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Modeling the Reliability and Availability Characteristics of a System with Three Stages of Deterioration

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ABSTRACT

Many authors have studied maintenance or optimization models of deteriorating system without taking into consideration the kind or the degree of such deterioration on the system performance. Also little or no attention is found on the various stages of such deterioration and their effect on some measures of system effectiveness. In this study, we considered a system with three stages of deterioration: slow, medium or fast. Measures of system effectiveness like mean time to system failure (MTSF) and system availability were discussed. We analyzed the system using kolmogorov's forward equations method and developed explicit expressions for mean time to system failure and steady state availability for the system. Some particular cases have also been discussed graphically to see the effect of deterioration rates on MTSF and system availability. The results have indicated that deterioration rates decrease the mean time to system failure and system availability.

Keywords: *mean time to system failure (MTSF), availability, slow deterioration, medium deterioration, fast deterioration*

1. INTRODUCTION

During operation, the strengths of systems are gradually deteriorated, until some point of deterioration failure, or other types of failures. As the age of equipment increases, the equipment slowly deteriorates correspondingly. Deterioration failure is still the inevitable fate of the equipment. In many manufacturing situation, the condition of the system has significant impact on the quantity and quality of the unit produced. Most of these systems are subjected to random deterioration which can results in unexpected failures and disastrous effect on safety and the economy it is therefore important to find a way to slow down the deterioration rate, and to prolong equipment's service life.

Yusuf and Bala [5] deal with stochastic modeling of two unit parallel system under two types of failures, where the system works in normal mode, deterioration (slow, mild, or fast) in model I and normal and failure modes in model II, Marcous et al [4] deal with the modeling bridge deterioration, Wirahadikusumah et al [6] model deterioration of combined sewers.

In this paper, we studied a system consisting of three units' active parallel with three stages of deterioration: slow, medium and fast deterioration. We analyzed the system using kolmogorov's forward equations method. Measures of system effectiveness such MTSF and system availability are studied graphically to see the system behavior in terms of deterioration rates.

2. MATERIALS AND METHODS

Many researchers have studied maintenance optimization of deteriorating system without taking into consideration the stages of such deterioration before the system fail and the effect of deterioration rates on system performance such as MTSF and system availability.In this study, expressions for mean time to system failure and system availability are obtained. The system is analyzed using kolmogorov's forward equations method. Particular cases are studied graphically to see the effect of deterioration rates on mean time to system failure and system availability.

2.1 Notations

$\beta_{12}, \beta_{13}, \beta_{14}$: slow, medium and fast deterioration rates respectively from S_1 to S_2, S_3 and S_4

β_{23}, β_{34}: deterioration rates from S_2 to S_3 and S_3 to S_4 respectively

$\beta_{15}, \beta_{25}, \beta_{35}, \beta_{45}$ are failure rates

α_{51}, α_{54} : repair rates

$\alpha_{21}, \alpha_{31}, \alpha_{41}$: Major maintenance rates

α_{32}, α_{43}: Minor maintenance rates

2.2 Assumptions and Description of the System

a. State of the system can be: Perfect (S_1), slow deterioration (S_2), Medium deterioration (S_3), Fast (S_4) or failed state $\left(S_5\right)$

b. At any given time t the system is either in the operating state, deteriorating state or in the failed state.

c. The units operate simultaneously

d. State S_5 can be access from the previous state

e. The state of the system changes as time progresses

f. System/units work in S_1, S_2, S_3 and S_4

g. the deteriorate stages can be slow, medium or fast

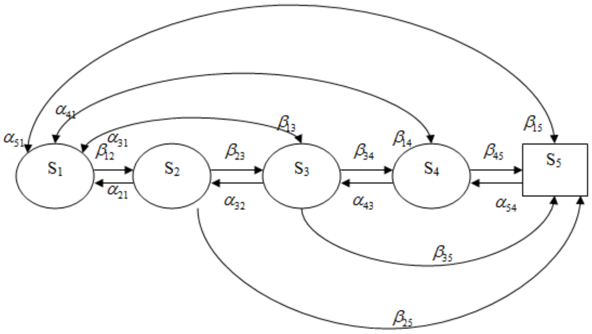

Fig. 1 Schematic diagram of the system

2.3 Mean Time to System Failure Analysis

From Fig. 1,let $P_i(t)$ be the probability that the system at time $t \geq 0$ is in state S_i. Let $P(t)$ be the probability row vector at time t with the initial conditions:

$$P(0) = [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)]$$
$$= \left[1, 0, 0, 0, 0\right]$$

$$\frac{dP_1(t)}{dt} = -(\beta_{12} + \beta_{13} + \beta_{14} + \beta_{15})P_1(t) + \alpha_{21}P_2(t) + \alpha_{31}P_3(t) + \alpha_{41}P_4(t) + \alpha_{51}P_5(t)$$

$$\frac{dP_2(t)}{dt} = -(\alpha_{21} + \beta_{23} + \beta_{25})P_2(t) + \beta_{12}P_1(t) + \alpha_{32}P_3(t)$$

$$\frac{dP_3(t)}{dt} = -(\alpha_{31} + \alpha_{32} + \beta_{34} + \beta_{35})P_3(t) + \beta_{13}P_1(t) + \beta_{23}P_2(t) + \alpha_{43}P_4(t)$$

$$\frac{dP_4(t)}{dt} = -(\alpha_{41} + \alpha_{43} + \beta_{45})P_4(t) + \beta_{14}P_1(t) + \beta_{34}P_3(t) + \alpha_{54}P_5(t)$$

$$\frac{dP_5(t)}{dt} = -(\alpha_{51} + \alpha_{54})P_5(t) + \beta_{15}P_1(t) + \beta_{25}P_2(t) + \beta_{35}P_3(t) + \beta_{45}P_4(t) \qquad (1)$$

The differential equations above can be expressed as:

$$\dot{P} = AP$$

where

$$A = \begin{bmatrix} X_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & X_{22} & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & X_{33} & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & X_{44} & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & X_{55} \end{bmatrix}$$

And

$X_{11} = -(\beta_{12} + \beta_{13} + \beta_{14} + \beta_{15})$, $X_{22} = -(\alpha_{21} + \beta_{23} + \beta_{25})$, $X_{33} = -(\alpha_{31} + \alpha_{32} + \beta_{34} + \beta_{35})$, $X_{44} = -(\alpha_{41} + \alpha_{43} + \beta_{45})$, $X_{55} = -(\alpha_{51} + \alpha_{54})$

It is difficult to evaluate the transient solutions hence following El-Said [1], Haggag [2], Wang et al [3], we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q. The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \quad (2)$$

This method is successful of the following relations:

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)\int_0^\infty e^{At}dt$$

$$\int_0^\infty e^{At}dt = -A^{-1}, \text{ for } A^{-1} < 0$$

Expression for MTSF can therefore be obtain from

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF = \frac{N_1}{D_1} \quad (3)$$

2.4 Availability Analysis

which in matrix form

For the analysis of availability case of Fig. 1 using the same initial conditions

$$P(0) = [P_1(0), P_2(0), P_3(0), P_3(0), P_5(0)]$$
$$= [1, 0, 0, 0, 0]$$

The differential equations above can be expressed as:

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \end{bmatrix} = \begin{bmatrix} X_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & X_{22} & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & X_{33} & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & X_{44} & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & X_{55} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

The system availability can be obtained from the solutions for $P_i(t)$, $i = 1, 2, ..., 5$. The states 0,1,2,3 and 4 in Fig. 1 are the only working states of the system. Following El-Said [1], Haggag [2] and Wang et al [3], the steady-state availability is given by :

$$A(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) \quad (4)$$

In the steady state, the derivatives of the state probabilities become zero so that

$$AP = 0 \quad (5)$$

$$\begin{bmatrix} X_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & X_{22} & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & X_{33} & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & X_{44} & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & X_{55} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition

$$P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) = 1 \qquad (6)$$

We substitute (6) in any of the redundant rows in (5) to give

$$\begin{bmatrix} X_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & X_{22} & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & X_{33} & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & X_{44} & \alpha_{54} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We solve for $P_1(\infty), P_2(\infty), P_3(\infty), P_4(\infty)$

The steady-state availability is given by:

$$A(\infty) = \frac{N_2}{D_2}$$

2.5 Busy Period Analysis

For the analysis of availability case of Fig. 1 using the same initial conditions

$$\begin{aligned} P(0) &= [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)] \\ &= [1, 0, 0, 0, 0, 0, 0] \end{aligned}$$

The differential equations above can be expressed as:

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \end{bmatrix} = \begin{bmatrix} X_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & X_{22} & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & X_{33} & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & X_{44} & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & X_{55} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

In the steady state, the derivatives of the state probabilities become zero so that

$$AP = 0$$

which in matrix form

$$\begin{bmatrix} X_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & X_{22} & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & X_{33} & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & X_{44} & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & X_{55} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition in (6) below:

$$P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) = 1$$

We substitute (6) in any of the redundant rows in (5) to give

$$\begin{bmatrix} X_{11} & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & X_{22} & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & X_{33} & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & X_{44} & \alpha_{54} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We solve for $P_1(\infty)$

The steady-state availability is given by :

$$B(\infty) = 1 - P_1(\infty) = \frac{N_3}{D_2} \qquad (7)$$

2.6 Particular Cases

Case I: To plot curves for β_{12} we fixed:
$\alpha_{21} = 0.3, \alpha_{32} = 0.7, \alpha_{43} = 0.6, \alpha_{54} = 0.9, \alpha_{31} = 0.2, \alpha_{41} = 0.45, \alpha_{51} = 0.8, \beta_{13} = 0.4, \beta_{14} = 0.6, \beta_{15} = 0.1,$
$\beta_{23} = 0.4, \beta_{24} = 0.5, \beta_{25} = 0.55, \beta_{35} = 0.2, \beta_{34} = 0.7$ and $\beta_{45} = 0.4$.

Case II: To plot curves for β_{13} we fixed:
$\alpha_{21} = 0.3, \alpha_{32} = 0.7, \alpha_{43} = 0.6, \alpha_{54} = 0.9, \alpha_{31} = 0.2, \alpha_{41} = 0.45, \alpha_{51} = 0.8, \beta_{12} = 0.1, \beta_{14} = 0.6, \beta_{15} = 0.1,$
$\beta_{23} = 0.4, \beta_{24} = 0.5, \beta_{25} = 0.55, \beta_{35} = 0.2, \beta_{34} = 0.7$ and $\beta_{45} = 0.4$.

Case III: To plot curves for β_{14} we fixed:

$\alpha_{21} = 0.3, \alpha_{32} = 0.7, \alpha_{43} = 0.6, \alpha_{54} = 0.9, \alpha_{31} = 0.2, \alpha_{41} = 0.45, \alpha_{51} = 0.8, \beta_{13} = 0.6, \beta_{12} = 0.4, \beta_{15} = 0.1,$ $\beta_{23} = 0.4, \beta_{24} = 0.5, \beta_{25} = 0.55, \beta_{35} = 0.2, \beta_{34} = 0.7$ and $\beta_{45} = 0.4$.

Fig.2 show the relationship between system availability and deterioration rates β_{12}, β_{13} and β_{14}

Fig.3 show the relationship between mean time to system failure (MTSF) and deterioration rates β_{12}, β_{13} and β_{14}

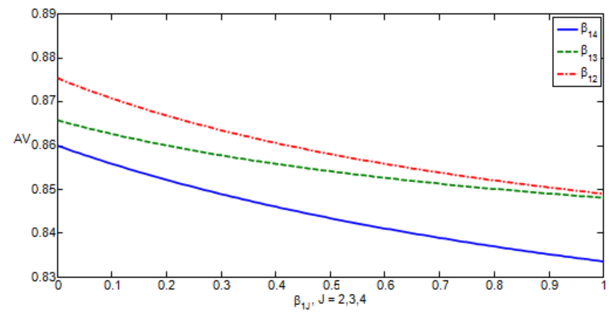

Fig. 2 effect of deterioration rates on Availability

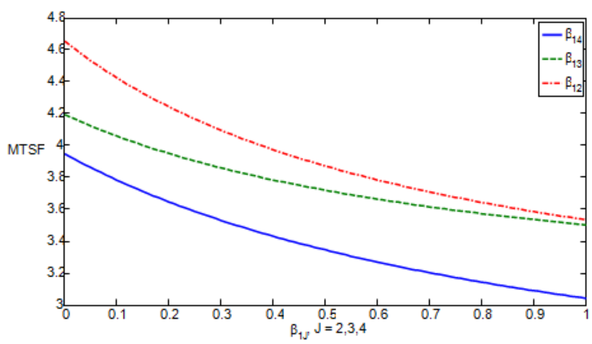

Fig. 3 effect of deterioration rates un MTSF

3. CONCLUSION

In this study, we developed explicit expressions for mean time to system failure and system availability. The mean time to system failure and system availability decreases with increase of slow, medium and fast deterioration as in Fig. 2 and Fig. 3. From Fig. 2 and it is observed that fast deteriorate rate decreases some of the measures of system effectiveness such as the system availability and mean time to system failure than medium and slow deterioration. It is therefore important to adopt preventive maintenance plan to check deterioration earlier before the condition of the system worsen.

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APPENDIX

$$N_1=(\beta_{23}\beta_{34}\beta_{45}+\alpha_{31}\alpha_{21}\alpha_{41}+\alpha_{31}\alpha_{21}\beta_{45}+\alpha_{31}\alpha_{41}\beta_{23}+\alpha_{31}\alpha_{21}\alpha_{43}+\alpha_{41}\beta_{34}\beta_{23}+\alpha_{31}\alpha_{43}\beta_{23}+\alpha_{31}\beta_{23}\beta_{45}+\alpha_{31}\alpha_{41}\beta_{25}$$
$$+\alpha_{31}\alpha_{43}\beta_{25}+\alpha_{31}\beta_{25}\beta_{45}+\alpha_{32}\alpha_{21}\alpha_{41}+\alpha_{32}\alpha_{21}\alpha_{43}+\alpha_{32}\alpha_{21}\beta_{45}+\alpha_{32}\alpha_{41}\beta_{25}+\alpha_{32}\alpha_{43}\beta_{25}+\alpha_{32}\beta_{25}\beta_{45}+\alpha_{41}\alpha_{21}\beta_{34}+$$
$$\alpha_{21}\beta_{34}\beta_{45}+\alpha_{41}\beta_{34}\beta_{25}+\beta_{34}\beta_{25}\beta_{45}+\alpha_{21}\alpha_{41}\beta_{35}+\alpha_{21}\alpha_{43}\beta_{35}+\alpha_{21}\beta_{35}\beta_{45}+\alpha_{41}\beta_{35}\beta_{23}+\alpha_{43}\beta_{35}\beta_{23}+\beta_{35}\beta_{23}\beta_{45}+$$
$$\alpha_{41}\beta_{35}\beta_{25}+\alpha_{43}\beta_{35}\beta_{25}+\beta_{35}\beta_{25}\beta_{45})+(\alpha_{31}\alpha_{41}\beta_{12}+\alpha_{31}\alpha_{43}\beta_{12}+\alpha_{31}\beta_{12}\beta_{45}+\alpha_{32}\alpha_{41}\beta_{12}+\alpha_{32}\alpha_{43}\beta_{12}+\alpha_{32}\beta_{12}\beta_{45}+$$
$$\alpha_{41}\beta_{12}\beta_{34}+\beta_{34}\beta_{12}\beta_{45}+\alpha_{41}\beta_{35}\beta_{12}+\alpha_{43}\beta_{35}\beta_{12}+\beta_{35}\beta_{12}\beta_{45}+\alpha_{32}\alpha_{41}\beta_{13}+\alpha_{32}\alpha_{43}\beta_{13}+\alpha_{32}\beta_{13}\beta_{45}+\alpha_{32}\alpha_{43}\beta_{14})+$$
$$(\alpha_{41}\beta_{23}\beta_{12}+\alpha_{43}\beta_{23}\beta_{12}+\beta_{23}\beta_{12}\beta_{45}+\alpha_{21}\alpha_{41}\beta_{13}+\alpha_{21}\alpha_{43}\beta_{13}+\alpha_{21}\beta_{13}\beta_{45}+\alpha_{21}\alpha_{43}\beta_{14}+\alpha_{41}\beta_{23}\beta_{13}+\alpha_{43}\beta_{23}\beta_{13}+$$
$$\beta_{23}\beta_{13}\beta_{45}+\alpha_{43}\beta_{23}\beta_{14}+\alpha_{41}\beta_{25}\beta_{13}+\alpha_{43}\beta_{25}\beta_{13}+\beta_{25}\beta_{13}\beta_{45}+\alpha_{43}\beta_{14}\beta_{25})+(\beta_{12}\beta_{34}\beta_{23}+\alpha_{21}\beta_{13}\beta_{34}+\alpha_{31}\alpha_{21}\beta_{14}+$$
$$\alpha_{32}\alpha_{21}\beta_{14}+\alpha_{21}\beta_{14}\beta_{34}+\alpha_{21}\beta_{14}\beta_{35}+\beta_{13}\beta_{34}\beta_{23}+\alpha_{31}\beta_{14}\beta_{23}+\beta_{34}\beta_{23}\beta_{14}+\beta_{14}\beta_{35}\beta_{23}+\beta_{13}\beta_{34}\beta_{25}+\alpha_{31}\beta_{14}\beta_{25}+$$
$$\alpha_{32}\beta_{14}\beta_{25}+\beta_{14}\beta_{25}\beta_{34}+\beta_{14}\beta_{35}\beta_{25})$$

$$D_1=\alpha_{31}\alpha_{21}\beta_{14}\beta_{45}+\alpha_{31}\beta_{14}\beta_{23}\beta_{45}+\alpha_{31}\beta_{14}\beta_{25}\beta_{45}+\alpha_{32}\alpha_{21}\beta_{14}\beta_{45}+\alpha_{32}\alpha_{43}\beta_{14}\beta_{25}+\alpha_{32}\beta_{14}\beta_{25}\beta_{45}+\alpha_{21}\beta_{14}\beta_{34}\beta_{45}+$$
$$\beta_{14}\beta_{34}\beta_{23}\beta_{45}+\beta_{14}\beta_{34}\beta_{25}\beta_{45}+\alpha_{21}\alpha_{43}\beta_{14}\beta_{35}+\alpha_{21}\beta_{14}\beta_{35}\beta_{45}+\alpha_{43}\beta_{14}\beta_{35}\beta_{23}+\beta_{14}\beta_{35}\beta_{23}\beta_{45}+\alpha_{43}\beta_{14}\beta_{35}\beta_{25}+$$
$$\beta_{14}\beta_{35}\beta_{25}\beta_{45}+\beta_{23}\beta_{34}\beta_{12}\beta_{45}+\alpha_{21}\beta_{13}\beta_{34}\beta_{45}+\beta_{13}\beta_{34}\beta_{23}\beta_{45}+\beta_{13}\beta_{34}\beta_{25}\beta_{45}+\alpha_{41}\beta_{23}\beta_{34}\beta_{15}+\beta_{23}\beta_{34}\beta_{15}\beta_{45}+$$
$$\alpha_{41}\beta_{35}\beta_{12}\beta_{23}+\alpha_{43}\beta_{35}\beta_{12}\beta_{23}+\beta_{35}\beta_{12}\beta_{45}\beta_{23}+\alpha_{21}\alpha_{41}\beta_{13}\beta_{35}+\alpha_{21}\alpha_{43}\beta_{13}\beta_{35}+\alpha_{21}\beta_{13}\beta_{45}\beta_{35}+\alpha_{41}\beta_{23}\beta_{13}\beta_{35}+$$
$$\alpha_{43}\beta_{23}\beta_{13}\beta_{35}+\beta_{23}\beta_{13}\beta_{45}\beta_{35}+\alpha_{32}\alpha_{41}\beta_{25}\beta_{13}+\alpha_{41}\beta_{25}\beta_{13}\beta_{35}+\alpha_{43}\alpha_{32}\beta_{25}\beta_{13}+\alpha_{43}\beta_{25}\beta_{13}\beta_{35}+\alpha_{32}\beta_{25}\beta_{13}\beta_{45}+$$
$$\beta_{25}\beta_{13}\beta_{45}\beta_{35}+\beta_{35}\beta_{12}\beta_{45}\beta_{25}+\alpha_{31}\alpha_{41}\alpha_{21}\beta_{15}+\alpha_{31}\alpha_{41}\beta_{15}\beta_{23}+\alpha_{31}\alpha_{41}\beta_{15}\beta_{25}+\alpha_{31}\alpha_{43}\alpha_{21}\beta_{15}+\alpha_{31}\alpha_{43}\beta_{15}\beta_{23}+$$
$$\alpha_{31}\alpha_{43}\beta_{15}\beta_{25}+\alpha_{31}\alpha_{21}\beta_{15}\beta_{45}+\alpha_{31}\beta_{15}\beta_{45}\beta_{23}+\alpha_{31}\beta_{15}\beta_{45}\beta_{25}+\alpha_{32}\alpha_{41}\alpha_{21}\beta_{15}+\alpha_{32}\alpha_{41}\beta_{15}\beta_{25}+\alpha_{32}\alpha_{43}\alpha_{21}\beta_{15}+$$
$$\alpha_{32}\alpha_{43}\beta_{15}\beta_{25}+\alpha_{32}\alpha_{21}\beta_{15}\beta_{45}+\alpha_{32}\beta_{15}\beta_{45}\beta_{25}+\alpha_{41}\alpha_{21}\beta_{35}\beta_{15}+\alpha_{41}\beta_{35}\beta_{15}\beta_{23}+\alpha_{41}\beta_{35}\beta_{15}\beta_{25}+\alpha_{43}\alpha_{21}\beta_{35}\beta_{15}+$$
$$\alpha_{43}\beta_{35}\beta_{15}\beta_{23}+\alpha_{43}\beta_{35}\beta_{15}\beta_{25}+\alpha_{21}\beta_{35}\beta_{15}\beta_{45}+\beta_{35}\beta_{15}\beta_{45}\beta_{23}+\beta_{35}\beta_{15}\beta_{45}\beta_{25}+\alpha_{31}\alpha_{41}\beta_{12}\beta_{25}+\alpha_{31}\alpha_{43}\beta_{12}\beta_{25}+$$
$$\alpha_{31}\beta_{12}\beta_{45}\beta_{25}+\alpha_{32}\alpha_{41}\beta_{12}\beta_{25}+\alpha_{32}\alpha_{43}\beta_{12}\beta_{25}+\alpha_{32}\beta_{12}\beta_{45}\beta_{25}+\alpha_{41}\beta_{35}\beta_{12}\beta_{25}+\alpha_{43}\beta_{35}\beta_{12}\beta_{25}+\alpha_{41}\beta_{34}\beta_{12}\beta_{25}+$$
$$\alpha_{41}\alpha_{21}\beta_{34}\beta_{15}+\alpha_{21}\beta_{34}\beta_{15}\beta_{45}+\beta_{34}\beta_{12}\beta_{45}\beta_{25}+\alpha_{41}\beta_{34}\beta_{15}\beta_{25}+\beta_{34}\beta_{15}\beta_{45}\beta_{25}$$

$$N_2=(\alpha_{21}\alpha_{41}\alpha_{54}\beta_{34}+\alpha_{41}\alpha_{54}\beta_{23}\beta_{34}+\alpha_{41}\alpha_{54}\beta_{25}\beta_{34}+\alpha_{21}\alpha_{41}\alpha_{51}\beta_{34}+\alpha_{21}\alpha_{51}\beta_{45}\beta_{34}+\alpha_{41}\alpha_{51}\beta_{23}\beta_{34}+\alpha_{51}\beta_{23}\beta_{45}\beta_{34}+$$
$$\alpha_{41}\alpha_{51}\beta_{25}\beta_{34}+\alpha_{51}\beta_{25}\beta_{45}\beta_{34}+\alpha_{31}\alpha_{21}\alpha_{41}\alpha_{54}+\alpha_{31}\alpha_{41}\alpha_{54}\beta_{23}+\alpha_{31}\alpha_{21}\alpha_{43}\alpha_{54}+\alpha_{41}\alpha_{54}\beta_{23}\beta_{35}+\alpha_{31}\alpha_{43}\alpha_{54}\beta_{23}+$$
$$\alpha_{31}\alpha_{41}\alpha_{54}\beta_{25}+\alpha_{31}\alpha_{54}\alpha_{43}\beta_{25}+\alpha_{32}\alpha_{21}\alpha_{41}\alpha_{54}+\alpha_{32}\alpha_{21}\alpha_{43}\alpha_{54}+\alpha_{32}\alpha_{41}\alpha_{54}\beta_{25}+\alpha_{21}\alpha_{41}\alpha_{54}\beta_{35}+\alpha_{41}\alpha_{54}\beta_{25}\beta_{35}+$$
$$\alpha_{31}\alpha_{21}\alpha_{43}\alpha_{51}+\alpha_{31}\alpha_{43}\alpha_{51}\beta_{23}+\alpha_{31}\alpha_{51}\beta_{23}\beta_{45}+\alpha_{31}\alpha_{41}\alpha_{51}\beta_{25}+\alpha_{31}\alpha_{43}\alpha_{51}\beta_{25}+\alpha_{31}\alpha_{51}\beta_{25}\beta_{45}+\alpha_{31}\alpha_{41}\alpha_{51}\beta_{23}+$$
$$\alpha_{32}\alpha_{21}\alpha_{41}\alpha_{51}+\alpha_{32}\alpha_{21}\alpha_{43}\alpha_{51}+\alpha_{32}\alpha_{21}\alpha_{51}\beta_{45}+\alpha_{32}\alpha_{41}\alpha_{51}\beta_{25}+\alpha_{32}\alpha_{43}\alpha_{51}\beta_{25}+\alpha_{32}\alpha_{51}\beta_{25}\beta_{45}+\alpha_{21}\alpha_{41}\alpha_{51}\beta_{35}+$$
$$\alpha_{21}\alpha_{43}\alpha_{51}\beta_{35}+\alpha_{21}\alpha_{51}\beta_{35}\beta_{45}+\alpha_{41}\alpha_{51}\beta_{35}\beta_{23}+\alpha_{43}\alpha_{51}\beta_{35}\beta_{23}+\alpha_{51}\beta_{35}\beta_{23}\beta_{45}+\alpha_{41}\alpha_{51}\beta_{35}\beta_{25}+\alpha_{43}\alpha_{51}\beta_{35}\beta_{25}+$$
$$\alpha_{51}\beta_{35}\beta_{25}\beta_{45}+\alpha_{31}\alpha_{21}\alpha_{41}\alpha_{51}+\alpha_{31}\alpha_{21}\alpha_{51}\beta_{45})+(\alpha_{32}\alpha_{43}\alpha_{54}\beta_{14}+\alpha_{31}\alpha_{41}\alpha_{54}\beta_{12}+\alpha_{31}\alpha_{43}\alpha_{54}\beta_{12}+\alpha_{32}\alpha_{41}\alpha_{54}\beta_{12}+$$
$$\alpha_{32}\alpha_{43}\alpha_{54}\beta_{12}+\alpha_{41}\alpha_{54}\beta_{12}\beta_{35}+\alpha_{32}\alpha_{43}\alpha_{51}\beta_{14}+\alpha_{31}\alpha_{41}\alpha_{51}\beta_{12}+\alpha_{31}\alpha_{43}\alpha_{51}\beta_{12}+\alpha_{31}\alpha_{51}\beta_{12}\beta_{45}+\alpha_{32}\alpha_{41}\alpha_{51}\beta_{12}+$$
$$\alpha_{32}\alpha_{43}\alpha_{51}\beta_{12}+\alpha_{32}\alpha_{51}\beta_{12}\beta_{45}+\alpha_{41}\alpha_{51}\beta_{35}\beta_{12}+\alpha_{43}\alpha_{51}\beta_{35}\beta_{12}+\alpha_{51}\beta_{35}\beta_{12}\beta_{45}+\alpha_{32}\alpha_{43}\alpha_{54}\beta_{15}+\alpha_{41}\alpha_{54}\beta_{34}\beta_{12}+$$
$$\alpha_{41}\alpha_{51}\beta_{34}\beta_{12}+\alpha_{51}\beta_{34}\beta_{12}\beta_{45}+\alpha_{32}\alpha_{43}\alpha_{54}\beta_{13}+\alpha_{32}\alpha_{41}\alpha_{51}\beta_{13}+\alpha_{32}\alpha_{43}\alpha_{51}\beta_{13}+\alpha_{32}\alpha_{51}\beta_{13}\beta_{45}+\alpha_{32}\alpha_{41}\alpha_{54}\beta_{13})+$$
$$(\alpha_{43}\alpha_{54}\beta_{23}\beta_{14}+\alpha_{41}\alpha_{54}\beta_{12}\beta_{23}+\alpha_{43}\alpha_{54}\beta_{23}\beta_{12}+\alpha_{21}\alpha_{43}\alpha_{54}\beta_{14}+\alpha_{43}\alpha_{54}\beta_{14}\beta_{25}+\alpha_{43}\alpha_{51}\beta_{23}\beta_{14}+\alpha_{54}\alpha_{43}\beta_{23}\beta_{15}+$$

$$\alpha_{43}\alpha_{54}\beta_{25}\beta_{15} + \alpha_{43}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{41}\alpha_{51}\beta_{23}\beta_{12} + \alpha_{43}\alpha_{51}\beta_{12}\beta_{23} + \alpha_{51}\beta_{23}\beta_{12}\beta_{45} + \alpha_{43}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{15} +$$
$$\alpha_{21}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{51}\beta_{13}\beta_{25}\beta_{45} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{43}\alpha_{54}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{13} +$$
$$\alpha_{21}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{13} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{51}\beta_{13}\beta_{23}\beta_{45} +$$
$$\alpha_{43}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{23}) + (\alpha_{54}\alpha_{31}\alpha_{21}\beta_{14} + \alpha_{31}\alpha_{54}\beta_{14}\beta_{23} + \alpha_{31}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{54}\alpha_{32}\alpha_{21}\beta_{14} + \alpha_{32}\alpha_{54}\beta_{14}\beta_{25} +$$
$$\alpha_{21}\alpha_{54}\beta_{14}\beta_{34} + \alpha_{54}\beta_{14}\beta_{34}\beta_{23} + \alpha_{54}\beta_{14}\beta_{34}\beta_{25} + \alpha_{21}\alpha_{54}\beta_{14}\beta_{35} + \alpha_{54}\beta_{14}\beta_{35}\beta_{23} + \alpha_{54}\beta_{14}\beta_{35}\beta_{25} + \alpha_{54}\beta_{23}\beta_{34}\beta_{12} +$$
$$\alpha_{21}\alpha_{54}\beta_{13}\beta_{34} + \alpha_{54}\beta_{13}\beta_{34}\beta_{23} + \alpha_{54}\beta_{13}\beta_{34}\beta_{25} + \alpha_{51}\beta_{23}\beta_{34}\beta_{14} + \alpha_{54}\beta_{23}\beta_{34}\beta_{15} + \alpha_{54}\beta_{12}\beta_{23}\beta_{35} + \alpha_{31}\alpha_{51}\beta_{14}\beta_{23} +$$
$$\alpha_{31}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{32}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{21}\alpha_{31}\alpha_{54}\beta_{15} + \alpha_{21}\alpha_{32}\alpha_{54}\beta_{15} + \alpha_{31}\alpha_{54}\beta_{23}\beta_{15} + \alpha_{54}\beta_{25}\beta_{35}\beta_{12} + \alpha_{31}\alpha_{54}\beta_{25}\beta_{15} +$$
$$\alpha_{32}\alpha_{54}\beta_{25}\beta_{15} + \alpha_{21}\alpha_{54}\beta_{35}\beta_{15} + \alpha_{21}\alpha_{51}\beta_{14}\beta_{35} + \alpha_{21}\alpha_{31}\alpha_{51}\beta_{14} + \alpha_{54}\beta_{23}\beta_{35}\beta_{15} + \alpha_{51}\beta_{23}\beta_{14}\beta_{35} + \alpha_{54}\beta_{25}\beta_{35}\beta_{15} +$$
$$\alpha_{32}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{51}\beta_{25}\beta_{14}\beta_{35} + \alpha_{31}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{32}\alpha_{21}\alpha_{51}\beta_{14} + \alpha_{51}\beta_{34}\beta_{12}\beta_{23} + \alpha_{51}\beta_{34}\beta_{25}\beta_{14} + \alpha_{21}\alpha_{54}\beta_{34}\beta_{15} +$$
$$\alpha_{54}\beta_{34}\beta_{25}\beta_{12} + \alpha_{54}\beta_{34}\beta_{25}\beta_{15} + \alpha_{21}\alpha_{51}\beta_{34}\beta_{14} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{34} + \alpha_{51}\beta_{13}\beta_{34}\beta_{25} + \alpha_{32}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{54}\beta_{13}\beta_{35} +$$
$$\alpha_{54}\beta_{13}\beta_{35}\beta_{23} + \alpha_{54}\beta_{13}\beta_{35}\beta_{25} + \alpha_{51}\beta_{13}\beta_{23}\beta_{34})$$

$$D_2 = \alpha_{31}\alpha_{21}\alpha_{54}\beta_{14} + \alpha_{31}\alpha_{21}\beta_{14}\beta_{45} + \alpha_{31}\beta_{14}\beta_{23}\beta_{45} + \alpha_{31}\beta_{14}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{32}\beta_{14}\beta_{45} + \alpha_{32}\alpha_{43}\beta_{14}\beta_{25} + \alpha_{32}\beta_{14}\beta_{25}\beta_{45} +$$
$$\alpha_{21}\beta_{14}\beta_{34}\beta_{45} + \beta_{14}\beta_{34}\beta_{23}\beta_{45} + \beta_{14}\beta_{34}\beta_{25}\beta_{45} + \alpha_{31}\alpha_{41}\beta_{12}\beta_{25} + \alpha_{31}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{31}\beta_{12}\beta_{45}\beta_{25} + \alpha_{32}\alpha_{41}\beta_{12}\beta_{25} +$$
$$\alpha_{32}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{32}\beta_{12}\beta_{45}\beta_{25} + \alpha_{41}\beta_{35}\beta_{12}\beta_{25} + \alpha_{43}\beta_{35}\beta_{12}\beta_{25} + \alpha_{51}\beta_{34}\beta_{12}\beta_{23} + \alpha_{51}\beta_{34}\beta_{25}\beta_{14} + \alpha_{41}\beta_{34}\beta_{12}\beta_{25} +$$
$$\alpha_{41}\alpha_{21}\beta_{34}\beta_{15} + \alpha_{41}\alpha_{54}\beta_{34}\beta_{12} + \alpha_{21}\alpha_{54}\beta_{34}\beta_{15} + \alpha_{54}\beta_{34}\beta_{25}\beta_{12} + \alpha_{21}\beta_{34}\beta_{15}\beta_{45} + \beta_{34}\beta_{12}\beta_{45}\beta_{25} + \alpha_{41}\alpha_{51}\beta_{34}\beta_{12} +$$
$$\alpha_{41}\beta_{34}\beta_{15}\beta_{25} + \alpha_{51}\beta_{34}\beta_{12}\beta_{45} + \alpha_{54}\beta_{34}\beta_{25}\beta_{15} + \alpha_{21}\alpha_{51}\beta_{34}\beta_{14} + \beta_{34}\beta_{15}\beta_{45}\beta_{25} + \alpha_{51}\beta_{13}\beta_{25}\beta_{45} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{13} +$$
$$\alpha_{41}\alpha_{51}\beta_{13}\beta_{32} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{54}\beta_{13}\beta_{35}\beta_{25} + \alpha_{51}\beta_{13}\beta_{23}\beta_{34} + \alpha_{43}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{25} +$$
$$\alpha_{32}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{34} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{13} + \alpha_{21}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{13} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{25} +$$
$$\alpha_{51}\beta_{13}\beta_{34}\beta_{25} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{13} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{32}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{51}\beta_{13}\beta_{23}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{13} + \alpha_{43}\alpha_{54}\beta_{13}\beta_{25} +$$
$$\alpha_{41}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{32}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{54}\beta_{13}\beta_{35} + \alpha_{54}\beta_{13}\beta_{35}\beta_{23} + \alpha_{21}\alpha_{43}\beta_{14}\beta_{35} + \alpha_{21}\beta_{14}\beta_{35}\beta_{45} + \alpha_{43}\beta_{14}\beta_{35}\beta_{23} +$$
$$\beta_{14}\beta_{35}\beta_{23}\beta_{45} + \alpha_{43}\beta_{14}\beta_{35}\beta_{25} + \beta_{14}\beta_{35}\beta_{25}\beta_{45} + \beta_{23}\beta_{34}\beta_{12}\beta_{45} + \alpha_{21}\beta_{13}\beta_{34}\beta_{45} + \beta_{13}\beta_{34}\beta_{23}\beta_{45} + \beta_{13}\beta_{34}\beta_{25}\beta_{45} +$$
$$\alpha_{31}\alpha_{54}\beta_{14}\beta_{23} + \alpha_{31}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{54}\alpha_{32}\alpha_{21}\beta_{14} + \alpha_{32}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{21}\alpha_{54}\beta_{14}\beta_{34} + \alpha_{54}\beta_{14}\beta_{34}\beta_{23} + \alpha_{54}\beta_{14}\beta_{34}\beta_{25} +$$
$$\alpha_{21}\alpha_{54}\beta_{14}\beta_{35} + \alpha_{54}\beta_{14}\beta_{35}\beta_{23} + \alpha_{54}\beta_{14}\beta_{35}\beta_{25} + \alpha_{54}\beta_{23}\beta_{34}\beta_{12} + \alpha_{21}\alpha_{54}\beta_{13}\beta_{34} + \alpha_{54}\beta_{13}\beta_{34}\beta_{23} + \alpha_{54}\beta_{13}\beta_{34}\beta_{25} +$$
$$\alpha_{41}\beta_{23}\beta_{34}\beta_{15} + \beta_{23}\beta_{34}\beta_{15}\beta_{45} + \alpha_{51}\beta_{23}\beta_{34}\beta_{14} + \alpha_{54}\beta_{23}\beta_{34}\beta_{15} + \alpha_{41}\beta_{35}\beta_{12}\beta_{23} + \alpha_{43}\beta_{35}\beta_{12}\beta_{23} + \beta_{35}\beta_{12}\beta_{45}\beta_{23} +$$
$$\alpha_{21}\alpha_{41}\beta_{13}\beta_{35} + \alpha_{21}\alpha_{43}\beta_{13}\beta_{35} + \alpha_{21}\beta_{13}\beta_{45}\beta_{35} + \alpha_{41}\beta_{23}\beta_{13}\beta_{35} + \alpha_{43}\beta_{23}\beta_{13}\beta_{35} + \beta_{23}\beta_{13}\beta_{45}\beta_{35} + \alpha_{41}\alpha_{32}\beta_{25}\beta_{13} +$$
$$\alpha_{41}\beta_{25}\beta_{13}\beta_{35} + \alpha_{43}\alpha_{32}\beta_{25}\beta_{13} + \alpha_{43}\beta_{25}\beta_{13}\beta_{35} + \alpha_{32}\beta_{25}\beta_{13}\beta_{45} + \beta_{25}\beta_{13}\beta_{45}\beta_{35} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{34} + \alpha_{41}\alpha_{54}\beta_{23}\beta_{34} +$$
$$\alpha_{41}\alpha_{54}\beta_{25}\beta_{34} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{34} + \alpha_{21}\alpha_{51}\beta_{45}\beta_{34} + \alpha_{41}\alpha_{51}\beta_{23}\beta_{34} + \alpha_{51}\beta_{23}\beta_{45}\beta_{34} + \alpha_{41}\alpha_{51}\beta_{25}\beta_{34} + \alpha_{51}\beta_{25}\beta_{45}\beta_{34} +$$
$$\alpha_{31}\alpha_{21}\alpha_{41}\alpha_{54} + \alpha_{31}\alpha_{41}\alpha_{54}\beta_{23} + \alpha_{31}\alpha_{21}\alpha_{43}\alpha_{54} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{41}\alpha_{54}\beta_{23}\beta_{35} + \alpha_{43}\alpha_{54}\beta_{23}\beta_{14} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{23} +$$
$$\alpha_{31}\alpha_{41}\alpha_{54}\beta_{25} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{25} + \alpha_{31}\alpha_{41}\alpha_{54}\beta_{12} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{12} + \alpha_{32}\alpha_{21}\alpha_{41}\alpha_{54} + \alpha_{32}\alpha_{21}\alpha_{43}\alpha_{54} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{25} +$$
$$\alpha_{32}\alpha_{41}\alpha_{54}\beta_{12} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{12} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{35} + \alpha_{41}\alpha_{54}\beta_{25}\beta_{35} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{35} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{23}\beta_{12} +$$
$$\alpha_{54}\beta_{12}\beta_{23}\beta_{35} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{43}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{31}\alpha_{21}\alpha_{43}\alpha_{51} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{43}\alpha_{51}\beta_{23}\beta_{14} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{23} +$$
$$\alpha_{31}\alpha_{51}\beta_{23}\beta_{45} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{25} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{25} + \alpha_{31}\alpha_{51}\beta_{25}\beta_{45} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{12} + \alpha_{31}\alpha_{51}\beta_{12}\beta_{45} +$$
$$\alpha_{31}\alpha_{51}\beta_{14}\beta_{23} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{23} + \alpha_{32}\alpha_{21}\alpha_{41}\alpha_{51} + \alpha_{32}\alpha_{21}\alpha_{43}\alpha_{51} + \alpha_{32}\alpha_{21}\alpha_{51}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{51}\beta_{25} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{25} +$$
$$\alpha_{32}\alpha_{51}\beta_{25}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{12} + \alpha_{32}\alpha_{51}\beta_{12}\beta_{45} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{35} + \alpha_{21}\alpha_{43}\alpha_{51}\beta_{35} + \alpha_{21}\alpha_{51}\beta_{35}\beta_{45} +$$
$$\alpha_{31}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{32}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{21}\alpha_{31}\alpha_{54}\beta_{15} + \alpha_{21}\alpha_{32}\alpha_{54}\beta_{15} + \alpha_{31}\alpha_{54}\beta_{23}\beta_{15} + \alpha_{54}\beta_{25}\beta_{35}\beta_{12} + \alpha_{31}\alpha_{54}\beta_{25}\beta_{15} +$$

$$\alpha_{43}\alpha_{54}\beta_{23}\beta_{15} + \alpha_{43}\alpha_{54}\beta_{25}\beta_{15} + \alpha_{43}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{32}\alpha_{54}\beta_{25}\beta_{15} + \alpha_{41}\alpha_{51}\beta_{35}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{23} + \alpha_{51}\beta_{35}\beta_{23}\beta_{45} +$$
$$\alpha_{41}\alpha_{51}\beta_{35}\beta_{25} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{25} + \alpha_{51}\beta_{35}\beta_{25}\beta_{45} + \alpha_{41}\alpha_{51}\beta_{35}\beta_{12} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{12} + \alpha_{51}\beta_{35}\beta_{12}\beta_{45} + \alpha_{41}\alpha_{51}\beta_{23}\beta_{12} +$$
$$\alpha_{43}\alpha_{51}\beta_{23}\beta_{12} + \alpha_{51}\beta_{23}\beta_{12}\beta_{45} + \alpha_{21}\alpha_{54}\beta_{35}\beta_{15} + \alpha_{21}\alpha_{51}\beta_{14}\beta_{35} + \alpha_{21}\alpha_{31}\alpha_{51}\beta_{14} + \alpha_{54}\beta_{23}\beta_{35}\beta_{15} + \alpha_{51}\beta_{23}\beta_{14}\beta_{35} +$$
$$\alpha_{54}\beta_{25}\beta_{35}\beta_{15} + \alpha_{32}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{51}\beta_{25}\beta_{14}\beta_{35} + \alpha_{31}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{15} + \alpha_{43}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{15} +$$
$$\alpha_{21}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{31}\alpha_{21}\alpha_{41}\alpha_{51} + \alpha_{32}\alpha_{21}\alpha_{51}\beta_{14} + \alpha_{31}\alpha_{21}\alpha_{51}\beta_{45} + \beta_{35}\beta_{12}\beta_{45}\beta_{25} + \alpha_{31}\alpha_{41}\alpha_{21}\beta_{15} + \alpha_{31}\alpha_{41}\beta_{15}\beta_{23} +$$
$$\alpha_{31}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{31}\alpha_{43}\alpha_{21}\beta_{15} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{23} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{31}\alpha_{21}\beta_{15}\beta_{45} + \alpha_{31}\beta_{15}\beta_{45}\beta_{23} + \alpha_{31}\beta_{15}\beta_{45}\beta_{25} +$$
$$\alpha_{32}\alpha_{41}\alpha_{21}\beta_{15} + \alpha_{32}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{32}\alpha_{43}\alpha_{21}\beta_{15} + \alpha_{32}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{32}\alpha_{21}\beta_{15}\beta_{45} + \alpha_{32}\beta_{15}\beta_{45}\beta_{25} + \alpha_{41}\alpha_{21}\beta_{35}\beta_{15} +$$
$$\alpha_{41}\beta_{35}\beta_{15}\beta_{23} + \alpha_{41}\beta_{35}\beta_{15}\beta_{25} + \alpha_{43}\alpha_{21}\beta_{35}\beta_{15} + \alpha_{43}\beta_{35}\beta_{15}\beta_{23} + \alpha_{43}\beta_{35}\beta_{15}\beta_{25} + \alpha_{21}\beta_{35}\beta_{15}\beta_{45} + \beta_{35}\beta_{15}\beta_{45}\beta_{23} +$$
$$\beta_{35}\beta_{15}\beta_{45}\beta_{25}$$