\infty}^{\infty} \frac{k}{|k|} e^{-|k|x} e^{-\iota ky}\, dk = \frac{-2\iota y}{y^2 + x^2},$$

$$\int_{-\infty}^{\infty} k e^{-|k|x} e^{-\iota ky}\, dk = \frac{-4\iota xy}{(y^2 + x^2)^2},$$

$$\int_{-\infty}^{\infty} \frac{1}{k} e^{-|k|x} e^{-\iota ky}\, dk = -2\iota \tan^{-1}\left(\frac{y}{x}\right),$$

$$\int_{-\infty}^{\infty} \frac{1}{|k|} e^{-|k|x} e^{-\iota ky}\, dk = -\log(y^2 + x^2)$$

REFERENCES

[1] Aki,K. and Richards, P.G. , 1980,Quantitave Seismology.Theory and Methods W.H.Freeman,San Francisco,932.

[2] Ben-Menahem, A. and Singh, S. J., 1981, Sesmic Waves and Sources, Springer-Verlag, New York, pp.1108.

[3] Bell M.L. and Nur A., 1978, Strength changes due to reservoir-induced pore pressure and stresses and application to Lake Oroville, J.Geophys. Res., 83, 4469-4483.

[4] Biot, M. A., (1941a), General theory of three dimensional consolidation. J.Appl.Phys.,12, 155-164.

[5] Biot, M. A.,1956, General solutions of the equations of elasticity and consolidation for a porous material, J.Appl.Mech., 78, 91-98.

[6] Chau, K. T., 1996, Fluid point source and point forces in linear elastic diffusive half-spaces, Mech.Mater. , 23,241-253.

[7] Chugh,Samta, ,Madan,D.K. and Singh, Kuldip, (2011), Plane strain deformation of an initially unstressed elastic medium, Applied Mathematics and Computation, 217, 21, 8683-8692.

[8] Debnath, L., 1995,Integral Transforms and their Application, CRC Press. Inc., New York.

[9] Garg,N.R., Goel, A., Miglani, A. and Kumar, R.,2003, Plane strain deformation of an orthotropic elastic medium using an eigenvalue approach, Earth Planets Space,55,3-9.

[10] Heaton, T. H. and Heaton, R. E., 1989, Static deformation from point sources and point force couples located in welded elastic Poissonian half-spaces: implication for sesmic moment tensors, Bull. Seismological Soc. Am., 79,813-841.

[11] Kumari, G., Singh S. J. and Singh, K., 1992, Static deformation of two welded half-spaces caused by a point dislocation source, Phys. Earth Planet.Int., 73, 53-76.

[12] Kumar, R., Miglani, A. and Garg, N.R., (2000), Plane strain problem of poroelasticity using Eigen value approach, Proceeding Indian Acad. Sci. (Earth Planet. Sci.), 109, 371-380.

[13] Kumar, R., Miglani, A. and Garg, N.R., (2002), Response of an anisotropic liquid saturated porous medium due to two dimensional sources, proceeding Indian Acad. Sci. (Earth Planet. Sci.), 111,143-151.

[14] Love, A.E.H., 1944, A Treatise on the Mathematical Theory of Elasticity, Dover Publications, New York.

[15] Maruyama, T., 1966, On two dimensional elastic dislocations in an infinite and semi-infinite medium; Bull. Earthq. Res. Inst., 44, 811-871.

[16] Pan, E., 1999, Green's functions in layered poroelastic half spaces, Int J. Numer. Anal. Meth. Geomech. 23, 1631-1653.

[17] Rajapakse, R. K. N. D. and Seenjuntichai T., 1993, Fundamental solutions for a poroelastic half space with compressible constituents, J.Appl. Mech., 60,844-856.

[18] Rani S., Singh, S.J., 2007, Quasi-static deformation due to two-dimensional seismic sources embedded in an elastic half-space in welded contact with a poroelastic half-space, J. Earth Syst. Sci., 116, 99-111.

[19] Rice, J. R. and Clearly, M. P., 1976, Some basic stress diffusion solutions for fluid saturated elastic porous media with compressible constituents, Rev. Geophys. Space Phys., 14,227-241.

[20] Rudnicki, J.W., 1987, Plane strain dislocations in linear elastic diffusive solids, J.Appl. Mech., 54, 545-552.

[21] Rudnicki, J.W. and Roeloffs, E., 1990, Plane strain shear dislocations moving steadily in linear elastic diffusive solids, J.Appl. Mech., 57, 32-39.

[22] Saada, A.S., 1974, Elasticity-theory and application, Pergamon Press Inc., New York.

[23] Selim, M.M. and Ahmed, M.K., (2006), Plane strain deformation of an initially stressed orthotropic elastic

medium, Applied Mathematics and Computation,175, 221-237.

[24] Selim, M.M., (2007), Static deformation of an irregular initially stressed medium, Applied Mathematics and Computation, 188,1274-1284.

[25] Singh, S. J. and Rani, S., 2006, Plane strain deformation of a multilayered poroelastic half-space by surface loads, J. Earth Sys.Sci. , 115,685-694.

[26] Singh, S. J., Rani, S. and Garg N. R., 1992,Displacements and stresses in two welded half-spaces due to two dimensional sources ,Phys. Earth Planet. Inter.,70, 90-101.

[27] Sneddon, I.N., 1951, Fourier transforms, McGraw-Hill, New York.

[28] Wang, H. F., 2000, Theory of Linear Poroelasticity, Princeton Univ. Press, Princeton.