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Second Low Analysis of Annular Sector Ducts in Fully Developed Laminar Flow under Constant Wall Heat Flux

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ABSTRACT

In this paper, the entropy generation of a fully developed laminar flow in annular sector ducts with constant wall heat flux is investigated. Entropy generation is obtained for various aspect ratio (ε), various sector angels (2φ), various wall heat flux and various Reynolds number. It is found that with the increasing aspect ratio (ε) and sector angels (2φ) values, total entropy generation and pumping power at fixed Reynolds number increases and with increasing wall heat flux values, total entropy generation increases, however, pumping power decreases.

Keywords: *Entropy; Annular sector duct; Laminar flow; Pumping power.*

1. INTRODUCTION

In a thermodynamic process the loss of exergy is primarily due to the associated irreversibilities which generate entropy. Most convective heat transfer processes are characterized by two types of exergy losses, e.g. losses due to fluid friction and those due to heat transfer across a finite temperature difference. The above two interrelated phenomena are manifestations of thermodynamic irreversibility and investigation of a process from this standpoint is known as second law analysis. However, there exists a direct proportionality between the wasted power (the rate of available work lost) and the entropy generation rate. [1,2]. For efficient optimal thermodynamic design entropy generation must be reduced. In this context, geometry of duct (cross-sectional area) is an important parameter on entropy generation. Various cross-sectional ducts are used in heat transfer devices due to the size and volume constraints to enhance heat transfer with passive method. Pressure drop and heat transfer analysis in various shaped ducts were summarized by Shah and London [3].

Sahin [4] presented the second law analysis for different shaped duct such as triangular, sinusoidal etc, in laminar flow and constant wall temperature boundary conditions. He made a comparison between these ducts to find an optimum shape. He found that the circular duct geometry is the favorable one among them. He made another study in order to investigate the constant heat flux effects on these cross-sectional ducts without taking into account the viscosity variation in the analysis [5]. Viscosity variation was considered by Sahin [6] for turbulent flow condition for circular ducts [7].

Recently, Oztop [8] made a study on entropy generation in semicircular ducts. Dagtekin et al.[9]investigated the problem for circular duct with different shaped longitudinal fins for laminar flow using the similar methods of Sahin [5]. Oztop et al. [10] made a study on entropy generation in rectangular ducts with semicircular ends ducts with two boundary conditions: constant wall temperature and constant wall heat flux and Jarungthammachote [11] investigated entropy generation for hexagonal duct subjected to constant heat flux. The main aim of this study is to investigate entropy generation through annular sector ducts. To the best of the author's knowledge the entropy generation in annular sector ducts with constant wall heat flux has not yet been investigated. The present paper reports an analytical study of entropy generation in laminar flow. The effect of Reynolds number, wall heat flux and geometrical dimensions on entropy generation and pumping power are analyzed.

2. PHYSICAL MODEL OF PROBLEM

The physical model of annular sector duct is depicted in Fig. 1. The hydraulic diameter of any duct given by

$$D_h = \frac{4A_c}{p} \qquad (1)$$

Where A_c is cross-sectional area and P is perimeter. The hydraulic diameter for annular sector cross-sectional area can be written as

$$D_h = \frac{2\sqrt{2}\,\sqrt{(2\varphi)\,(1-\varepsilon^2)}}{2(1-\varepsilon)+(2\varphi)\,(1+\varepsilon)}\,\sqrt{A_c} \qquad (2)$$

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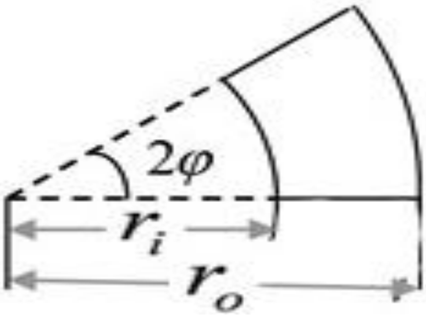

Figure 1. Cross section of annular sector duct

3. ENTROPY GENERATION ANALYSIS

The total entropy generation within a control volume of thickness dx , shown in Fig. 2, along the duct can be written as follows

$$d\dot{S}_{gen} = \dot{m}\, ds - \frac{\delta\dot{Q}}{T_w} \tag{3}$$

Where $\delta\dot{Q} = qPdx$ is heat transfer rate to the fluid flowing in this system for an incompressible fluid we have

$$T\, ds = C_P\, dT - v\, dP \tag{4}$$

Substituting ds from Eq. 4 into Eq. 3, $d\dot{S}_{gen}$ can be written as

$$d\dot{S}_{gen} = \dot{m} C_P \left(\frac{T_w - T}{T\, T_w} dT - \frac{1}{\rho C_P T} dP \right) \tag{5}$$

Pressure drop in Eq. 5 is given in the following equation.

$$dP = -\frac{f \rho U^2}{2D_h} dx \tag{6}$$

The bulk temperature variation of fluid along a duct is given in the following equation.

$$T = T_o + (4q / \rho U D_h C_P)\, x \tag{7}$$

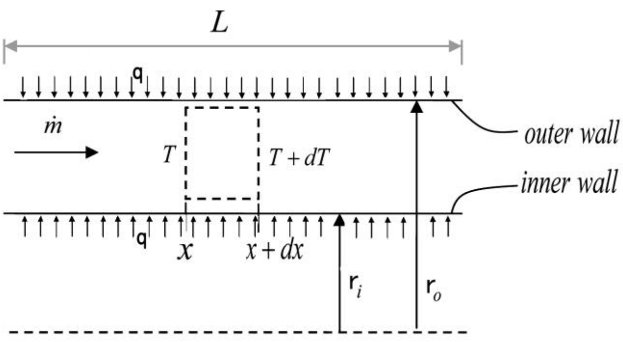

Figure 2.control volume for entropy generation

And then, for the constant wall heat flux boundary conditions, the total entropy generation is obtained by integration of Eq. (5) using Eqs. (2) and (7). The dimensionless total entropy generation based on the flow stream heat capacity rate $(\dot{m}/C_P)$ is defined as

$$Ns = \ln\left[\frac{(\text{Re} + \tau\eta_1)(1+\tau)}{(\text{Re} + \tau\,\text{Re} + \tau\eta_1)}\right] + \eta_2\, \text{Re}^2 \ln\left[\frac{\text{Re} + \tau\eta_1}{\text{Re}}\right] \tag{8}$$

In this equation η_1 and η_2 are,

$$\eta_1 = {4Nu\,\lambda}/{\text{Pr}} \tag{9}$$

$$\eta_2 = {\mu^3 f\, \text{Re}}/{8\rho^2 D_h{}^3 q} \tag{10}$$

In these equations some parameters can be made dimensionless as follows

$$St = \frac{h}{\rho U C_P} = \frac{Nu}{\text{RePr}} \tag{11}$$

$$\tau = \frac{T_W - T}{T_o} \tag{12}$$

$$\lambda = \frac{L}{D_h} \tag{13}$$

The value of $(f\,\text{Re})$ for fully developed laminar flow is given by shah and London [3] and the value of Nu for fully developed laminar flow is given by Ben-Ali et al. [12] and Soliman [13] for variety of duct geometries.

4. PUMPING POWER TO HEAT TRANSFER RATIO

The power required to overcome the fluid friction in the duct in dimensionless form is

$$P_r = \frac{A_c \Delta P U}{\dot{Q}} \tag{14}$$

For constant wall heat flux boundary conditions, the pumping power to heat transfer ratio for fully developed laminar flow becomes

$$P_r = \eta_2\, \text{Re}^2 \tag{15}$$

5. RESULT AND DISCUSSION

A second law analysis is conducted for annular sector duct in laminar flow regime. Water has been used as working

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fluid. The thermophysical properties used are shown in Table 1.

Fig. 3 shows dimensionless entropy generation for different aspect ratio (ε) of annular sector duct at different Reynolds number values. In this figure, total entropy generation decreases considerably while the Reynolds number is increased. As the value of aspect ratio (ε) is increased total entropy generation increased for fixed Reynolds number. Fig. 4 shows variation of pumping power for different aspect ratios (ε) and Reynolds numbers. As the aspect ratio (ε) is increased pumping power to heat transfer ratio increases. With increasing of Reynolds number pumping power to heat transfer ratio values also increase. For a fixed Reynolds number as aspect ratios (ε) values are increased pumping power to heat transfer ratio values are increased, especially for higher values of Reynolds number. These results indicate that for larger aspect ratios friction losses are higher than that of lower aspect ratios as expected.

Table 1. Thermophysial properties of water

SYMBOL	QUANTITY
$C_P\,(J/kgK)$	4182
Pr	7
$T_w\,(K)$	293
$\mu(Ns/m^2)$	9.93×10^{-4}
$\rho(kg/m^3)$	998.2

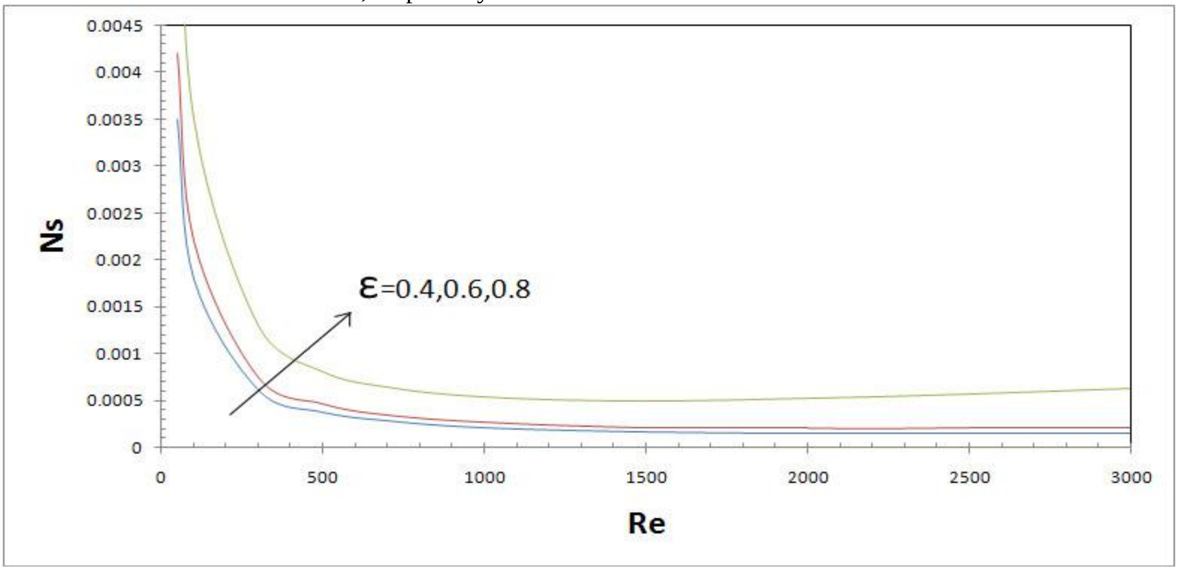

Figure 3. Variation of dimensionless entropy generation for different ε values and Reynolds number (τ=0. 1 and 2φ=60^o)

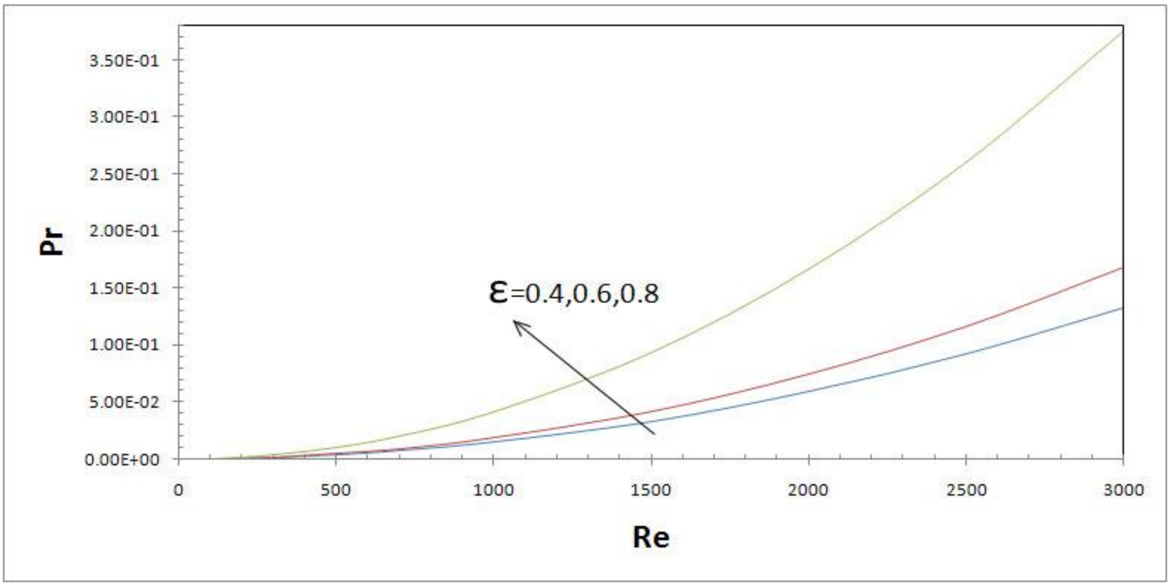

Figure 4. Variation of pumping power for different ε values and Reynolds number (τ=0. 1 and 2φ=60^o)

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Fig. 5 shows dimensionless entropy generation for different sector angel (2φ) of annular sector duct at different Reynolds number values. As the sector angel (2φ) is increased total entropy generation increases for fixed Reynolds number. With increasing of Reynolds number total entropy generation values also decrease. For a fixed Reynolds number as sector angel (2φ) values are increased total entropy generation values are decreased, especially for lower values of Reynolds number. Fig. 6 shows variation of pumping power for different sector angel (2φ) and Reynolds numbers. As the sector angel (2φ) is increased pumping power to heat transfer ratio increases. With increasing of Reynolds number pumping power to heat transfer ratio values also increase. For a fixed Reynolds number as sector angel (2φ) are increased pumping power to heat transfer ratio values are increased, especially for higher values of Reynolds number. These results indicate that for larger sector angels (2φ) friction losses are higher than that of lower sector angels (2φ) as expected.

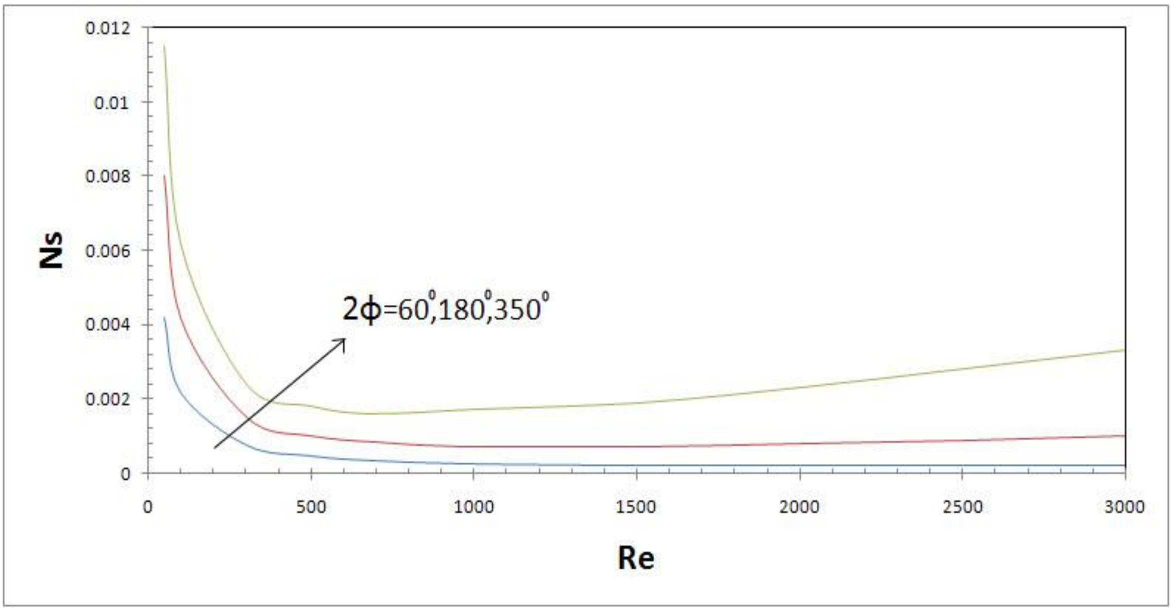

Figure 5. Variation of dimensionless entropy generation for different 2φ values and Reynolds number (τ=0. 1 and ε=0.6)

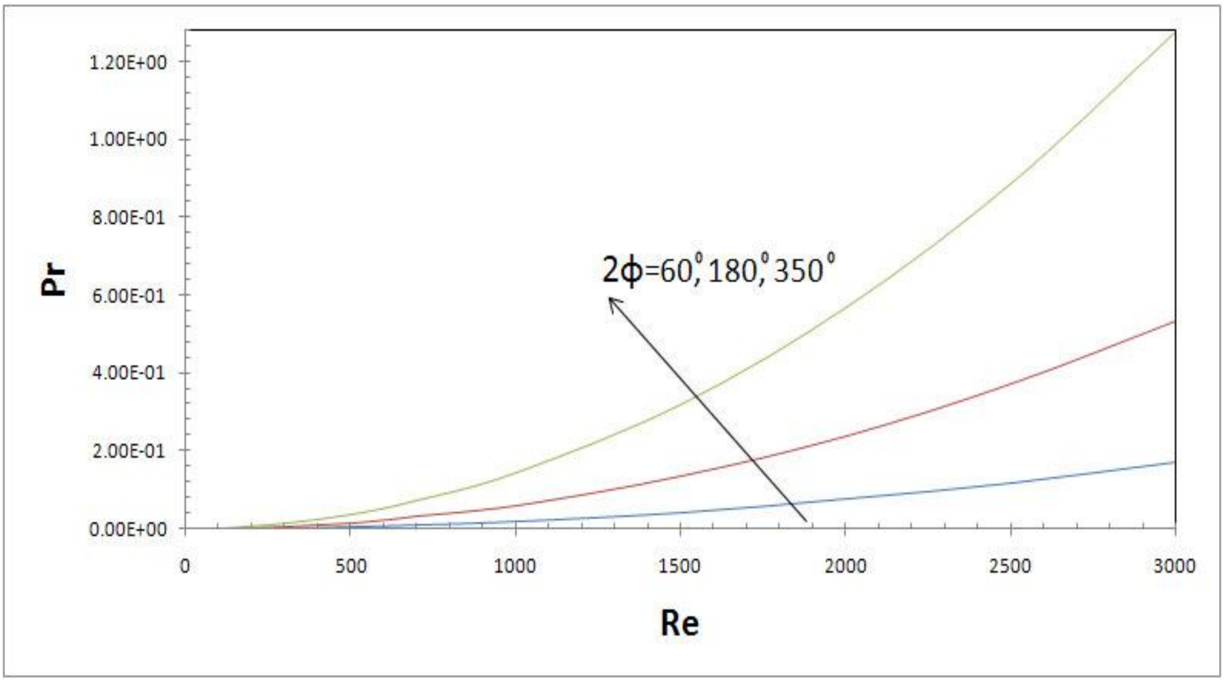

Figure 6. Variation of pumping power for different 2φ values and Reynolds number (τ=0. 1 and ε=0.6)

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Fig. 7 shows the effects of heat flux value on entropy generation at different Reynolds numbers for a fixed aspect ratio $(\varepsilon = 0.6)$ and sector angel $(2\varphi = 60^o)$, namely, for a fixed cross sectional area. Entropy generation is influenced by the wall heat flux. As Reynolds number is increased, the dimensionless entropy generation decreases, especially for higher q values. Thus, lower entropy generation is obtained for higher value of q (e.g. $q = 1000W/m^2$). The decrease of total entropy generation depends on the increase of wall heat flux as expected. Fig. 8 shows variation of pumping power to heat transfer ratio for various wall heat flux and Reynolds numbers. Increasing Reynolds number yields higher pumping power to heat transfer ratio values for fixed q values. As the wall heat flux is increased pumping power to heat transfer ratio values decreases for fixed Reynolds number, especially for higher values of Reynolds number. These results indicate that for lower wall heat flux friction losses are higher than that of higher wall heat flux as expected.

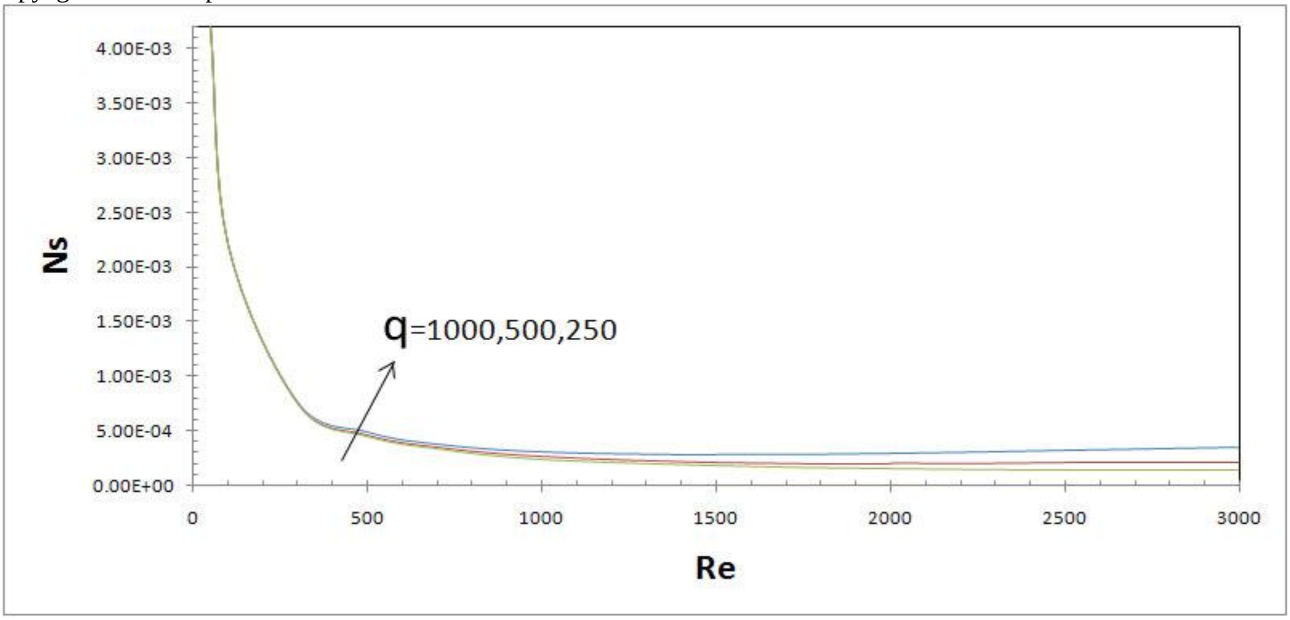

Figure 7. Variation of dimensionless entropy generation for various wall heat flux values and Reynolds number (ε=0. 6 and 2φ=60°)

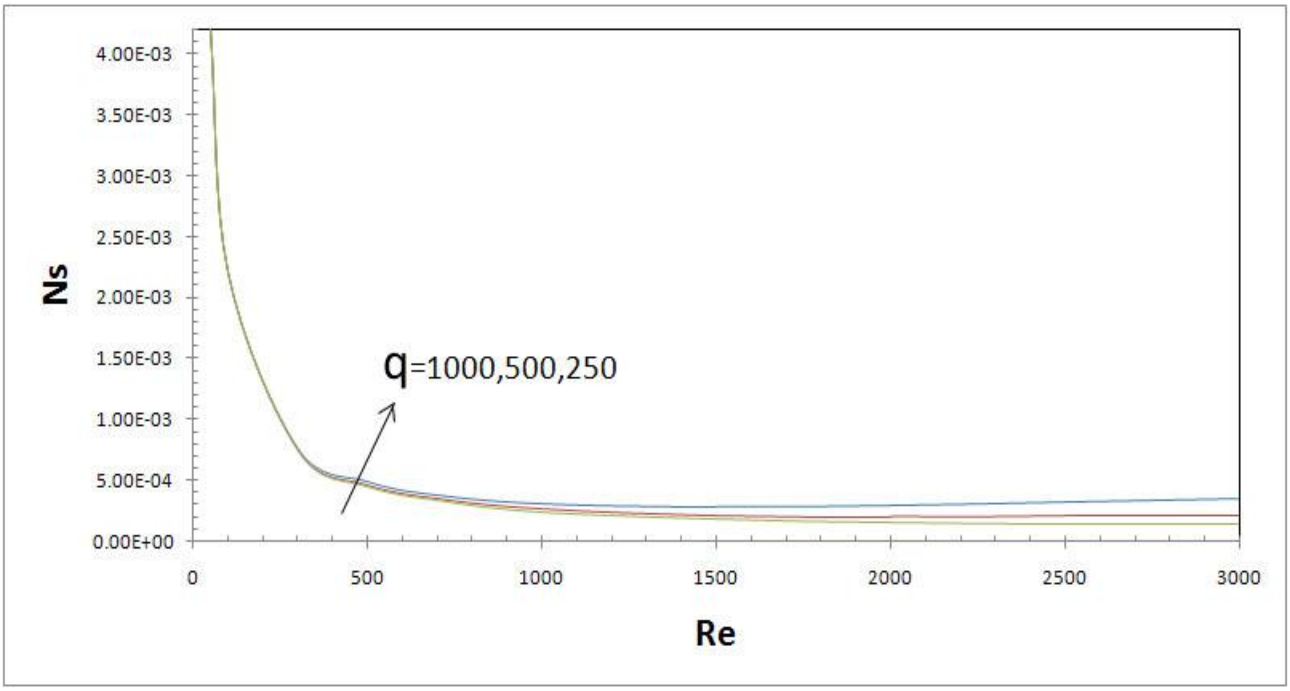

Figure 8. Variation of pumping power for various wall heat flux values and Reynolds number (ε=0. 6 and 2φ=60o)

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6. CONCLUSION

In this study second law analysis of laminar flow subjected to constant wall heat flux has been obtained for annular sector ducts. Some conclusions can be given as follows:

As Reynolds number is increased total entropy generation is decreasing. As the values of aspect ratio (ε) and sector angel (2φ) are increased total entropy generation for fixed Reynolds number. The pumping power ratio increases as the aspect ratio (ε), sector angel (2φ) and Re number are increased.

When wall heat flux (q) is increased total entropy generation and pumping power ratio are decreasing for fixed Reynolds number.

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REFERENCES

[1] Bejan A. Entropy generation through heat and fluid flow. NY: John Wiley; 1994.

[2] Bejan A. Entropy generation minimization: the new thermodynamics of finite size devices and finite-time processes. Journal Applied Physics 1996;79 (3):1191–218.

[3] R.K. Shah, Laminar flow forced convection in ducts. Academic Press; 1978.

[4] Sahin AZ. A second law comparison for optimum shape of subjected to constant wall temperature and laminar flow. Heat and Mass Transfer 1998;33: 425–430.

[5] Sahin AZ. Irreversibilities in various duct geometries with constant wall heat flux and laminar flow. Energy 1998;23:465–473.

[6] Sahin AZ. Second lawanalysis of laminar viscous flow through a duct subjected to constant wall temperature. Journal Heat Transfer 1998;120:76–83.

[7] Sahin AZ. Entropy generation in turbulent liquid flow through a smooth duct subjected to constant wall temperature. International Journal Heat and Mass Transfer 2000;43:1469–1478.

[8] Ozotop HF. Effective parameters on second law analysis for semicircular ducts in laminar flow and constant wall heat flux. International Communications in Heat and Mass Transfer 2005;32:266–274.

[9] Ozotop HF. Effective parameters on second law analysis for semicircular ducts in laminar flow and constant wall heat flux. International Communications in Heat and Mass Transfer 2005;32:266–274.

[10] Dagtekin I, Ozotop HF, Sahin AZ. An analysis of entropy generation through a circular duct with different shaped longitudinal fins for laminar flow. International Communications in Heat and Mass Transfer 2005;48: 171–181.

[11] Ozotop HF, Dagtekin I, Sahin AZ. Second law analysis of fully developed laminar flow for rectangular ducts with semicircular ends. International Communications in Heat and Mass Transfer 2009;36:725–730.

[12] Jarungthammachote S. Entropy generation analysis for fully developed laminar convection in hexagonal duct subjected to constant heat flux. Energy 2010;35:5374-5379.

[13] Ben-Ali TM, Soleiman HM, and Zariffeh EK. Further results for laminar heat transfer in annular section and circular sector ducts. Journal Heat Transfer 1989;111:1090-1093.

[14] Soleiman HM. Laminar heat transfer in annular sector ducts. Journal Heat Transfer 1987;109: 247-249.

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