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Conjugate Effects of Radiation and Viscous Dissipation on Natural Convection flow over a Sphere with Pressure Work

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ABSTRACT

The effects of viscous dissipation and radiation on natural convection flow along a sphere with pressure work have been investigated. The governing equations are transformed into dimensionless non-similar equations by using set of suitable transformations and solved numerically by the finite difference method along with Newton's linearization approximation. We have focused our attention on the evaluation of velocity profiles, temperature profiles, shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number for different values of radiation parameter, Prandlt number, heat generation parameter, magnetic parameter, joule heating parameter and viscous dissipation parameter and the numerical results have been shown graphically.

Keywords: *Natural convection, Radiation heat transfer, Prandtl number, Nusselt number, Viscous dissipation parameter and Pressure work.*

1. INTRODUCTION

The conjugate effects of viscous dissipation and radiation on natural convection flow over a sphere with pressure work have been analyzed extensively. Many researchers have studied the problems of free convection boundary layer flow over or on a various types of geometrical shapes. Amongst them Nazar et al. [1] have studied free convection boundary layer on an isothermal sphere in a micropolar fluid. Hossain and Takhar [2] have analyzed the effects of radiation using the Rosseland diffusion approximation which leads to non-similar solutions for free convection flow past a heated vertical plate. Akhter and Alim [3] studied the effects of radiation on natural convection flow around a sphere with uniform surface heat flux. Limitations of this approximation are discussed briefly in Özisik [4]. The transformed boundary layer equations are solved numerically using finite difference method describe by Keller [5] and later by Cebeci and Bradshaw [6] along with Newton's linearization approximation. Miraj et al. [7] studied the effects of pressure work and radiation on natural convection flow around a sphere with heat generation and also Miraj et al. [8] have studied the problem of effects of pressure work and radiation on natural convection flow around a sphere with heat generation. Alam et al. [9] studied the viscous dissipation effects with MHD natural convection flow on a sphere in presence of heat generation. Amin [10] also analyzed the influences of both first and second order resistance, due to the solid matrix of non-darcy porous medium, Joule heating and viscous dissipation on forced convection flow from a horizontal circular cylinder under the action of transverse magnetic field. Joshi and Gebhart [11] have shown that the effect of pressure stress work and viscous dissipation in some natural convection flows. Zakerullah [12] has been investigated the viscous dissipation and pressure work effects in axisymmetric natural convection flows. The influence and importance of viscous stress work effects in laminar flows have been examined by Gebhart [13] and Gebhart and Mollendorf [14]. Numerical results have been obtained in terms of local skin friction, rate of heat transfer, velocity profiles, as temperature profiles as well in presence of pressure work and shown graphically.

2. FORMULATION OF THE PROBLEM

The natural convection boundary layer flow from an isothermal sphere of radius *a*, which is immersed in a viscous and incompressible optically dense fluid with radiation heat loss is considered. It is assumed that the constant temperature at the surface of the sphere is T_w, where $T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity and (U, V) are velocity components along the (X, Y) axes. The physical configuration considered is as shown in Fig.1

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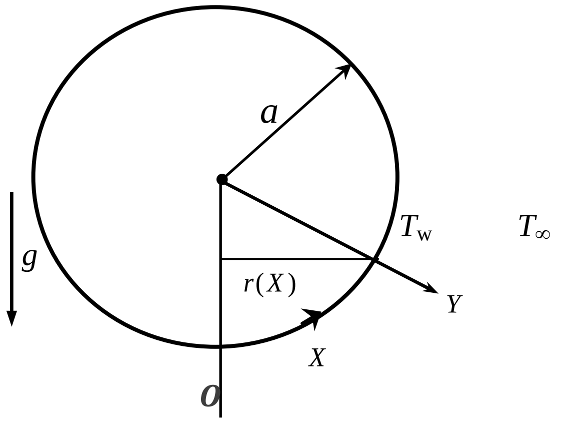

Figure 1: Physical model and coordinate system

According to the above assumption, the governing equations continuity, momentum and energy for steady two-dimensional laminar boundary layer flow problem under consideration can be written as

$$\frac{\partial}{\partial X}(rU)+\frac{\partial}{\partial Y}(rV)=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\nu\frac{\partial^2 U}{\partial Y^2}+g\beta(T-T_\infty)\sin\left(\frac{X}{a}\right) \tag{2}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial Y^2}-\frac{1}{k}\frac{\partial q_r}{\partial Y}\right)+\frac{T\beta}{\rho c_P}U\frac{\partial P}{\partial X}+\frac{\nu}{\rho c_p}\left(\frac{\partial U}{\partial Y}\right)^2 \tag{3}$$

We know for hydrostatic pressure, $\frac{\partial P}{\partial X}=\rho g$

With the boundary conditions

$$\begin{aligned} &U=V=0,\quad T=T_\infty \quad at \quad Y=0\\ &U\to 0,\quad T\to T_\infty \quad as \quad Y\to\infty \end{aligned} \tag{4}$$

where $r(X)=a\sin\left(X/a\right)$ is the radial distance from the centre to the surface of the sphere, k is the thermal conductivity, β is the coefficient of thermal expansion, $\nu\,(=\mu/\rho)$ is the kinematic viscosity, μ is the viscosity of the fluid, ρ is the density and c_p is the specific heat due to constant pressure.
The above equations are non-dimensionalised using the following new variables:

$$\xi=\frac{X}{a},\quad \eta=\frac{Y}{a}Gr^{\frac{1}{4}},\quad u=\frac{aU}{\nu}Gr^{-\frac{1}{2}},\quad v=\frac{aV}{\nu}Gr^{-\frac{1}{4}} \tag{5}$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\quad Gr=g\beta(T_w-T_\infty)\frac{a^3}{\nu^2} \tag{6}$$

$$\theta_w=\frac{T_w}{T_\infty},\ \Delta=\theta_w-1=\frac{T_w}{T_\infty}-1=\frac{T_w-T_\infty}{T_\infty} \tag{7}$$

where Gr is the Grashof number, θ is the non-dimensional temperature function, θ_w is the surface temperature parameter and q_r is the radiation heat flux. Thus the Rosseland diffusion approximation proposed by Siegel and Howell [15] is given by simplified radiation heat flux term as:

$$q_r=-\frac{4\sigma}{3(a_r+\sigma_s)}\frac{\partial T^4}{\partial Y} \tag{8}$$

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where a_r is the Rosseland mean absorption co-efficient, σ_s is the scattering co-efficient and σ is the Stefan-Boltzmann constant. Substituting (5), (6) and (7) in the continuity Equation (1), the momentum Equation (2) and the energy Equation (3) leads to the following non-dimensional equations:

$$\frac{\partial}{\partial \xi}(ru) + \frac{\partial}{\partial \eta}(rv) = 0 \tag{9}$$

$$u\frac{\partial u}{\partial \xi} + v\frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} + \theta \sin\xi \tag{10}$$

$$u\frac{\partial \theta}{\partial \xi} + v\frac{\partial \theta}{\partial \eta} = \frac{1}{Pr}\frac{\partial}{\partial \eta}\left[\left\{1+\frac{4}{3}Rd\left(1+(\theta_w-1)\theta\right)^3\right\}\frac{\partial \theta}{\partial \eta}\right] + Ge\left(\theta + \frac{T_\infty}{T_w - T_\infty}\right)u + Vd\left(\frac{\partial u}{\partial \eta}\right)^2 \tag{11}$$

where $Pr = \dfrac{\mu c_p}{k}$ is Prandtl number, $Ge = \dfrac{g\beta a}{C_P}$ is pressure work parameter, $Vd = \dfrac{\nu^2 Gr}{\rho a^2 c_P (T_w - T_\infty)}$ is viscous dissipation and $Rd = \dfrac{4\sigma T_\infty^3}{k(a_r + \sigma_s)}$ is radiation parameter.

With the boundary conditions (4) become

$$\begin{aligned} &u = v = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0 \\ &u \to 0, \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{12}$$

To solve equations (10) and (11) with the help of following variables

$$\psi = \xi r(\xi) f(\xi,\eta)\ ,\ \theta = \theta(\xi,\eta),\ r(\xi) = \sin\xi \tag{13}$$

where ψ is stream function defined by

$$u = \frac{1}{r}\frac{\partial \psi}{\partial \eta}, \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial \xi} \tag{14}$$

$$u = \frac{1}{r}\frac{\partial \psi}{\partial \eta} = \frac{1}{r}\frac{\partial}{\partial \eta}(\xi r f) = \xi\frac{\partial f}{\partial \eta} = \xi f', \ \text{where}\ f' = \frac{\partial f}{\partial \eta}$$

$$v = -\frac{1}{r}\frac{\partial \psi}{\partial \xi} = -\frac{1}{r}\frac{\partial}{\partial \xi}(\xi r f) = -\frac{1}{r}\left(rf + \xi r' f + \xi r\frac{\partial f}{\partial \xi}\right) = -f - \frac{\xi}{\sin\xi}\ f\cos\xi - \xi\frac{\partial f}{\partial \xi}$$

Using the above values in Equation (10), we get the following equation

$$\frac{\partial^3 f}{\partial \eta^3} + \left(1 + \frac{\xi}{\sin\xi}\cos\xi\right) f\frac{\partial^2 f}{\partial \eta^2} + \theta\frac{\sin\xi}{\xi} - \left(\frac{\partial f}{\partial \eta}\right)^2 = \xi\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \xi \partial \eta} - \frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2}\right) \tag{15}$$

Putting the values of u and v in Equation (11), we get the following equation

$$\begin{aligned} \xi f'\frac{\partial \theta}{\partial \xi} + \left(-f - \frac{\xi}{\sin\xi}\ f\cos\xi - \xi\frac{\partial f}{\partial \xi}\right)\frac{\partial \theta}{\partial \eta} &= \frac{1}{Pr}\frac{\partial}{\partial \eta}\left\{\left(1+\frac{4}{3}Rd(1+(\theta_w-1)\theta)^3\right)\frac{\partial \theta}{\partial \eta}\right\} \\ &+ Ge\left(\theta + \frac{T_\infty}{T_w - T_\infty}\right)\xi f' + Vd\,\xi^2\left(\frac{\partial^2 f}{\partial \eta^2}\right)^2 \end{aligned} \tag{16}$$

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Along with boundary conditions

$$\begin{aligned} & f = f' = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0 \\ & f' \to 0, \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{17}$$

where primes denote the differentiation of the function with respect to η.

It can be seen that near the lower stagnation point of the sphere, i.e., $\xi \approx 0$, Equations (15) and (16) reduce to the following ordinary differential equations:

$$f''' + 2f\,f'' - f'^2 + \theta = 0 \tag{18}$$

$$\frac{1}{Pr}\left[\left\{1 + \frac{4}{3}Rd\left(1 + (\theta_w - 1)\theta\right)^3\right\}\theta'\right]' + 2f\theta' = 0 \tag{19}$$

Subject to the boundary conditions

$$\begin{aligned} & f(0) = f'(0) = 0, \; \theta(0) = 1 \\ & f' \to 0, \; \theta \to 0 \; \text{as} \; \eta \to \infty \end{aligned} \tag{20}$$

In practical applications, the physical quantities of principal interest are shearing stress in terms of local skin friction coefficient C_f and the rate of heat transfer in terms of local Nusselt number Nu, which can be written in non- dimensional form as

$$C_f = \frac{a^2 Gr^{-\frac{3}{4}}}{\mu\nu}\tau_w \quad \text{and} \quad Nu = \frac{aGr^{-\frac{1}{4}}}{k(T_w - T_\infty)}(q_c + q_r)_{Y=0} \tag{21}$$

where $\tau_w = \mu\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$ is the shearing stress, $q_c = -k\left(\frac{\partial T}{\partial Y}\right)_{Y=0}$ is the conduction heat flux, k being the thermal conductivity of the fluid and q_r is the radiation heat flux.

The heat flux q_w is defined by

$$q_w = (q_c)_{Y=0} + (q_r)_{Y=0} = -k\left(\frac{\partial T}{\partial Y}\right)_{Y=0} + (q_r)_{Y=0}$$

Using equations (5), (6), boundary condition (20) and putting the values of τ_w and q_w in (21), we get the following equations

$$Nu = -\left(1 + \frac{4}{3}Rd\,\theta_w{}^3\right)\theta'(\xi, 0) \tag{22}$$

$$C_f = \xi f''(\xi, 0) \tag{23}$$

3. RESULTS AND DISCUSSION

Conjugate effects of radiation and viscous dissipation on natural convection flow over a sphere with pressure work have been investigated. Solutions are obtained in terms of velocity profiles, temperature profiles, skin friction coefficient and the rate of heat transfer and presented graphically for selected values of the radiation parameter *Rd*, Prandtl number *Pr*, pressure work parameter *Ge* and viscous dissipation parameter *Vd*. The effects of radiation parameter *Rd* on the velocity and temperature profiles with Prandtl number *Pr* = 0.72, pressure work parameter *Ge* = 1.0 and viscous dissipation parameter *Vd* =0.1 are shown in Figures 2(a) and 2(b) respectively.

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It is observed that in presence of pressure work and viscous dissipation velocity profiles increases as the radiation parameter *Rd* increases in Fig 2(a), which is physically realizable as higher radiation occurs when temperature is higher and eventually velocity rises. However, in Fig 2(b) the temperature profiles cross the sides in between η = 1.90430 and η = 1.99188 due to pressure work and viscous dissipation with increasing values of radiation parameter *Rd*. This is to be noted that, the temperature profiles having the higher values for lower values of radiation parameter and the temperature profiles increases comparatively slower along η-direction than the lower values for radiation parameter. Finally, the velocity boundary layer thickness as well as thermal boundary layer thickness increase for the increasing values of radiation parameter. In Figures 3(a) and 3(b) show that the effects of Prandtl number *Pr* on the velocity profiles and temperature profiles with radiation parameter *Rd* = 1.0, pressure work parameter *Ge* =1.0 and viscous dissipation parameter *Vd* = 0.1. It is observed that for increasing values of Prandtl number *Pr* the velocity profiles decreases and the temperature profiles cross the sides in between η = 1.11440 and η = 1.17520 in presence of pressure work. The velocity boundary layer thickness and thermal boundary layer thickness decrease for the increasing values of Prandtl number. The variation of pressure work parameter *Ge* on velocity profiles and temperature profiles while radiation parameter *Rd* =1.0, Prandtl number *Pr* = 0.72 and viscous dissipation parameter *Vd* =0.1 are shown in Figures 4(a) and 4(b). We observed that due to the change of pressure work parameter *Ge* from 0.1 to 3.0 the velocity increases sharply. For a particular value of pressure work we observe that velocity profile increases sharply but decrease gradually and it gives us a maximum value of that in Figure 4(a). Whereas, in Figure 4(b) shows that the temperature profile increases along with the pressure work parameter. In Figure 4(b) for a particular small value of pressure work parameter *Ge* gives the temperature profile gradually decreases along η-direction but for the higher particular value of pressure work the temperature profile gradually increase up to a certain level after that it decreases along η-direction and finally approaches to the asymptotic value (zero). Both the velocity and the temperature profiles accumulate nearly at the some positions of η, *i.e.,* there is no change of the velocity boundary layer thickness and thermal boundary layer thickness as well. Viscous dissipation parameter *Vd* on the velocity profile and temperature profile increase in Figures 5(a) and 5(b). We observe that in Figure 5(a) the velocity profile increases with the increase of the viscous dissipation parameter *Vd*. While in Figure 5(b) the temperature profile increases along with increase of the viscous dissipation parameter *Vd*. Moreover, we observe that there is no change of velocity boundary layer thickness and thermal boundary layer thickness. The effect of different values of radiation parameter *Rd* on the skin friction coefficient C_f and the rate of heat transfer *Nu* while Prandtl number *Pr* = 0.72, pressure work parameter *Ge* = 1.0 and viscous dissipation parameter *Vd* =0.1 are shown in Figures 6(a) and 6(b). Here, as the radiation parameter *Rd* increases, the skin friction coefficient C_f increases up to a certain level and then decreases gradually in Figure 6(a) but for a particular value of a radiation parameter the skin friction coefficient monotonically increases. On the other hand, the rate of heat transfer *Nu* increases up to the certain position of ξ after that the rate of heat transfer *Nu* decreases with the increase of radiation parameter in Figure 6(b) but for a particular value of a radiation parameter the rate of heat transfer monotonically decreases in presence of pressure work. It is seen in Figure 7(a), the skin friction coefficient C_f decreases first up to the certain position of ξ and then the skin friction coefficient C_f increases as the Prandtl number *Pr* increases . In Figure 7(b), the rate of heat transfer *Nu* increases up to the certain position of ξ and from that the rate of heat transfer *Nu* decreases for increasing values of the Prandtl number *Pr*. In Figures 8(a) and 8(b) show that the skin friction coefficient C_f increases and the rate of heat transfer *Nu* decreases for increases of the pressure work parameter *Ge* with radiation parameter *Rd* =1.0, Prandtl number *Pr* = 0.72 and viscous dissipation parameter *Vd* =0.1 . It is observed that the pressure work has great influence on skin friction coefficient C_f and the rate of heat transfer *Nu* as well. Frictional force at the wall becomes much higher towards the downstream for higher values of pressure work parameter *Ge* and the rate of heat transfer *Nu* as shown in Figure 8(b) gradually decreased for higher values of pressure work parameter *Ge*. In Figure 9(a) shows that the skin friction coefficient C_f increases as the viscous dissipation parameter *Vd* increases with radiation parameter *Rd* =1.0, Prandtl number *Pr* = 0.72 and pressure work parameter *Ge* = 1.0. Frictional force at the wall becomes much higher towards the downstream for higher values of *Vd* and the rate of heat transfer as shown in 9(b) gradually decreased for higher values of viscous dissipation parameter.

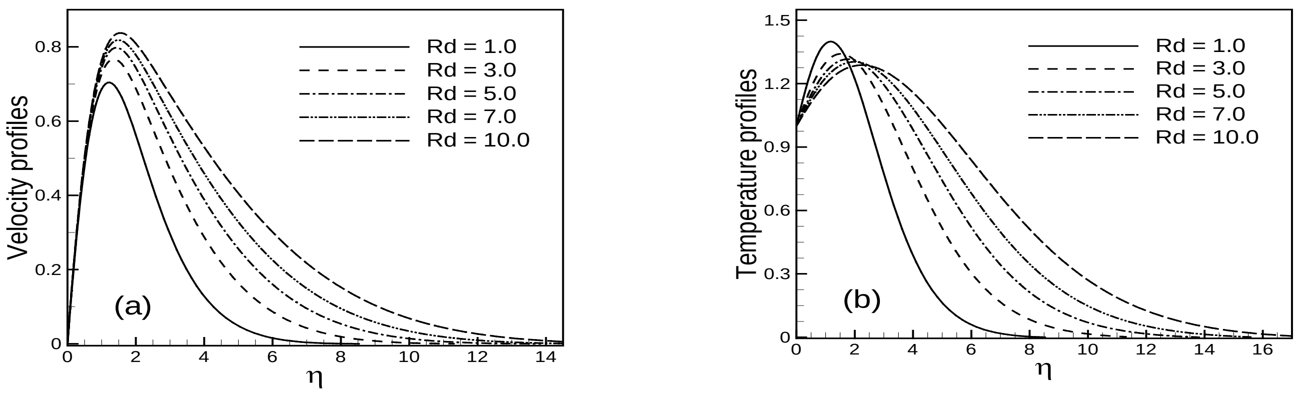

Figure 2: (a) Velocity profiles and (b) Temperature profiles for different values of *Rd* when *Pr* = 0.72, *Ge* = 1.0 and *Vd* = 0.1

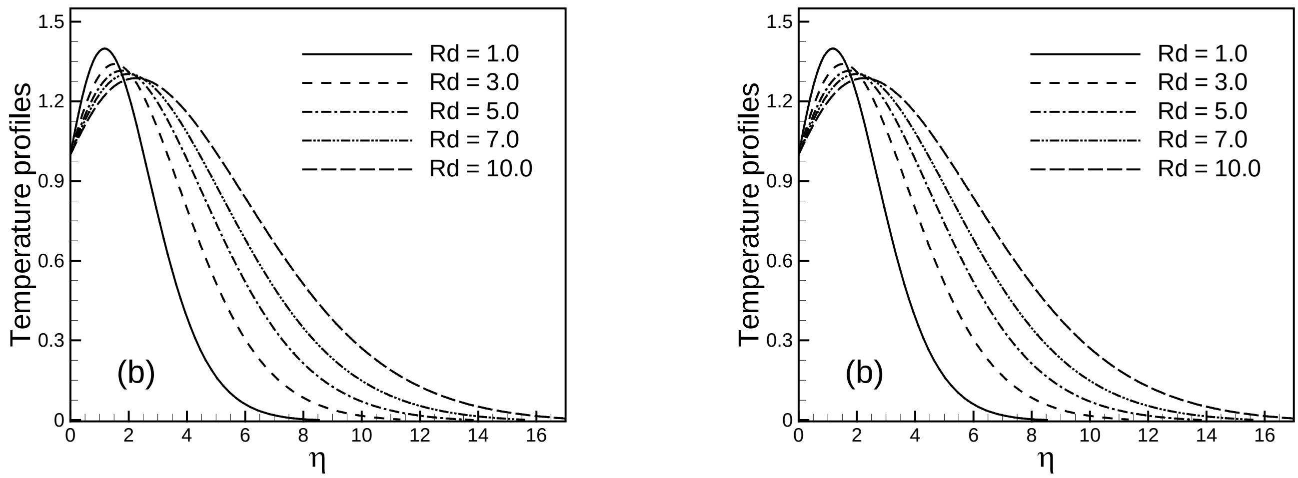

Figure 2: (a) Velocity profiles and (b) Temperature profiles for different values of Rd when Pr = 0.72, Ge = 1.0 and Vd = 0.1

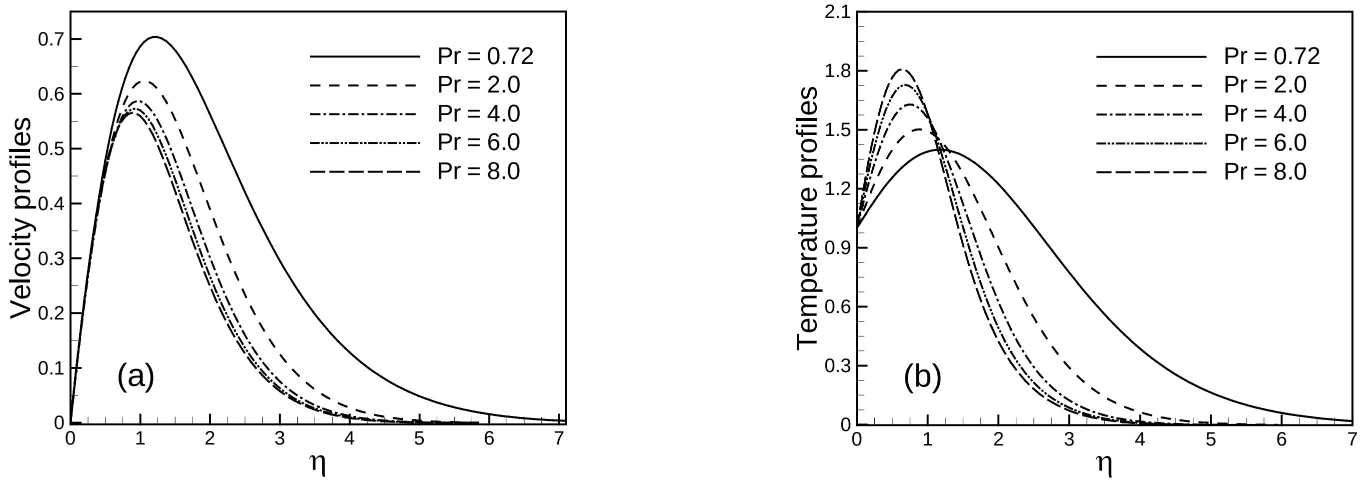

Figure 3: (a) Velocity profiles and (b) Temperature profiles for different values of *Pr* when *Rd* = 1.0, *Ge* = 1.0 and *Vd* = 0.1.

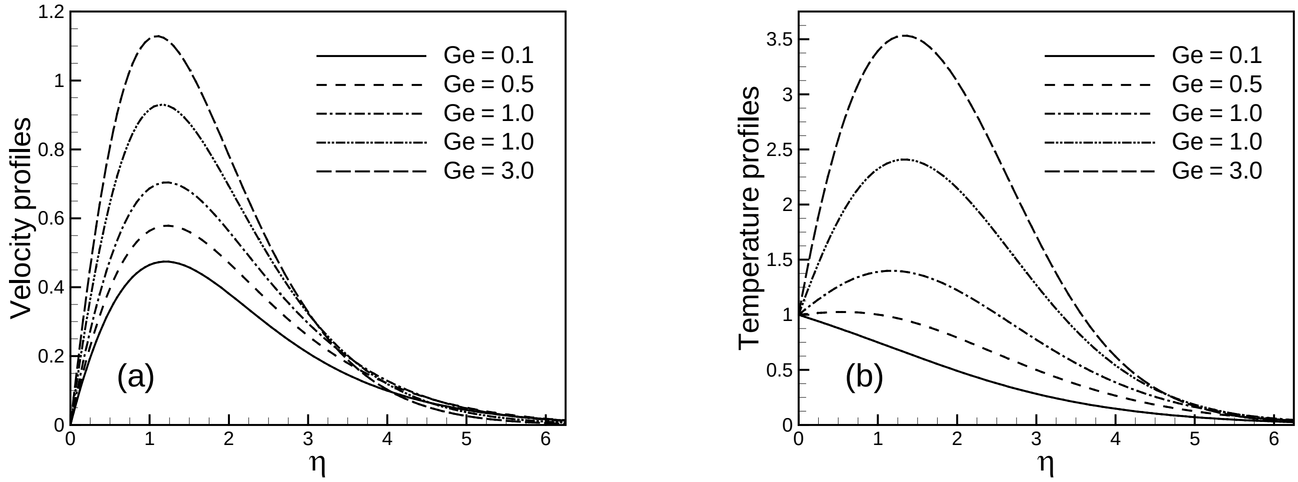

Figure 4: (a) Velocity profiles and (b) Temperature profiles for different values of *Ge* when *Rd* = 1.0, *Pr* = 0.72, and *Vd* = 0.1.

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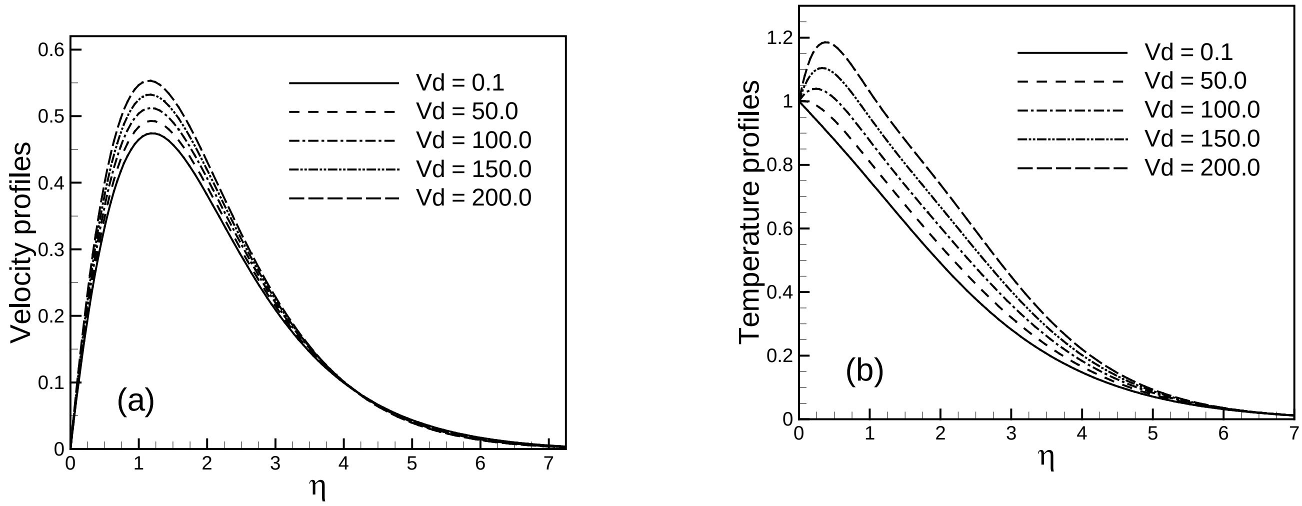

Figure 5: (a) Velocity profiles and (b) Temperature profiles for different values of *Vd* when *Rd* = 1.0, *Pr* = 0.72 and *Ge* = 1.0

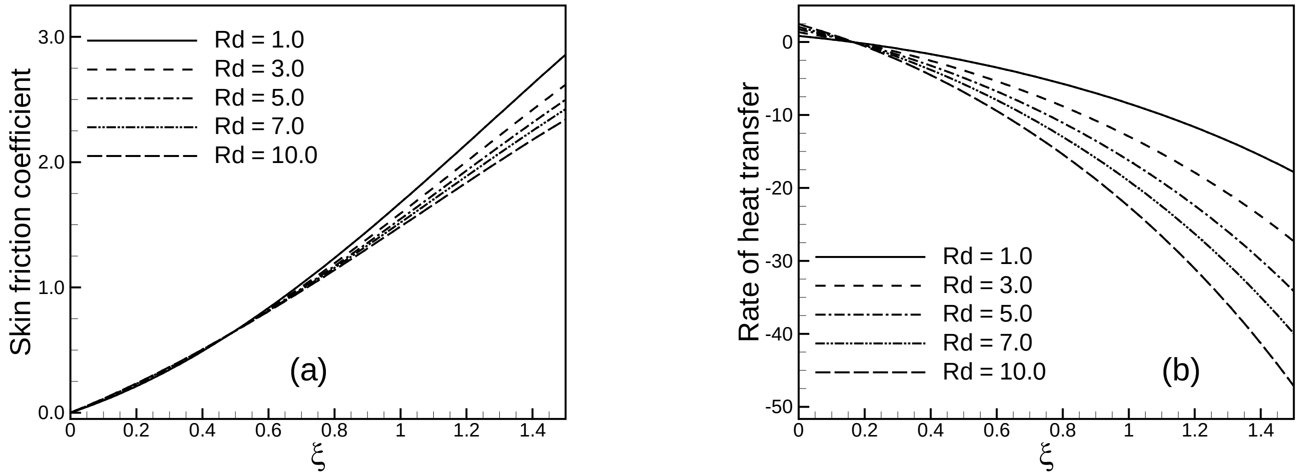

Figure 6: (a) Skin friction coefficient and (b) Rate of heat transfer for different of *Rd* when *Pr* = 0.72, *Ge* = 1.0 and *Vd* = 0.1

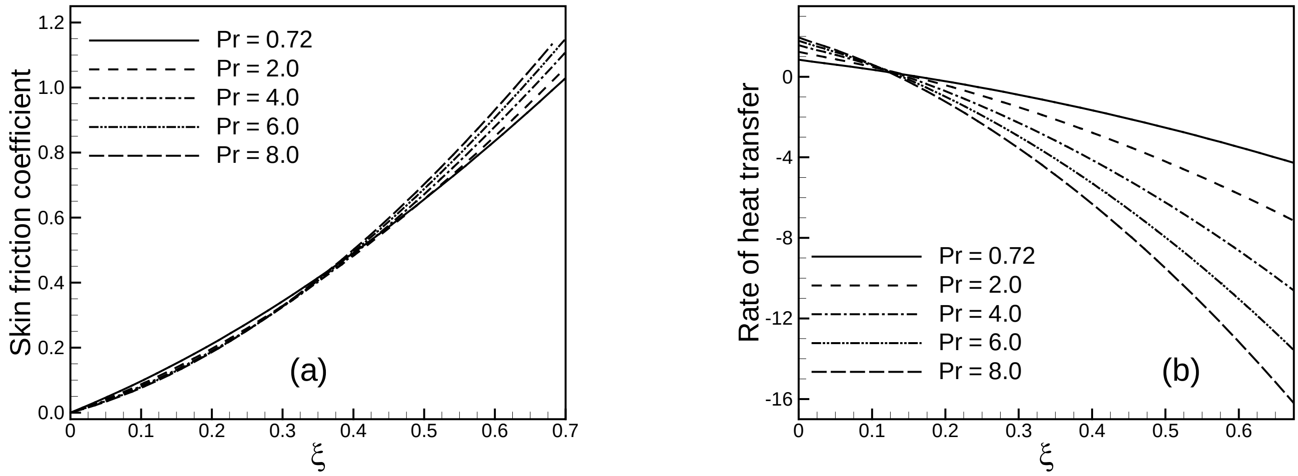

Figure 7: (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *Pr* when *Rd* = 1.0, *Ge* = 1.0 and *Vd* = 0.1

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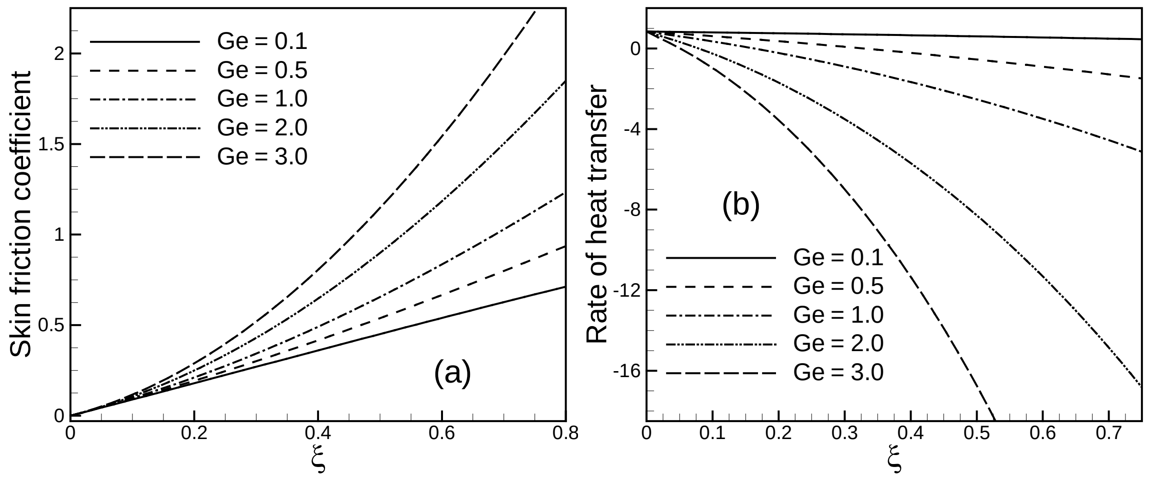

Figure 8: (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *Ge* when *Rd* = 1.0, *Pr* = 0.72, and *Vd* = 0.1

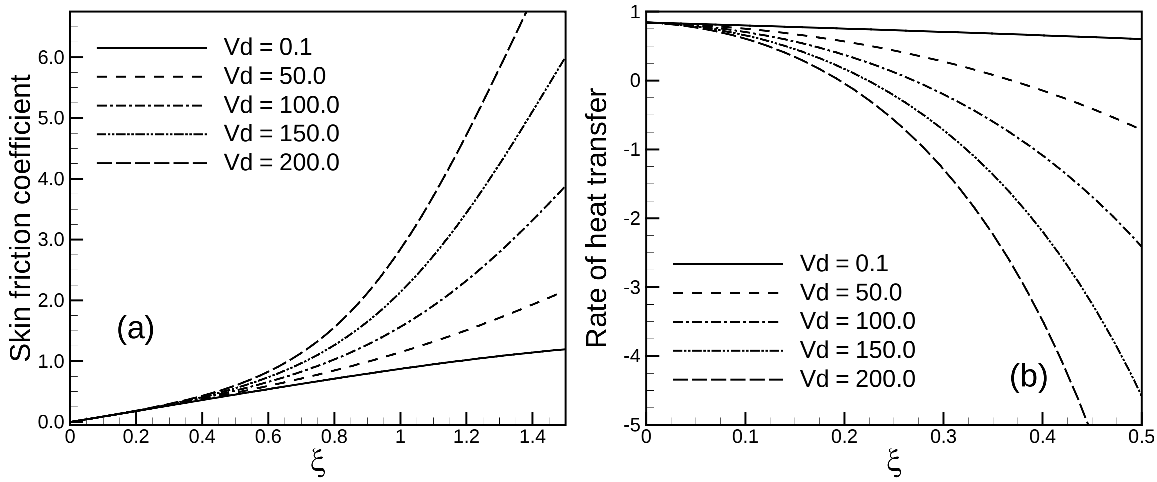

Figure 9: (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *Vd* when *Rd* = 1.0, *Pr* = 0.72 and *Ge* = 1.0

4. CONCLUSION

The conjugate effects of viscous dissipation and radiation on natural convection flow over a sphere with pressure work have been analyzed extensively for different values of relevant physical parameters. From the present investigation the following conclusions may be drawn:
Velocity profiles increases for increasing values of radiation parameter *Rd*, pressure work parameter *Ge* and viscous dissipation parameter *Vd* on the other hand velocity profiles decreases for increasing values of Prandlt number *Pr*. Temperature profiles increases for increasing values of pressure work parameter *Ge* and viscous dissipation parameter *Vd.* Skin friction coefficients C_f increases contrary the rate of heat transfer *Nu* decreases for increasing values of pressure work parameter *Ge* and viscous dissipation parameter *Vd*. Temperature profiles, skin friction coefficient and rate of heat transfer crosses the side for different values of radiation parameter *Rd* and Prandlt number *Pr*.

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Nomenclature

a	Radius of the sphere [m]	U	Velocity component along the surface [ms^{-1}]
a_r	Rosseland mean absorption co-efficient [cm^3/s]	V	Velocity component normal to the surface [ms^{-1}]
C_f	Skin-friction coefficient	Vd	Viscous dissipation parameter
C_P	Specific heat at constant pressure [Jkg^{-1}k^{-1}]	u	Dimensionless velocity along the surface
f	Dimensionless stream function	v	Dimensionless velocity normal to the surface
g	Acceleration due to gravity [ms^{-2}]	X	Coordinate along the surface [m]
Ge	Pressure work parameter	Y	Coordinate normal to the surface [m]
Gr	Grashof number	**Greek symbols**	

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k	Thermal conductivity [$wm^{-1}k^{-1}$]
Nu	Nusselt number
Pr	Prandtl number
q_c	Conduction heat flux [w/m^2]
q_r	Radiative heat flux [w/m^2]
q_w	Heat flux at the surface [w/m^2]
Rd	Radiation parameter
r	Radial distance from the symmetric axis to the surface [m]
T	Temperature of the fluid in the boundary layer [K]
T_∞	Temperature of the ambient fluid [K]
T_w	Temperature at the surface [K]
β	Coefficient of thermal expansion [K^{-1}]
θ	Dimensionless temperature
μ	Dynamic viscosity of the fluid [$kgm^{-1}s^{-1}$]
ν	Kinematic viscosity [m^2/s]
ρ	Density of the fluid [kgm^{-3}]
σ	Stephan Boltzmann constant [$js^{-1}m^{-2}k^{-4}$]
σ_0	Electrical conductivity [$mho.m^{-1}$]
σ_s	Scattering coefficient [m^{-1}]
τ_w	Shearing stress at the wall [N/m^2]
ξ	Dimensionless coordinate along the surface
η	Dimensionless coordinates normal to the surface
ψ	Stream function [m^2s^{-1}]

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