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Remark on Some Operations of Intuitionistic Fuzzy Sets

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ABSTRACT

We show by example that Definition 3.3 and Proposition 3.5 in [7] can only preserve the property of intuitionistic fuzzy set if the hesitation margin is negligible. We also obtain a result from the Proposition.

Keywords: *Fuzzy sets, Intuitionistic fuzzy sets, Modal operators.*

1. INTRODUCTION

In [2, 3], Intuitionistic Fuzzy Set (IFS) was introduced as a generalization of fuzzy set earlier proposed in [1]. IFS attracted much attention due to its significant in tackling vagueness or the representation of imperfect knowledge. There are volume of literatures involving the fundamentals, theory and applications of IFS. The cardinal aim of this work is to proffer solution to some errors pin-pointed by [8] in Definition 3.3 and Proposition 3.5 of [7] by introducing a restriction to the definition of IFS in [3].

Definition 1: Fuzzy set [1]

Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x)$: $X \longrightarrow [0,1]$ is the membership function of the fuzzy set A.

Definition 2

Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu A x,\ \nu A(x)$: $X \longrightarrow [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$. Furthermore, we have $\prod_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\prod_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\prod_A(x) \in [0,1]$ i.e., $\prod_A(x): X \longrightarrow [0,1]$ for every $x \in X$. $\prod_A(x)$, expresses the lack of knowledge of whether x belongs to IFS A or not.

Definition 3: [8]

If A and B are two IFSs of the set X, then;

(i) $A \subseteq B$ iff $\forall x \in X$, $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$

(ii) $A = B$ iff $\forall x \in X$, $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$

(iii) $\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$

(iv) $\Diamond A = \{\langle x, 1 - \nu_A(x),\ \nu_A(x), \rangle : x \in X\}$

Definition 4: Normalization [7]

The normalization of an intuitionistic fuzzy set A of the universe X, denoted by NORM (A) is defined as:

$$\text{NORM}\ (A) = \{\langle x,\ \mu_{NORM(A)}(x_i),\ \nu_{NORM(A)}(x_i)\rangle : x \in X\},$$

where $\mu_{NORM(A)}(x_i) = \frac{\mu_{(A)}(x_i)}{sup\left(\mu_{(A)}(x_i)\right)}$ and $\nu_{NORM(A)}(x_i) = \frac{\nu_{(A)}(x_i) - \inf(\nu_{(A)}(x_i))}{1 - \inf(\nu_{(A)}(x_i))}$ for $x_i \in X$, $i = 1, \ldots, n$ and $\inf\left(\nu_{(A)}(x_i)\right) \ne 0$.With this definition, we have the following proposition:

Proposition 1 [7]

For any IFS A of the universe X,

(i) if $\prod_A(x_i) = 0$, then $\prod_{NORM(A)}(x_i) = 0$

(ii) *NORM*($\Box A$) = $\Box$(*NORM* (A)),

(iii) *NORM* ($\Diamond A$) = $\Diamond$(*NORM*(A)).

In [8], Definition 3.3 and Proposition 3.5 of [7] were disproved with a given counter example but we observed that whenever Π is negligible i.e. $\Pi \approx 0$, both Definition 3.3 and Proposition 3.5 of [7] are valid. That is, the normalization of an IFS A becomes IFS whenever we restrict the parameters to such that Π does not exist. That is, $\mu_A(x) + \nu_A(x) = 1$. We verify this assertion with an example.

2. EXAMPLE

Let the universe $X = \{x_1, x_2, x_3\}$ and let an IFS $A = \{\langle 0.6, 0.4\rangle, \langle 0.8, 0.2\rangle, \langle 0.7, 0.3\rangle\}$ where Π= 0

Then $sup\ (\mu_{(A)}(x_i)) = 0.8$, $inf\left(\nu_{(A)}(x_i)\right) = 0.2$. We have $\mu_{NORM(A)}(x_1) = \frac{0.6}{0.8} = 0.75$, $\mu_{NORM(A)}(x_2)=\frac{0.8}{0.8} = 1.0$, $\mu_{NORM(A)}(x_3)=\frac{0.7}{0.8} = 0.875$, $\nu_{NORM(A)}(x_1) = \frac{0.2}{0.8} = 0.25$, $\nu_{NORM(A)}(x_2) = \frac{0.0}{0.8} = 0$, $\nu_{NORM(A)}(x_3) = \frac{0.1}{0.8} = 0.125$,

Then

$$NORM(A) = \{\langle 0.75, 0.25\rangle, \langle 1.0, 0\rangle, \langle 0.875, 0.125\rangle\}$$

Clearly, $\mu_{NORM(A)}(x_i) + \nu_{NORM(A)}(x_i) = 1$ for $i = 1,2,3$ which satisfies the property of IFS Therefore, *NORM(A)* is an IFS whenever Π ≈0 .

Next, we verify the Proposition 3.5 in [7].

(i) If $\prod_A(x_i) = 0$, then $\prod_{NORM(A)}(x_i) = 0$

This is obvious since $\mu_{NORM(A)}(x_i) + \nu_{NORM(A)}(x_i) = 1$ for $i = 1,2,3$

(ii) *NORM*(□*A*) = □*(NORM* (*A*)),

As Π⟶ 0, $\mu_A(x) = 1 - \nu_A(x)$ and $\nu_A(x) = 1 - \mu_A(x)$.Then, it follows that $\square A = A$ and $\lozenge A = A$ i.e. $\square A = \lozenge A = A$.

Therefore, *NORM*(*A*) = *NORM*(□*A*).

Also, if *NORM*(*A*) ={⟨0.75,0.25⟩,⟨1.0,0.0⟩,⟨0.875,0.125⟩}, then

□*NORM*(*A*) ={⟨0.75,0.25⟩,⟨1.0,0.0⟩,⟨0.875,0.125⟩}from Definition 3(iii).

Hence, *NORM* (□*A) =* □ *(NORM(A)).*

(iii) *NORM (◊A) = ◊(NORM(A)).*

The result follows as in (*ii)*

Corollary: For any IFS *A* of the universe *X*;

(i) NORM (□*A) = NORM (◊A),*

(ii) □ *(NORM(A)) =◊(NORM(A)).*

Proof

Given that $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$ .Since Π ≈0 ,it means that $\mu_A(x) = 1 - \nu_A(x)$ and $\nu_A(x) = 1 - \mu_A(x)$ for every $x \in X$.

From Definition 3, $\square A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle: x \in X\}$=$\{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$ =A. Also, from Definition 3, $\lozenge A$=$\{\langle x, 1 - \nu_A(x), \nu_A(x), \rangle: x \in X\}$=$\{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$=$A$. Then, we conveniently say that $\square A = \lozenge A = A$ *as* Π ≈0. *So* NORM (□*A) = NORM (◊A)*

Next, we show that □ *(NORM(A)) =◊(NORM(A)).*

From Definition 4,

NORM $(A) = \{\langle x, \mu_{NORM(A)}(x), \nu_{NORM(A)}(x)\rangle : x \in X$ }. So $\mu_A(x) + \nu_A(x) = 1$ for Π ≈0.

Also,

□*(NORM(A))=*{⟨*x*, $\mu_{NORM(A)}(x)$, 1-$\mu_{NORM(A)}(x)$⟩*:* $x \in X$} =$\{\langle x, \mu_{NORM(A)}(x), \nu_{NORM(A)}(x)\rangle : x \in X\}$= *NORM(A).*

But◊*(NORM(A))=*{⟨*x*, 1-$\nu_{NORM(A)}(x)$, $\nu_{NORM(A)}(x)$⟩*:* $x \in X$ }

=$\{\langle x, \mu_{NORM(A)}(x), \nu_{NORM(A)}(x)\rangle : x \in X\}$= *NORM(A).*

Therefore, □ *(NORM(A)) =◊(NORM(A)).*

3. CONCLUSION

It is apparent that, both Definition 3.3 and Proposition 3.5 in [7] are valid in fuzzy mathematics whenever the hesitation margin tends to zero.

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