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A Different Look at the Pol Erdos-Mordell Theorem

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ABSTRACT

In this study a different demonstration of the theorem regarding "the distance to the corners of a point placed inside a triangle (x_1, x_2, x_3 represent the distance of P to the points A, B, and C) which was proven by Luis Mordell and introduced by Pol Erdos in 1935 has been defined. With this theorem, U has been designated to constitute for the half perimeter of the triangle and by way of using

$$U<x_1< x_2 <x_3 <2U$$

The theorem has been invalidated.

Keywords: *Triangle,half perimeter*

1. INTRODUCTION

Let the letter P represent a random point at the inside of the $\overset{\Delta}{ABC}$. Let x_1, x_2, x_3 in turn to represent the distance of P to the points A, B, and C. Let the distance of P to the $[BC]$, $[AC]$, and $[AB]$ sides of the triangle in turn be represented by d_1, d_2, d_3.

$$x_1 + x_2 + x_3 \geq \left(\frac{a}{b}+\frac{b}{a}\right)\cdot d_3 + \left(\frac{b}{c}+\frac{c}{b}\right)\cdot d_1 + \left(\frac{a}{c}+\frac{c}{a}\right)\cdot d_2$$

İnequality is present.

This theorem has been first introduced by Pol Erdos in year 1935 and been proven by Luis Mordell in the same year for the first time. Many proof of this inequality has been introduced since than. Adre Aves by using the Ptolemi theorem, Leon Bankof by using similar triangles and angle calculations, W. Komorning by using the inequality of the area, Mordell and Barrow by using trigonometry have all proven this inequality.

We on the other hand, formed a proof as follows by taking advantage of the concepts such as "bisector and symmetrical in accordance to perpendicular" stated in the article written by Nicolaos Dergiades concerning this theorem.

Proof:

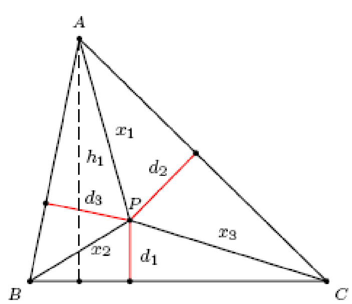

Figure 1

Let's draw up a vertical h_1 line from A corner to the BC side. (Figure 1) According to the Area Theorem in triangle;

$$\frac{a.h_1}{2} = \frac{a.d_1}{2} + \frac{b.d_2}{2} + \frac{c.d_3}{2} \tag{1}$$

At the APDH vertical trapezoid for $\forall d_1 \in R$

It is

$$x_1 + d_1 \geq h_1. \tag{2}$$

$$P \in h_1 \Rightarrow x_1 + d_1 = h_1$$

provided.

From the expression of (2) it is

$$ax_1 + ad_1 \geq ah_1.$$

Here when we place the result in the expression (1) in its place

$$ax_1 + ad_1 \geq ah_1 = ad_1 + bd_2 + cd_3$$

And

$$ax_1 \geq bd_2 + cd_3$$

$$x_1 \geq \frac{b}{a}d_2 + \frac{c}{a}d_3 \ldots \quad (3)$$

And if the same operations are applied to the sides of AB and AC in turn;

$$x_2 \geq \frac{a}{b}d_1 + \frac{c}{b}d_3 \ldots.$$

$$x_3 \geq \frac{a}{c}d_1 + \frac{b}{c}d_2 \ldots$$

And the last three inequalities give us

$$x_1 + x_2 + x_3 \geq d_1\left(\frac{a}{b} + \frac{a}{c}\right) + d_2\left(\frac{b}{a} + \frac{b}{c}\right) + d_3\left(\frac{c}{a} + \frac{c}{b}\right) \quad (4)$$

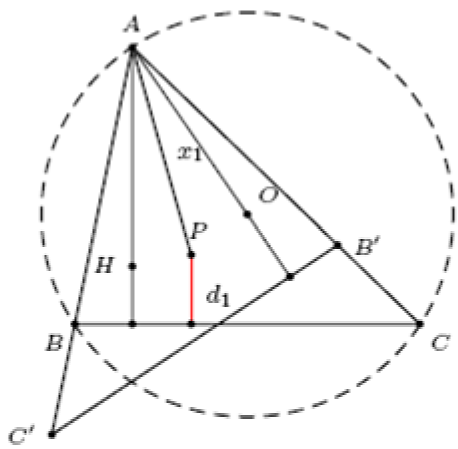

Figure 2

Let the P point be on the bisector belonging to the A corner on the ABC triangle. (Figure2)

The symmetrical of this triangle according to the A inner bisector is

$$\frac{a.h_1}{2} = \frac{a.d_1}{2} + \frac{b.d_3}{2} + \frac{c.d_2}{2}$$

in the expression (1) In this case; (5)

And when we put into the place the (5) expression stated in (2)

$$ax_1 + ad_1 \geq ah_1 = ad_1 + bd_3 + cd_2 \quad (6)$$

$$ax_1 \geq bd_3 + cd_2$$

$$x_1 \geq \frac{b}{a}d_3 + \frac{c}{a}d_2 \quad (7)$$

When we write x_2 and x_3 on the other sides

$$x_2 \geq \frac{a}{b}d_3 + \frac{c}{b}d_1$$

$$x_3 \geq \frac{a}{c}d_2 + \frac{b}{c}d_1$$

$$x_1 \geq \frac{b}{a}d_3 + \frac{c}{a}d_2$$

And when we add these three expressions, it is;

$$\geq d_3\left(\frac{a}{b} + \frac{b}{a}\right) + d_1\left(\frac{c}{b} + \frac{b}{c}\right) + d_2\left(\frac{c}{a} + \frac{a}{c}\right) \quad (8)$$

$d_1 + d_2 + d_3 \rangle\ 0$, $\frac{a}{b} + \frac{b}{a} \geq 2$, $\frac{b}{c} + \frac{c}{b} \geq 2$ and $\frac{a}{c} + \frac{c}{a} \geq 2$

And when this expression is placed in its place in the (8) expression it is,

$$x_1 + x_2 + x_3 \geq 2(d_1 + d_2 + d_3)$$

When $d_1=d_2=d_3$, P point becomes the center of the inner circle.

Therefore;

$$x_1 + x_2 + x_3 \geq 2(3r) = 6r$$

And the validity of the inequality

$$u < x_1 + x_2 + x_3 < 2u$$

no longer exists.

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