



## Improvement Modal Testing in Dynamics Revolves Structures to Investigate the Gyroscope Effect Performance

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### ABSTRACT

Revolve dynamic systems require careful consideration in their dynamic models as they include localized non-proportional damping and frequency-dependent gyroscopic effects. Devices such as engines, motors, disc drives and turbines all develop characteristic inertia effects that can be analyzed to improve the design and decrease the possibility of failure. At higher speeds, the inertia effects of the rotating parts must be represented in order to predict the rotor behavior. The natural frequency versus the spin-speed diagram or the interference diagram, often known as the Campbell diagram, is one of the important design tools to obtain critical speeds of rotating machineries. It is basically the graphical representation of the speed-dependent frequency equation. This variation is important when the effect of the gyroscopic couple is considered, which causes coupling of the rotor motion in two orthogonal planes. Campbell diagrams are commonly plotted either through numerical analyses or through modal testing methods and whirl orbital motion can be detected in this research.

It is important to identify all the critical speeds within the range of operation and analyse the damping effects, stiffness and other phenomena also their effects in the safe operation. There are several phenomena need to be detected such as gyroscopic effect which would create complexity in the mathematical procedures in modal analysis which need to be addressed and interpreted appropriately before they could be used in modal testing of rotating machinery.

The experimental technique used thus far is called Modal Testing. The technique has recently been applied to rotating structures and some research papers been published, however the full implementation of Modal Testing in active structures and the implications are not fully understood and are therefore in need of much further and more in depth investigations. Mathematical model of the system from the test data can be assembled.

The raw data obtained from experiment was used in finite element (FE) model for comparison. It has good capability for Eigen analysis and also good graphical facility and obtained good result.

**Keywords:** *Campbell diagram; Whirl motion; Modal test; Gyroscopic effect; Finite element method.*

### NOMENCLATURE

$I_d$	Mass moment of inertia of the disc about it-self.
$M \bar{l}^2$	Added inertia due to parallel axis shift.
$M$	Mass of disc = 0.8 kg.
$m$	Mass of shaft.
$d$	Diameter of disk.
$l$	Length of the shaft.
$H_{yy}$	Real and imaginary part.
$\Omega$	Is rotational speed of the rotor.
$[M],[C],[K]$	Mass, damping, and stiffness matrices.
$FRF$	Frequency response functions.
$x, f$	Displacement, external force vectors.
$Y \& Z$	Directions of movement for rotating machinery.
$I_o$	Is the polar mass moment of inertia.
$J$	Is the diametral mass moment of inertia of the disk.
$\omega_o$	Is the stationary natural frequency (equal to vertical & horizontal modes in our case).
$\omega_\Omega$	Is the natural frequency of the rotor at a given speed.
$\omega_1, \omega_2$	Are backward and forward whirl natural frequency of the system

## 1. INTRODUCTION

The first vibrational theory developed for rotor dynamic analysis was conducted by Foppl (1895) and Jeffcott (1919). Foppl mathematically showed that operation above the first critical speed is possible and stable using an undamped flexible rotor. Jeffcott added damping to the rotor system analysis. Continuing research led to the understanding and causes of rotor instabilities such as internal damping, oil whip, oil whirl, cross-coupling stiffness in bearings, seals and gyroscopics. The Jeffcott, a single mass flexible, rotor was used by rotor dynamics despite its simplistic formulation and assumptions to analyze complex rotor systems and effects. However, with the development and needs for analysis of more complex systems such as gas turbines and jet engines, the Jeffcott rotor would not suffice to accurately model these more complex systems, thus alternative methods were developed [1, 2].

The inability and difficulty of deriving analytical models, such as the Jeffcott rotor, to model complex rotor systems led to the implementation of computational models for solving rotor dynamic systems. Developments in computing capabilities and a transition from analytical modeling to modeling the actual system geometry arose in the 1960's and 1970's. The two methods developed for such analytical modeling were the Transfer Matrix method (TMM) and the Finite Element method (FEM)[3].

Rotating machines such as steam or gas turbines, turbo-generators, internal combustion engines, motors, and disc drives can develop inertia effects that can be analyzed to improve the design and decrease the possibility of failure. Current trends in rotating equipment design focus on increased speeds, which increase operational problems caused by vibration. At higher rotational speeds, the inertia effects of rotating parts must be consistently represented to accurately predict rotor behavior.

Inertia effects in rotating structures are usually caused by gyroscopic moment introduced by the precise motions of the vibrating rotor as it spins. As spin velocity increases, the gyroscopic moment acting on the rotor becomes critical [4]. Not accounting for inertia effects at the design level can lead to bearing and support structure damage. It is also important to consider bearing stiffness, support structure flexibility, and damping characteristics to understand the stability of a vibrating rotor. The best method for measurement of dynamic performance of machine is to obtain the frequency response functions at appropriate points [5]. The database consisting of complete mathematical model of the machine which includes both the supporting structures as well as the moving parts and is based on the structural dynamics characteristics of the

system. Linear structures for the purpose of vibration analysis may be modelled either as a continuous system or a discrete one. In the case of the latter method, the mass of the system is lumped into a finite number of masses connected by springs and dampers representing the stiffness and damping in the system respectively. The system is then said to possess an infinite number of degrees of freedom. Damping mechanism comes in different forms such as viscous, structural, hysteretic and coulomb.. The equations of motion of the discrete model consists of a finite number of second order ordinary differential equations coupled together, in which case they have to be solved simultaneously [6, 7]. The process becomes progressively more difficult as the number of degrees of freedom increases.

There is a way of de-coupling the equations of motion using the orthogonality properties of the natural modes. In this method, the system is transformed from generalised coordinates to a new set of coordinates called principal coordinates, in which case the equations are solved independently and inverse transformed into the generalised coordinates. The method of de-coupling the equations of motion is called modal analysis [6, 8 & 9].

In modal analysis applied to passive structures (non-rotating) it is assumed that the system matrices are symmetric, which simplifies the transformation. While the symmetric assumption of system matrices is valid for passive structures, it is not suitable for structures containing rotating elements. If the theory of modal analysis for passive structures is applied to active structures the full de-coupling is not achieved. However it is possible to use a different technique to achieve full de-coupling of the differential equations of motion [10, 11 & 12].

### 1.1 Gyroscopic Effect

For a structure spinning about an axis  $\Delta$ , if a rotation about an axis perpendicular to  $\Delta$  (a precession motion) is applied to the structure, a reaction moment appears. That reaction is the gyroscopic moment [13, 14]. Its axis is perpendicular to both the spin axis  $\Delta$  and the precession axis. The resulting gyroscopic matrix couples degrees of freedom that are on planes perpendicular to the spin axis. The resulting gyroscopic matrix,  $[G]$ , will be skew symmetric. Gyroscopic effect occurs whenever the mode shape has an angular (conical/rocking) component. Also at the same section it is mentioned that having an overhung load will make the gyroscopic occurs in both, 1<sup>st</sup> and 2<sup>nd</sup> mode of vibration that mean gyroscopic effect occurs at the lower frequencies. It is sufficient to allocate the experiment in the 1<sup>st</sup> mode, [13, 15], see Figure 1, Gyroscopic effect and picture 1, Gyroscopic body.

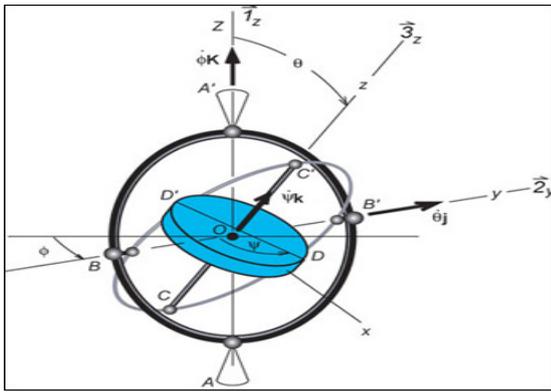
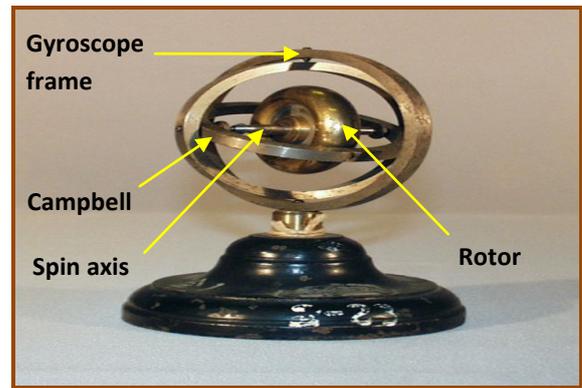


Figure 1. The gyroscopic effect [13, 15].



Picture 1. Gyroscopic body.

### 1.1.1 Whirl

When a rotating structure vibrates at its resonant frequency, points on the spin axis undergo an orbital motion, called whirling. Whirl motion can be a forward whirl (FW) if it is in the same direction as the rotational velocity or backward whirl (BW) if it is in the opposite direction [4, 16].

## 2. OBJECTIVE

Gyroscopic action occurs whenever the axis of the rotating body is made to change its direction. Rotating components produce a gyroscopic couple, which can lead to undesirable effects; for example, when a road vehicle travels round a bend the gyroscopic couple produced by turning the axes of the wheels tends to overturn the vehicle. In an aircraft changing direction, the gyroscopic couple produced due to the rotating components of the engine causes the aircraft to pitch up or down. In ship pitching, the turbine rotor axis of rotation pitches with the ship and it tends to make the ship swing sideways. Gyroscopic effects can also be used to advantage as in the case of gyro-stabilizers in which the undesirable effect is resisted and provides stabilization so we study all behaviors of gyroscopic.

## 3. METHODS

### 3.1 System Mathematical Model

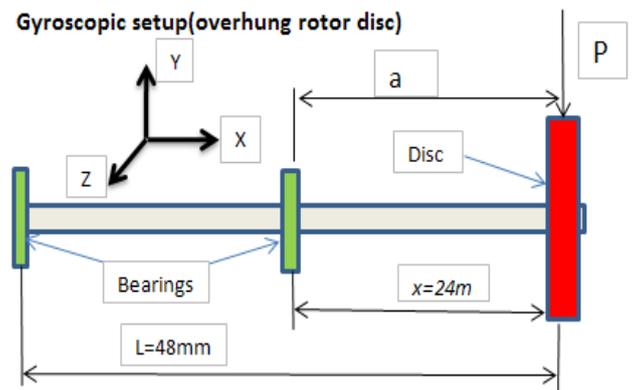
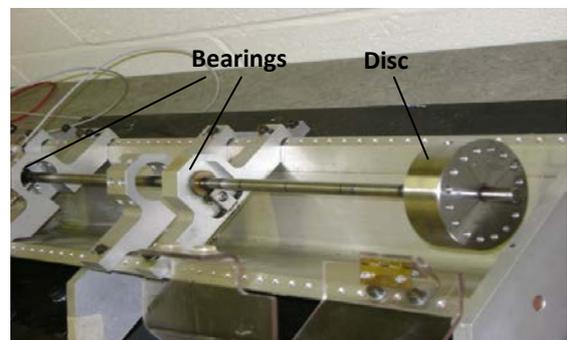
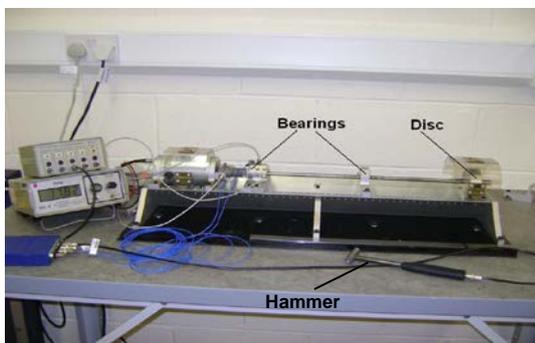


Figure 2. Gyroscopic setup parameter [11]



Picture 2. Experimental setup for the modal testing

### 3.1.1 Equations of movement

The general equations of motion for a multi-degree of freedom vibratory system may be written as [6, 9 &15]:

$$[M]\{\ddot{q}(t)\} + [G + C](\Omega)\{\dot{q}(t)\} + [B] + [K](\Omega)\{q(t)\} = \{F(t)\} \quad (1)$$

In the above equations the mass, damping and stiffness matrices are speed dependent. The damping matrix also includes the gyroscopic effect [G], of the rotating system, so the equation of motion for a rotor consisting of shaft and a single over hanged disc can be written as follow:-

$$\begin{bmatrix} \frac{I_o}{l^2} & 0 \\ 0 & \frac{I_o}{l^2} \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \ddot{z} \end{Bmatrix} + \begin{bmatrix} c_y & \\ & -\frac{J\Omega}{l^2} \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{z} \end{Bmatrix} + \begin{bmatrix} \frac{J\Omega}{l^2} & \\ & c_z \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{z} \end{Bmatrix} + \begin{bmatrix} k_y & 0 \\ 0 & k_z \end{bmatrix} \begin{Bmatrix} y \\ z \end{Bmatrix} = \begin{Bmatrix} f_y \\ f_z \end{Bmatrix} = \begin{Bmatrix} F_y \\ F_z \end{Bmatrix} e^{i\omega t} \quad (2)$$

The frequency response function (point and transfer) due to a single excitation and single response point obtained by the following expression [6, 12]

$$H_{yy}(\omega) = \frac{Y}{F_{yy}} = \frac{(k l^2 + i \omega c l^2 - \omega^2 I_o)}{(k^2 l^2 + 2 i \omega c k l^2 - \omega^2 (c^2 l^2 + 2 I_o k + (J \Omega / l)^2) - 2 i \omega^3 c I_o + \omega^4 (I_o / l)^2)} \quad (3)$$

$$H_{yz}(\omega) = \frac{(i \omega J \Omega)}{(k^2 l^2 + 2 i \omega c k l^2 - \omega^2 (c^2 l^2 + 2 I_o k + (J \Omega / l)^2) - 2 i \omega^3 c I_o + \omega^4 (I_o / l)^2)} \quad (4)$$

Where  $H_{yy} = f(\omega, \Omega) + f(\omega, \Omega) i$  Real and imaginary part.

By using the Equation (4).The frequency response function (FRF) of the rotor was found, as this is a graphical method shown in Figure 9.

The polar mass moment of inertia ( $I_o$ ) about the point o can be calculated as follows:

$$I_o = I_d + M l^2 + \frac{m l^2}{3} \quad (5)$$

$$I_d = \frac{\pi d^4}{64}, \quad \frac{m l^2}{3}, \quad \gamma = \frac{J}{I_o}$$

$$\omega_a = \sqrt{\left(\frac{k_y l^2}{I_o}\right)}$$

Represent the vertical mode.

$$\omega_b = \sqrt{\left(\frac{k_z l^2}{I_o}\right)}$$

Represent the horizontal mode.

For rotating conditions the natural frequencies has to be calculated from the equations below,

$$\omega_{\Omega}^2 = \omega_o^2 + \left(\frac{\gamma^* \Omega}{2}\right)^2 \tag{6}$$

$$\omega_{1,2} = \omega_{\Omega} \pm \left(\frac{\gamma^* \Omega}{2}\right) \tag{7}$$

### 3.2 Rotate Acquisition and Analysis in Experimental

The Rotate Acquisition and Analysis software package is part of (m + p international's). It is designed for troubleshooting and analyzing noise or vibration problems related to the speed characteristics of rotating or reciprocating components of a machine in operation [17, 18].

#### 3.2.1 Applications

- Rotating machinery diagnostics
- Analysis of noise and vibration problems related to rotational forces, e.g. in automotive and aircraft engines, transmissions, gearboxes, compressors, pumps, turbines and electric motors
- Order analysis

#### 3.2.2 Test setup

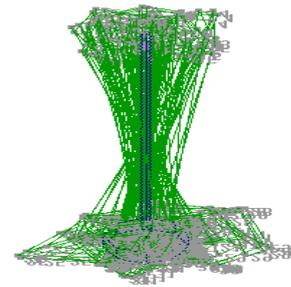
The test rotor is shown in picture (2). Basically, the rotor consisted of a shaft with a nominal diameter of 10 mm, with an overall length of 610 mm. Two plain bearings, RK4 Rotor Kit made by Bentley Nevada (the advanced power systems energy services company) are used to extract the necessary information for diagnostic of rotating machinery, such as turbines and compressor. The testing of the process will be conducted on the rotary machine as the project is based on rotary dynamics reach practical results for the purpose of subsequently applied machinery rotary by using (Smart office program), and then do the experimental testing using the impact test, installed fix two accelerometer (model 333B32), sensitivity (97.2 & 98.6) mV/g in Y & Z direction and roving the hammer (model 4.799.375, S.N24492) on each point for the purpose of generating strength of the movement for the vibration body and the creation of vibration for that with creating a computer when taking readings in public that he was dimensions and introducing it with the data within the program (Smart office), [18, 19]. Configuration for testing on the machines with rotary machine the creation of all necessary equipment for that purpose with geometry design wizard (see, Figure 3).



A-At start rotation.



B- After rotation, (first mode shape).



C- After rotation, (second mode shape).

Figure 3: Geometry design for modal (gyroscopic effect), experimental test using smart office.

### 3.3 Simulation of Model (FE)

A model of rotor system with an overhung disc with multi degree of freedom (Y and Z directions) has been used to demonstrate the above capability see Figure 4. A program has been written in (ANSYS 12), Postprocessing commands (/POST1). For preparing of Campbell diagrams, (CAMPBELL) command was used in input file. Applying of gyroscopic effect to

rotating structure was carried by using (CORIOLIS) command. This command also applies the rotating damping effect. Another command which was used in input file (SYNCHRO) that Specifies whether the excitation frequency is synchronous or asynchronous with the rotational velocity of a structure in a harmonic analysis; (PLCAMP) plots Campbell diagram data, (PRCAMP) prints Campbell diagram data [20, 21 & 22].

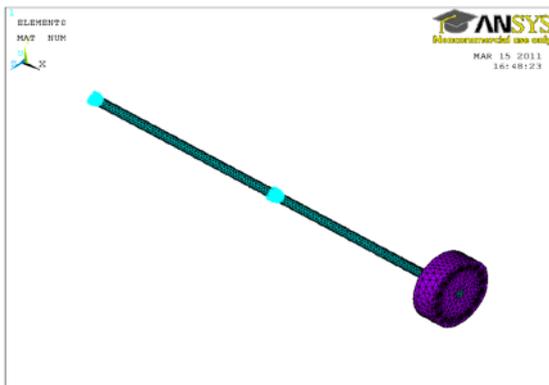


Figure 4: A- Finite element model (FE) (gyroscopic geometry) in ANSYS (APDL), B-(ANSYS) work bench.

#### 3.3.1 Campbell diagram

A modal analysis with multiple load steps corresponding to different angular velocities  $\omega$ , a Campbell diagram (PLCAMP or PRCAMP) shows the evolution of the natural frequencies [21]. ANSYS determines Eigen frequencies at each load step. The plot showing the variation of Eigen frequency with respect to rotational speed may not be readily apparent. For example, if the gyroscopic effect is significant on an Eigen mode, its frequency tends to split so much that it crosses the other frequency curves as the speed increases.

#### 3.3.2 Critical speeds

The PRCAMP command also prints out the critical speeds for a rotating synchronous (unbalanced) or asynchronous force. The critical speeds correspond to the intersection points between frequency curves and the added line  $F=s.\omega$  (where  $s$  represents  $SLOPE > 0$  as specified via PRCAMP). Because the critical speeds are determined graphically, their accuracy depends upon the quality of the Campbell diagram [21, 23]. To retrieve and store critical speeds as parameters, use the GET command.

### 3.3.3 Whirls and stability

As Eigen frequencies split with increasing spin velocity, ANSYS identifies forward (FW) & backward (BW) whirls, and unstable frequencies [4]. Because of the orbit shape the shaft makes when rotating, this mode is sometimes referred to as a “Cylindrical “mode. If it is viewed from the front, the shaft appears to be bouncing up and down. Therefore this mode is also

known as “bounce” or “translator” mode, [4, 24 & 25]. Not to forget that mean while the rotor is also rotating. Therefore the whirling motion of the rotor (orbit shape path) can be in the same direction as the shaft s rotation or can be in an opposite direction. This gives rise to the labels “forward whirl” FW and “backward whirl” BW, see Figure 5, Shows rotor cross sections over the course of time for both synchronous forward and synchronous backward whirl.

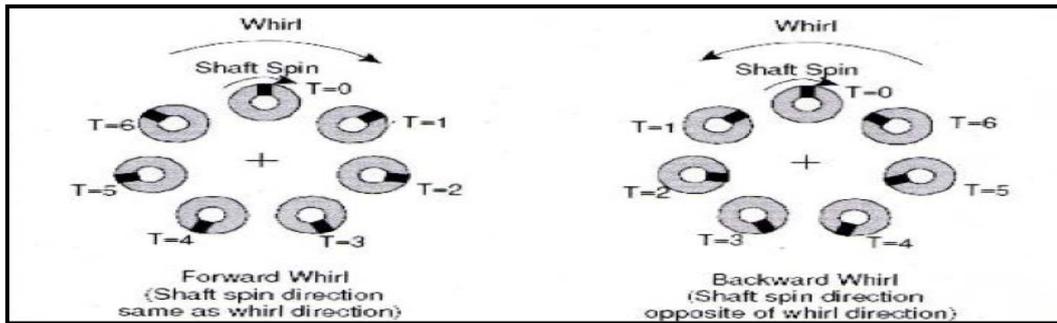
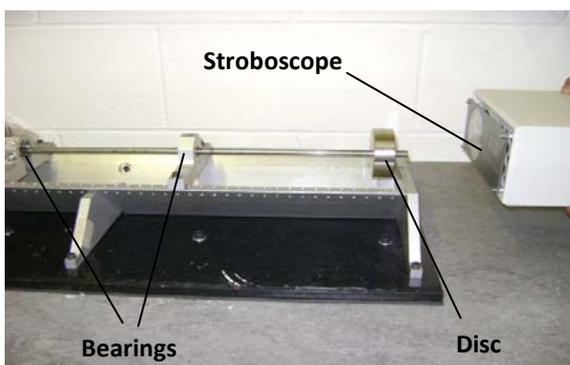


Figure 5: Whirl sense, [4, 20]

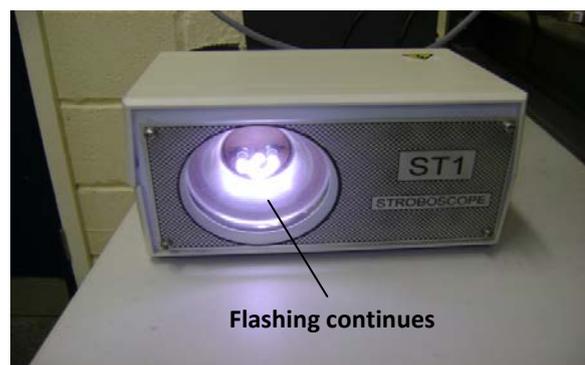
Note that for forward whirl, a point on the surface of the rotor moves in the same direction as the whirl. Thus,for synchronous forward whirl (i.e. unbalance excitation), a point at the outside of the rotor remains to the outside of the whirl orbit, [4, 20]. With backward whirl, on the other hand, a point at the surface of the rotor moves in the opposite direction as the whirl to the inside of the whirl orbit during the whirl.

## 4. RESULTS (TABLES & FIGURES)

We can use the STROBOSCOPE (model ST1, supply 1, volts 230, 50Hz) When triggering the stroboscope at flashing continues see Picture 3, B to recognize the forward (FW) and backward (BW) whirl components, that respectively increase and decrease in frequency with increased rotor speed.



A- FW, BW Whirl detection

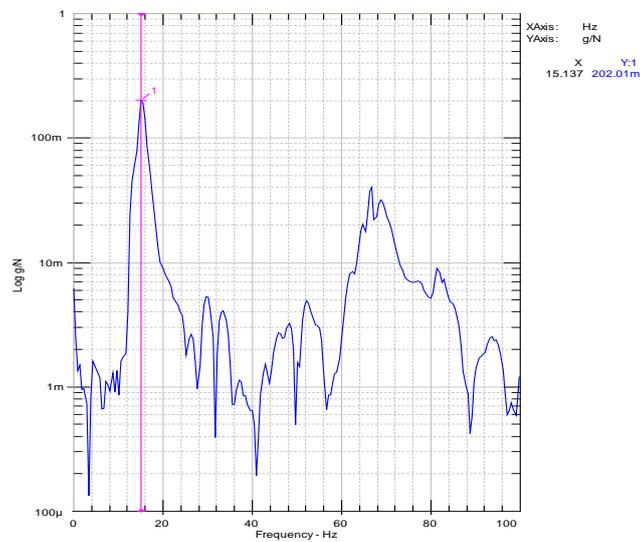


B- Stroboscope body

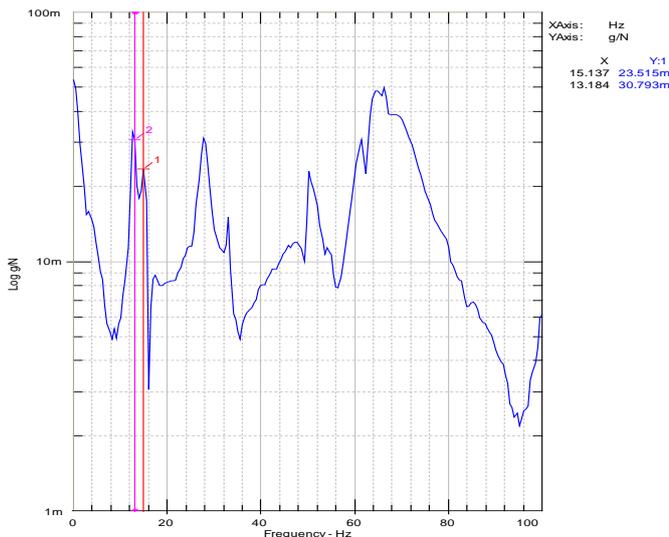
Picture 3: Experimental setup method for the gyroscopic to detection FW, BW whirl;

As it can be seen from the experimental Figures 6, 7 & graphical calculation Figure 9. Follows the direct receptance, which assume the excitation and response measurement are shown in figures at various speeds. The natural frequency of the rotor splits up into two parts, as the rotor starts rotating. According to the Equations, (6)-(7) it shows the critical speed of the stationary rotor; this then splits into two critical speeds all value shown in Table.1, as it can be seen from the Figures 6, 9. The gap widens up as the speed increases. Like it was discussed, this phenomenon is known as gyroscopic effect and occurs because of the Backward (BW) and forward (FW) whirl of the rotor. The Eigen vectors would show these phenomena, as the speed

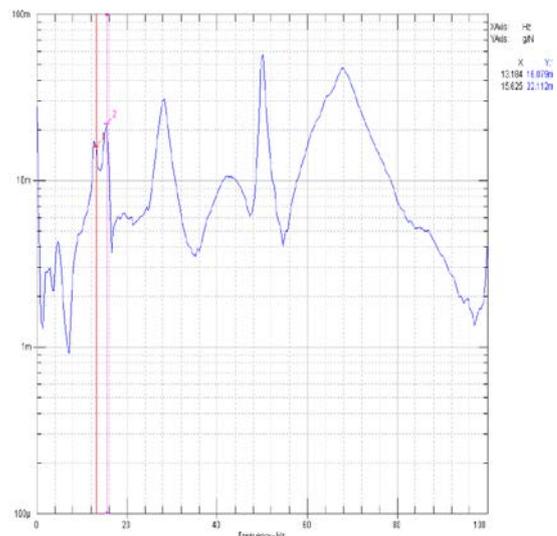
increases the gap between the two frequencies becomes larger, the anti-resonance between the two peaks is due to the fact that this is a point or direct measurement, a phenomenon in modal testing. Figure 8-A, show Experiment & Theoretical calculation value of relation between the (Natural frequencies, Hz) with respect increasing speed of rotation for the 1<sup>st</sup> mode, when the speed increasing the two lines are divergence the same reason before. All figure in experimental obtained frequency response functions (FRF) of the geometry by merging all the nodal response function into a single (FRF), by using the calculator function of the smart office software are shown in Figure 6, for different speed of rotation.



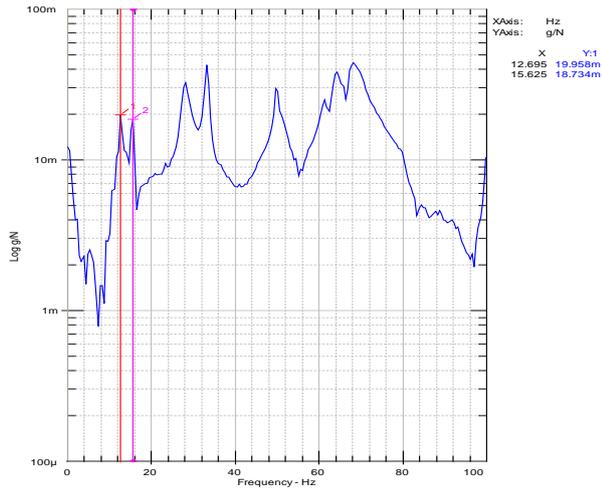
**A-Experimental result speed of rotation (30) RPM, (stationary).**



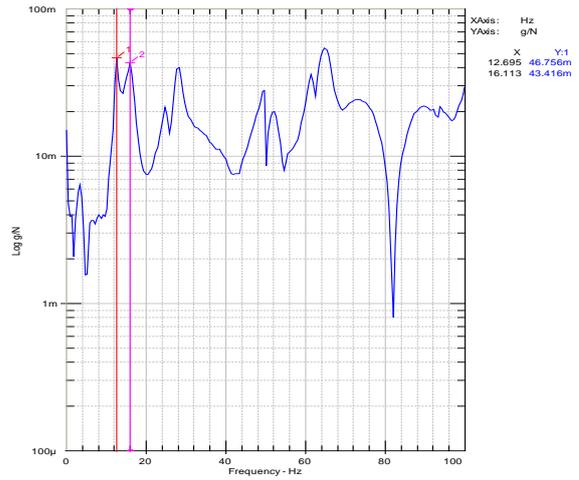
**B-Experimental result speed (2000) RPM.**



**C-Experimental result speed (3000) RPM.**

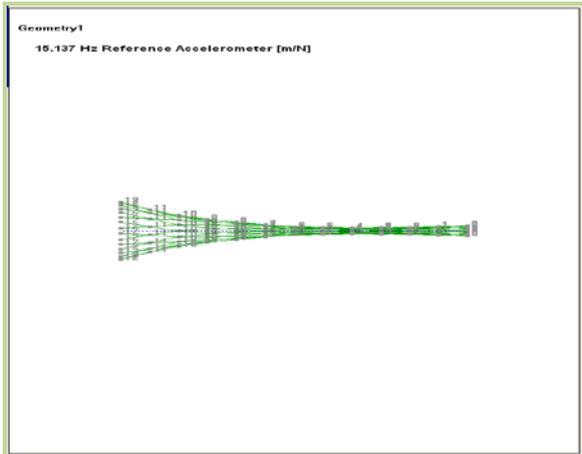


D-Experimental result speed (4000) RPM.

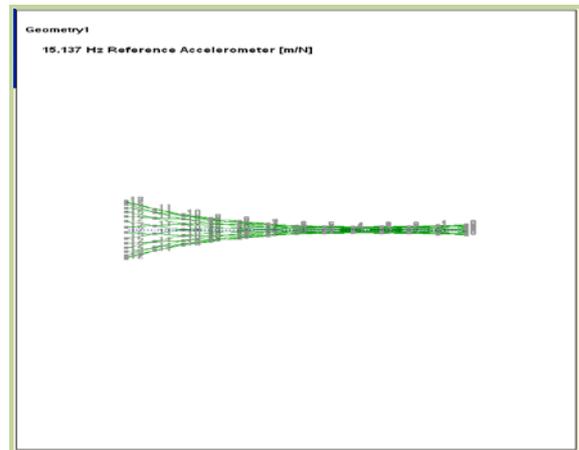


E-Experimental result speed (6000) RPM.

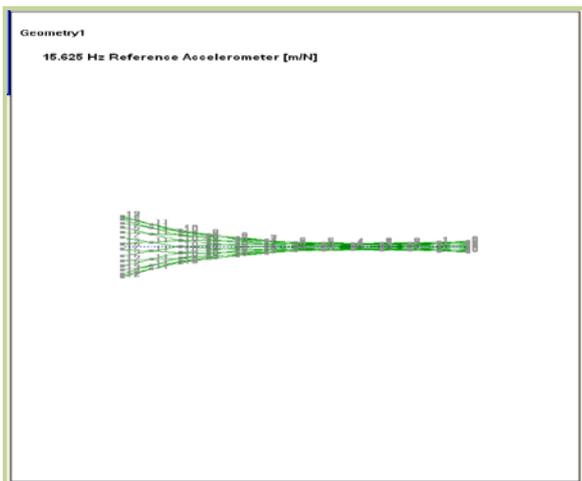
Figure 6: Experimental result show frequency response function curve (FRF) versus natural frequency (Hz) for different speed of rotation;



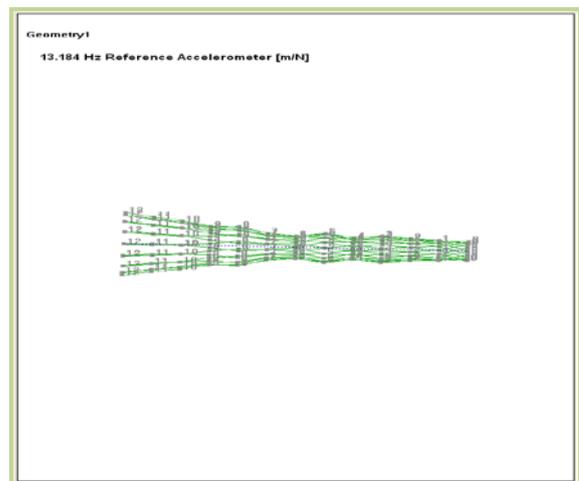
A-(15.13 Hz) at speed 30, 2000 rpm (FW).



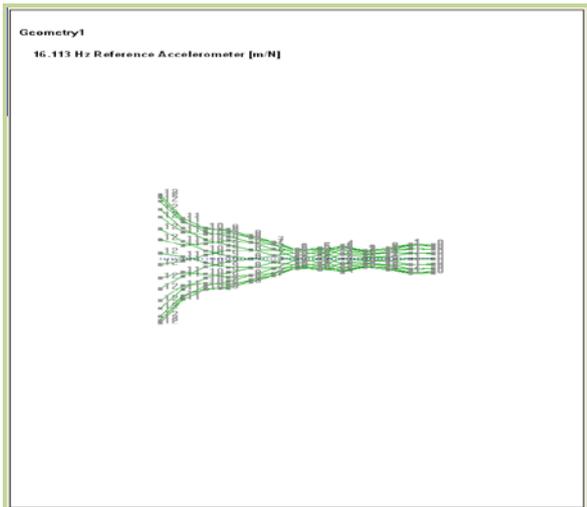
B-(15.13 Hz) at speed 30 rpm (BW).



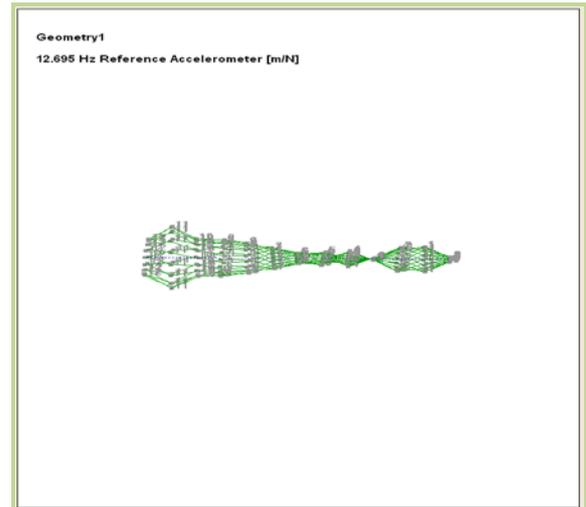
C-(15.62 Hz) at speed 3000, 4000 rpm (FW).



D-(13.18 Hz) at speed 2000, 3000rpm (BW).



E-(16.11Hz) at speed 6000 rpm (FW).



F-(12.69 Hz) at speed 4000, 6000 rpm (BW).

Figure 7: Experimental result show first mode shape for different speed of rotation (Gyroscopic effect) using smart office;

Table 1 Backward & Forward Natural Frequencies in Experimental and Theoretical Calculations of Gyroscopic Effect.

Speed of Rotation (RPM)	Experimental Results (Hz), BW	Experimental Results (Hz), FW	Theoretical Values, $\omega_1$ (Hz), BW	Theoretical Values, $\omega_2$ (Hz), FW
30	15.13	15.13	14.3	14.3
2000	13.18	15.13	13.5	15.2
3000	13.18	15.62	13.13	15.61
4000	12.69	15.62	12.76	16.07
6000	12.69	16.11	12.05	17.01

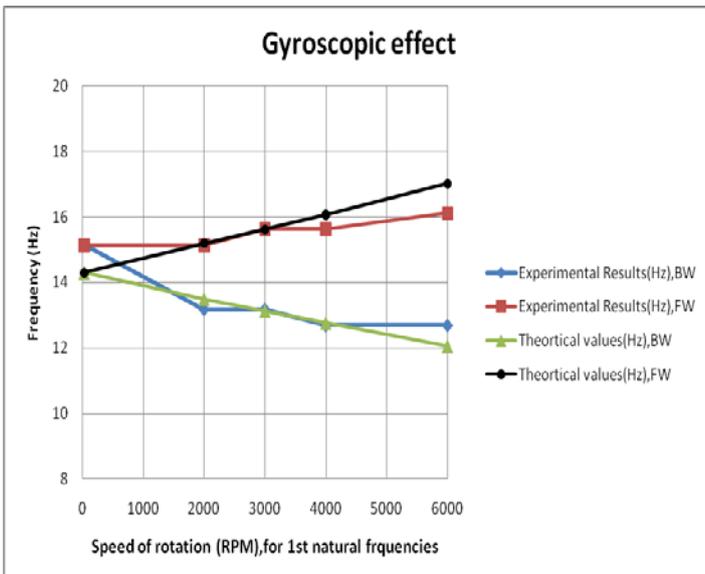
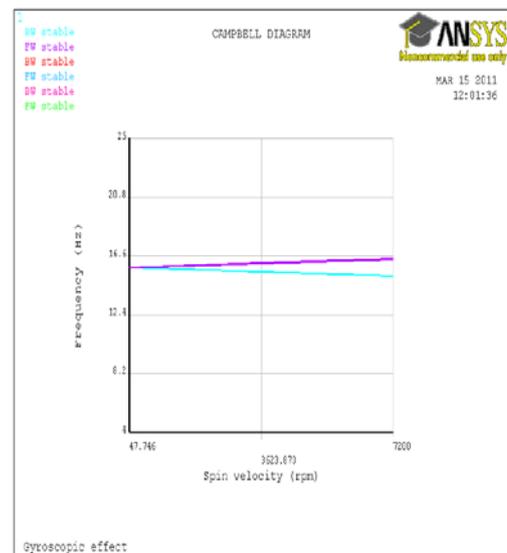


Figure 8. A- Experimental & Theoretical natural frequencies versus rotation for 1<sup>st</sup> natural frequency (Campbell diagram).



B-ANSYS, Campbell diagram speed of calculations gyroscopic effect.

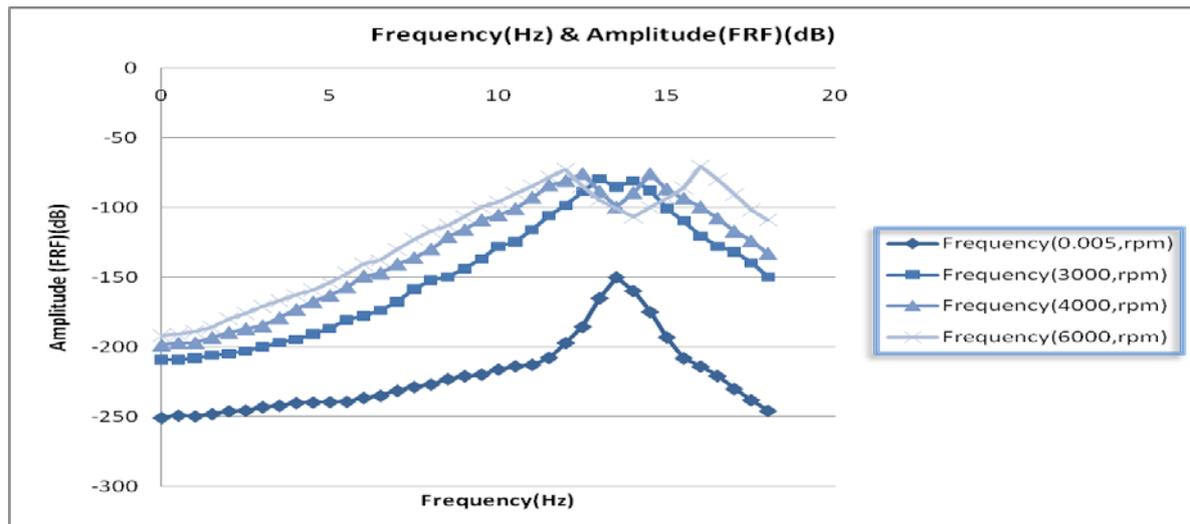


Figure 9: More Frequency response function (FRF) versus natural frequency (Hz), graphical calculations.

## 5. DISCUSSION AND CONCLUSION

In this paper investigate the behaviour of a rotor system with gyroscopic effect has been careed out , by increasing the rotional speed, the distance between the forward whirl and backward whirl increasees related to the mode-shape and the ratio of the polar mass moment of inertia ( $I_0$ ) to the diametral mass moment of inertia(J). Draw the campbell diagram and the outcom result Figure 8-A, B .

The calculation values obtained from the Equation (7), are perfectly nearby to the values obtained from the experimental that shown in Table 1, take cognizance Figure 8-A., and comparison with ANSYS Figure 8-B (Campbell diagram). By using Equation (4), frequency response function (FRF) of the rotor was found as this is a graphical method see Figure 9. In this figure we see the approval to experimental and ANSYS simulation the gap widens up as the speed increases due to (BW& FW) whirl.

In this paper a simple mathematical model has been assembled, however more elaborate models based on a much large degree of freedom may be used based on suppleness or stiffness influence coefficients. The mathematical models may also be used to refine the measured data and help in removal of contaminated data. It is therefore feasible to create a mathematical model as a database for various systems for condition monitoring during their life time of the machines.

### 5.1 Summaries What Have Learned

The rotor whirl is either in the same direction as rotation or against rotation, which results into both forward and backward whirl mode. The frequencies are

affected by both the mass ( $m$ ) and diametral mass moment of inertia ( $J$ ).

The gyroscopic effect enhances are significantly by lowering between the overhung load and the inward bearing.

By increasing the rotional speed ,the distance between the forward whirl and backward whirl increases related to the mode-shape and the ratio of the polar mass moment of inertia ( $I_0$ ) to the diameter mass moment of inertia ( $J$ ).

The explanation for this surprising behaviour is a gyroscopic effect that occurs whenever the mode shape has an angular (conical / rocking) component.

Consider the forward whirl as shaft speed increases, the gyroscopic effects essentially act like an increasingly stiff spring on the central disc for the rocking motion. Increasing stiffness acts to increase the natural frequency.

For backward whirl, the effect is reversed. Increasing rotor spin speed acts to reduce the effective stiffness, thus reducing the natural frequency (as a side note, the gyroscopic terms are generally written as a skew-symmetric matrix added to the damping matrix-the net result, though, is a stiffening /softening effect). In the case of the first modes of the machines (cylindrical modes), very little effect of the gyroscopic terms was noted, because the centre disc was whirling without any conical motion. So it can be said that, without the conical motion, the gyroscopic effects does not appear. Therefore, for the soft bearing case, which has a very cylindrical motion, no effect was observed, while for the stiff bearing case, which has a bulging cylinder and (thus conical type motion near the bearing ), a slight effect was noted, [9, 26].

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