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Analytical Modeling of the Terrestrial Albedo Torque Applied on Satellites of Various Shapes

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ABSTRACT

This paper is devoted to construct a simple model to investigate the terrestrial albedo torque applied on satellites of different geometries. Assuming that the Sun lies on the equator, the albedo irradiance is calculated using a numerical model in which irradiance depends on latitude, longitude and satellite altitude. However, in the present work the longitude dependency is disregarded. A detailed model for the optical properties of the satellite surface is used to evaluate the applied force. This force is formulated using a geocentric equatorial system in which the Earth is considered as oblate spheroid. The albedo torque is formulated for cylindrical satellite, spherical satellite and for satellite of complex shape. Based on the Earth's reflectivity data measured by NASA Total Ozone Mapping Spectrometer (TOMS project), the results show that the albedo torque has a significant contribution on the satellites on low Earth orbit. It has a maximum value in the order of 10^{-8} for satellite lies on altitude of 200 Km. However, it has a minimum value in the order of 10^{-13} for satellite lies on altitude of 2000 Km.

Keywords: *Terrestrial albedo, NASA Total Ozone Mapping Spectrometer (TOMS Project), radiative force, radiation Torque, geocentric equatorial , spherical and cylindrical coordinate systems and LEO satellites.*

1. INTRODUCTION

Modeling of the radiation forces and torques exerted on satellites are required for extremely accurate satellite positioning and orientation. The main contribution of the radiation is due to the direct solar radiation, so several models were constructed to estimate its force and torque [2], [6], [7] and [13-18]. The second main contribution of radiation forces is the Earth reflected radiation known as the Earth's albedo. It is an extremely complex phenomenon shows relevant spatial and temporal variations. Albedo depends upon the reflectivity of the illuminated surface of the Earth that is visible to the spacecraft, the solar angle, and the position of the spacecraft in space. Moreover, it depends on seasonal variations and geographical longitude and latitude of the Earth surface that illuminated by the Sun and seen by the satellite [1], [3] and [11].

The main issue of this work is to construct a detailed model of Earth's albedo torque applied on LEO satellites of different geometrical shapes.

2. MODELING OF THE ALBEDO TORQUE

2.1 The Albedo Irradiance

The albedo irradiance, which reaches to the satellite surface, is determined using a numerical model. This model is based on partitioning the Earth surface into a number of cells forming a grid. Then the incident solar irradiance on each cell is used to calculate the total radiant flux. Given the Sun and satellite positions w.r.t. that grid cells, the total albedo irradiance, S , at the satellite surface is given by [3]:

$$S = \begin{cases} \dfrac{\rho(\varphi_g,\theta_g) E_{AM0} A_C (\hat{r}_{sun}^T \hat{n}_C)(\hat{r}_{sat}^T \hat{n}_C)}{\pi \left\| \bar{r}_{Sat} \right\|^2} & \text{if } (\varphi_g,\theta_g)\ V_{Sun} \cap V_{Sat} \\ 0 & \text{else} \end{cases} \tag{1}$$

where $v_{Sun} \cap v_{Sat}$ is the set of grid points that are illuminated by the Sun and visible by the satellite, $\rho(\varphi_g,\theta_g)$ is the reflectivity of grid point of latitude φ_g and longitude θ_g , $E_{AM0} = 1367\ W/m^2$ is the incident solar irradiance, $\hat{n}_C$ is the cell normal and $\hat{r}_{Sun}$ and $\hat{r}_{Sat}$ are unit vectors directed from the grid center to the Sun and satellite respectively. The cell area $A_C(\varphi_g)$ is found using the surface revolutions as [3]:

$$A_C(\varphi_g) = \theta_g\, r_E^2\,[\cos(\varphi_g - \frac{\Delta\varphi_g}{2}) - \cos(\varphi_g + \frac{\Delta\varphi_g}{2})], \qquad (2)$$

where $\Delta\theta_g = 1.25^o$, $\Delta\varphi_g = 1^o$ and r_E is the Earth mean radius.

It was obvious that the albedo irradiance had a large dependency on the geographical latitude of the grid points. The maximum albedo is observed over the poles and decreased by moving away of them and towards the shadow side of the Earth. Moreover, there is a significant dependency on the longitude and the solar angle, where albedo has maximum values for low solar angles [4],[11] and [12]. However, In the present work, longitude dependency is disregarded.

2.2 The Albedo Force

The total radiant force exerted on a flat non-perfectly reflecting surface is given by [9]:

$$d\bar{f} = \frac{S}{c}[\Theta_1 \hat{n} + \Theta_2\, \hat{u}\,]dA \,, \qquad (3)$$

where c is the speed of light and S is the radiation irradiance at the satellite surface. $\hat{n}$ and $\hat{u}$ are unit vectors along the satellite surface normal and the direction of incident radiation respectively. The functions Θ_1 and Θ_2 clarify the various contributions of the surface physical properties and the satellite orientation w.r.t the incident radiation.

$$\Theta_1 = 2\rho'\beta\cos^2\eta + (B_f\rho'(1-\beta) + \alpha'\frac{\varepsilon_f B_f - \varepsilon_b B_b}{\varepsilon_f + \varepsilon_b})\cos\eta \qquad (4)$$
$$\Theta_2 = (1-\rho'\beta)\cos\eta$$

where η is the radiation incident angle to the satellite surface normal, β is the satellite surface specularity, ρ' is the satellite surface reflectivity, B_f and B_b are the non-Lambartian coefficient of the front and back surfaces of the spacecraft respectively, α' is the spacecraft absorption coefficient, ε_f and ε_b are the front and back satellite surface emissivity respectively.

2.3 The Coordinate Systems

The geocentric equatorial system is used with unit the vectors; $\hat{e}_x$ directed parallel to the Earth equatorial plane, $\hat{e}_y$ directed in the plane that contains the meridian of the sub-satellite point and $\hat{e}_z$ directed normal to the equatorial plane. As shown in Fig. 1, the incident radiation vector, $\bar{u}$, is given by:

$$\bar{u} = \bar{r} - \bar{r}_E \,, \qquad (5)$$

where $\bar{r}$ is the satellite position vector and $\bar{r}_E$ is the Earth radius vector. For oblate Earth, the Earth radius vector is given by [5]:

$$\bar{r}_E = \begin{pmatrix} G_1\cos\varphi_g\cos\theta \\ G_1\cos\varphi_g\sin\theta \\ G_2\sin\varphi_g \end{pmatrix}, \qquad (6)$$

with

$$G_1 = \frac{a_e}{(1-(2f_e' - f_e'^2)\sin^2\varphi_g)^{1/2}} + h \,,$$

$$G_2 = \frac{a_e(1-f_e')^2}{(1-(2f_e' - f_e'^2)\sin^2\varphi_g)^{1/2}} + h \,, \qquad (7)$$

where a_e is the Earth's Equatorial radius, f_e' is the Earth's flattening, φ_g is the geodetic latitude, h is the height above sea level and θ is the sidereal time.

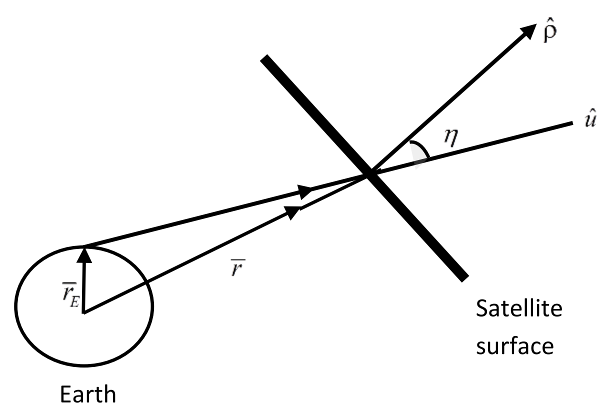

Fig. 1 Albedo irradiance at satellite *surface.*

The satellite position vector, $\bar{r}$, in the geocentric coordinate system, is given by [5]:

$$\bar{r} = r\begin{pmatrix} \cos\Omega\cos(\omega+\upsilon) - \sin\Omega\sin(\omega+\upsilon)\cos i \\ \sin\Omega\cos(\omega+\upsilon) + \cos\Omega\sin(\omega+\upsilon)\cos i \\ \sin(\omega+\upsilon)\sin i \end{pmatrix}, \qquad (8)$$

with

$$r = \frac{a(1-e^2)}{1+e\cos\upsilon} \,,$$

where Ω is the longitude of the ascending node, ω is the argument of perigee, υ is the true anomaly , i the inclination, a is the semi-major axis and e is the eccentricity. Using the geocentric coordinate system, the incident radiation vector is:

$$\bar{u} = u_x \hat{e}_x + u_y \hat{e}_y + u_z \hat{e}_z \quad (9)$$

With

$$u_x = r(\cos\Omega\cos(\omega+\upsilon) - \sin\Omega\sin(\omega+\upsilon)\cos i) - G_1\cos\varphi_g\cos\theta \quad (10)$$

$$u_y = r(\sin\Omega\cos(\omega+\upsilon) + \cos\Omega\sin(\omega+\upsilon)\cos i) - G_1\cos\varphi_g\sin\theta \quad (11)$$

$$u_z = r\sin(\omega+\upsilon)\sin i - G_2\sin\varphi_g . \quad (12)$$

The unit vector $\hat{n}$, normal to the satellite surface, can be expressed in terms of a cooordinate system with an orgin lies on the geometric center of the satellite and the vectors $\hat{e}_{x'}$, $\hat{e}_{y'}$ and $\hat{e}_{z'}$. Suppose that $\hat{n}$ is directed in the $\hat{e}_{z'}$ direction, therefore the components $n_{x'} = n_{y'} = 0$ and $n_{z'} = 1$. By transforming the normal vector $\hat{n}$ into geocentric coordinate system, its components will be:

$$n_x = r(\cos\Omega\cos(\omega+\upsilon) - \sin\Omega\sin(\omega+\upsilon)\cos i) \quad (13)$$

$$n_y = r(\sin\Omega\cos(\omega+\upsilon) + \cos\Omega\sin(\omega+\upsilon)\cos i) \quad (14)$$

$$n_z = 1 + r\sin(\omega+\upsilon)\sin i \quad (15)$$

2.4 The Albedo Torque

The radiation torque, $\hat{N}$, acting on a spacecraft is given by the general expression [2] and [15]:

$$\bar{N} = \int \bar{R} \times d\bar{f} , \quad (16)$$

where $\bar{R}$ is the vector from the spacecraft's center of mass to the element of satellite projected area dA. The geocentric components of terrestrial albedo torque, $\hat{N}$, acting on a spacecraft is given by:

$$N_x = \frac{S}{c}\int [R_y(\Theta_1 n_z + \Theta_2 u_z) - R_z(\Theta_1 n_y + \Theta_2 u_y)]dA \quad (17)$$

$$N_y = \frac{S}{c}\int [R_z(\Theta_1 n_x + \Theta_2 u_x) - R_x(\Theta_1 n_z + \Theta_2 u_z)]dA \quad (18)$$

$$N_z = \frac{S}{c}\int [R_x(\Theta_1 n_y + \Theta_2 u_y) - R_y(\Theta_1 n_x + \Theta_2 u_x)]dA \quad (19)$$

These equations represent the albedo contribution of a single Earth cell. However, the Sunlit area visible to the satellite is decomposed into a definite number of cells. So, the total albedo irradiance reaching to the satellite can be obtained by summing up the contribution of each cell. Consequently, the total torque applied over the whole spacecraft is obtained by vector sum of all cells contributions as follows [4] and [12]:

$$\bar{N}_{total} = \sum_{i=1}^{k} \bar{N}_i , \quad (20)$$

where k is the number of the illuminated Earth cells visible from the satellite.

In order to evaluate the previous integrals, the vector $\bar{R}$ must be determined. So, satellite geometry must be considered. In the next sections, three particular cases; circular cylindrical satellite, spherical satellite and satellite of complex shape will be studied.

2.5 Albedo Torque Applied on Circular Cylindrical Satellite

The position vector of the surface elements can be expressed interms of a cooordinate system with an orgin lies on the geometric center of the satellite and the vectors $\hat{e}_{x'}$, $\hat{e}_{y'}$ and $\hat{e}_{z'}$ as follows:

$$\bar{R} = \rho\cos\phi\,\hat{e}_{x'} + \rho\sin\phi\,\hat{e}_{y'} + z\,\hat{e}_{z'}$$

where ρ is the radius of the circular base and ϕ is the azimuthal angle. The position vector, $\bar{R}$, can be transformed into geocentric coordinate as follows:

$$\bar{R} = R_x\hat{e}_x + R_y\hat{e}_y + R_z\hat{e}_z$$

with

$$R_x = \rho\cos\phi + r(\cos\Omega\cos(\omega+\upsilon) - \sin\Omega\sin(\omega+\upsilon)\cos i) \quad (21)$$

$$R_y = \rho\sin\phi + r(\sin\Omega\cos(\omega+\upsilon) + \cos\Omega\sin(\omega+\upsilon)\cos i) \quad (22)$$

$$R_z = z + r\sin(\omega+\upsilon)\sin i \quad (23)$$

The illuminated surfaces of the cylinder satellite are a circular flat surface of radius ρ and an area $A_1 = \pi\rho^2$ in addition to a portion σ of the cylinder side of height H approximated and an area, $A_2 = 2\sigma\pi\rho H$, as in fig. 2.

Substituting into eqs. (17-19) and integrating over the areas A_1 and A_2, albedo torque applied on a cylindrical satellite is obtained.

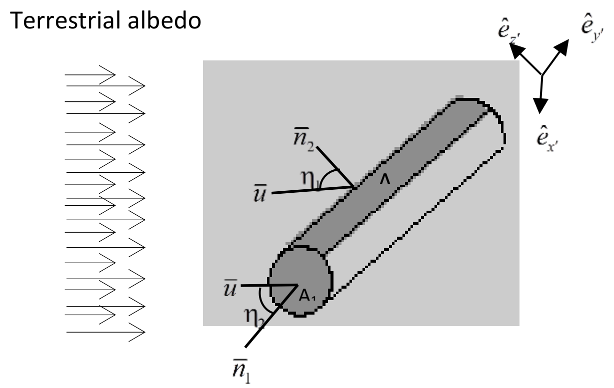

Fig. 2 The illuminated surfaces of the circular

2.6 Albedo Torque Applied on Spherical Satellite

The position vector of the surface elements can be expressed interms of a cooordinate system with an orgin lies on the geometric center of the satellite and the vectors $\hat{e}_{x'}$, $\hat{e}_{y'}$ and $\hat{e}_{z'}$ as follows:

$$\hat{R} = \rho\cos\vartheta\sin\phi\,\hat{e}_{x'} + \rho\sin\phi\sin\vartheta\,\hat{e}_{y'} + \rho\cos\phi\,\hat{e}_{z'}$$

where ρ is the radius of the sphere, ϑ is the polar angle and ϕ is the azimuthal angle. The position vector, $\hat{R}$, can be transformed into geocentric coordinate as follows:

$$\bar{R} = R_x\hat{e}_x + R_y\hat{e}_y + R_z\hat{e}_z$$

with

$$R_x = \rho\cos\vartheta\sin\phi + r(\cos\Omega\cos(\omega+\upsilon) - \sin\Omega\sin(\omega+\upsilon)\cos i) \quad (22)$$

$$R_y = \rho\sin\phi\sin\vartheta + r(\sin\Omega\cos(\omega+\upsilon) + \cos\Omega\sin(\omega+\upsilon)\cos i) \quad (23)$$

$$R_z = \rho\cos\phi + r\sin(\omega+\upsilon)\sin i \quad (24)$$

The illuminated surfaces of the spherical satellite can be considered as a hemisphere with an area $A = 2\pi\rho^2$, as shown in fig. 3.

Substituting into eqs. (17-19) and integrating over the area A, the albedo torque applied on a spherical satellite is obtained.

2.7 Albedo Torque Applied on Satellite of complex shape

In order to compute the torque applied on spacecraft of complex shape, we can follow the following scheme [7] and [8]:

a. Approximate each surface by means of simple geometric shapes (planes, cylinders, cones, spheres,…, etc.).

b. Determined the torque applied on each surface independently.

c. Then the total torque applied over the whole spacecraft is obtained by vector sum of all torques applied on each elementary surface.

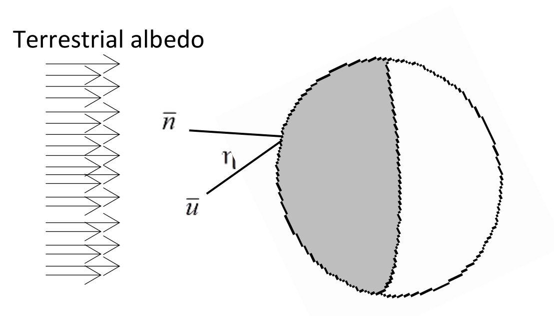

Fig. 3 The illuminated surfaces of the spherical satellite

3. NUMERICAL APPLICATION

Depending on the Earth's reflectivity data measured by NASA's Earth Probe satellite, which is part of the TOMS project (Total Ozone Mapping Spectrometer)[10]. The data shows Earth's reflectivity as a function of the latitude and as illustrated in fig. 4 .

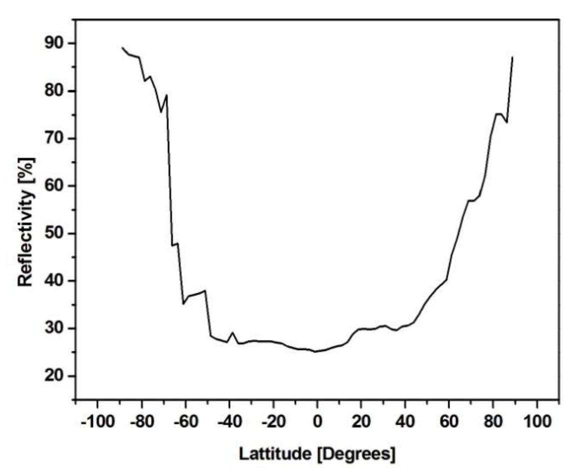

Fig. 4 Latitude dependency of Earth's reflectivity during the local summer

As illustrated in fig 1, the albedo has a maximum value $\sim 88\ \%$ over the latitude of 88.75 South. However, it has a minimum value $\sim 13.8\ \%$ over the latitude of 1.25 South.

The albedo algorithm is implemented for the satellite AAUSAT-II. It is LEO cubesat standard satellite with NORAD ID 32788 and of dimensions $1\,m \times .1\,m \times .113\,m$. Its area to mass retio is $\sim 8.69 \times 10^{-5}\ m^2/kg$ with mass of $\sim 750\ gm$. In the present work, the satellite AAUSAT-II is assumed to be on altitude of 200 Km and the torque model algorithm is applied under the following postulates:

a. The Sun lies on the equator (i.e. low solar angle in order to attain the highest reflectance) and the radiation fall normal to the Earth's surface.

b. The radiation fall normal to the satellite surface (i.e. $\eta = 0$).

c. The satellite surface is perfectly reflecting. So, the albedo force will be duplicated, where the satellite surface specularity $\beta = 1$ and the satellite surface reflectivity $\rho' = 1$. While the spacecraft absorption coefficient $\alpha' = 0$, consequently the functions $\Theta_1 = 2$ and $\Theta_2 = 0$.

Given a specific time during the local summer and based on the previous postulates, the albedo torque algorithm is applied on the satellite AAUSAT-II. The results are illustrated in the following graphs:

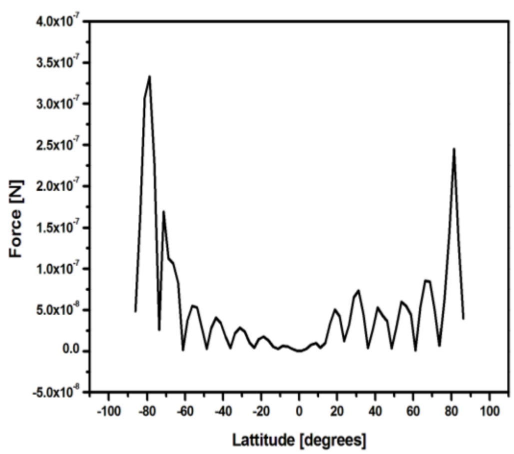

Figure. 5 Albedo force affecting on a LEO satellite that lies on 200 km altitude

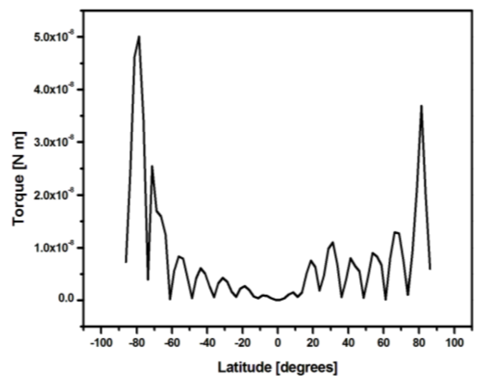

Fig. 6 Albedo torque affecting on a LEO satellite that lies on 200 km altitude

As illustrated in fig 5 for satellite at altitude of 200 km, the maximum albedo force is $\sim 3.3 \times 10^{-7}\ N$ and the minimum value is $\sim 3.5 \times 10^{-10}\ N$. Moreover as illustrated in fig 6, albedo torque had a maximum value of $\sim 5 \times 10^{-8}\ Nm$ as the satellite passed over latitude of $\sim$ 88 South. However, it had a minimum value of $\sim 5.3 \times 10^{-11}\ Nm$ as the satellite passed over the latitude of $\sim$ 1.25 South.

Also the algorithm is applied on the same satellite assuming that it lies on LEO of different altitudes and the results are tabulated in the following table:

Table 1 albedo torque affecting on LEO satellites lie on different altitudes

Altitude[km]	Maximum Torque[N m]	Minimum Torque[N m]
400	1.3×10^{-8}	1.4×10^{-11}
800	3.4×10^{-9}	3.6×10^{-12}
1200	1.6×10^{-9}	1.7×10^{-12}
1600	1×10^{-9}	1×10^{-12}
2000	6.4×10^{-10}	6.7×10^{-13}

4. CONCLUSION

A simple analytical model of the terrestrial albedo torque affecting on satellites of various shapes is constructed w.r.t. the geocentric equatorial coordinate system. This model can be used for advanced studies of satellite rotational motion.

For LEO satellites, the current numerical test confirms that the albedo force has significant dependency on the altitude. It decreases as the satellite altitude increases, where it is in the order of 10^{-7} at altitude 200 km and it decreases to 10^{-10} at altitude of 2000 km. So, we can conclude that the albedo force is of the same order of magnitude of air drag force, radiation pressure and the effect of Sun and moon on the satellite.

The results obtained by applying the torque model show that the albedo torque has a maximum value in the order of 10^{-8} for satellite lies on altitude of 200 Km. However, it has a minimum value in the order of 10^{-13} for satellite lies on altitude of 2000 Km. So, we can conclude that the albedo torque has a significant contribution on the LEO satellites also, it has a great dependency on satellite altitude.

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