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Bounds on Connected Domination in Squares of Graphs

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ABSTRACT

Let $G^2 = (V, E(G^2))$ be a square graph of a graph G. A set D of G^2 is said to be connected dominating set of G^2, if every vertex not in D is adjacent to at least one vertex in D and the sub graph $\langle D \rangle$ is connected. The minimum cardinality of a connected dominating set of G^2 is called the connected domination number of square graph G^2 and is denoted by $\gamma_c(G^2)$. In this paper many bounds on $\gamma_c(G^2)$ are found in terms of elements of G but not the elements of G^2. Also its relationship with other different domination parameters were obtained. Further we develop the relation between G and G^2 in terms of domination parameters.

Keywords: *Graph, Square graph, Dominating set, Connected domination number*

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1. INTRODUCTION

In this paper we follow the notations of [1]. All the graphs considered here are finite, non-trivial, undirected and connected without loops or multiple edges.

In general, we use $\langle X \rangle$ to denote the sub graph induced by the set of vertices X and $N(v)$ and $N[v]$ denote the open and closed neighborhoods of a vertex v, respectively. Also the term $\alpha_0(G)(\alpha_1(G))$ is the minimum number of vertices (edges) in a vertex (edge) cover of G.

The square of a graph G, denoted by G^2, has the same vertices as in G and the vertices u and v are joined in G^2, if and only if they are joined in G by a path of length one or two. The concept of squares of graphs was introduced in [2].

Let $G = (V, E)$ be a graph. In this paper we denote the number of vertices of G as p and q the number of edges. A set $D' \subseteq V$ is said to be a dominating set of G, if every vertex in $(V - D')$ is adjacent to some vertex in D'. The minimum cardinality of vertices in such a set is called the domination number of G and is denoted by $\gamma(G)$.

A dominating set D' of G is said to be a connected dominating set, if the sub graph $\langle D' \rangle$ induced by D' is connected in G. The minimum cardinality of vertices in such a set is called the connected domination number of G and is denoted by $\gamma_c(G)$. Domination related parameters are now well studied in graph theory [3].

A dominating set D' of G is said to be an independent dominating set if no two vertices in D' are adjacent in G. The minimum cardinality of an independent dominating set is called an independent domination number of G and is denoted by $\gamma_i(G)$.

A dominating set D' is said to be total dominating set of G, if $N(D') = V$ or equivalently, if for every vertex $v \in V$, there exists a vertex $u \in D', u \neq v$, such that u is adjacent to v. The total domination number of G, denoted by $\gamma_t(G)$ is the minimum cardinality of a total dominating set of G.

Further $\beta_0(G)$ and $\beta_1(G)$ represents vertex independence number and edge independence number.

A set D of G^2 is said to be connected dominating set of G^2 if every vertex not in D is adjacent to at least one vertex in D and the set induced by D is connected. The minimum cardinality of a connected dominating set of G^2 is called connected domination number of G^2. Much effort has been made by many authors to establish the relationship among distance domination parameters, for the known results see a recent survey by M.A.Henning [4].

In this paper we establish some relation of $\gamma_c(G^2)$ with $\gamma_c(G)$, $\gamma(G^2)$, $\alpha_0(G)$ and also the results of $\gamma_c(G^2)$ are expressed not in terms of the elements of G^2, but expressed in terms of the elements of G.

2. RESULTS

Initially we provide an upper bound for $\gamma_c(G^2)$ in terms of $\gamma_c(G)$.

Theorem 2.1: For any connected graph G, $\gamma_c(G^2) \le \gamma_c(G)$. Equality holds for $diam(G) \le 2$, except for K_{p_1,p_2} with $p_1, p_2 \ge 2$.

Proof: For any connected graph G with $diam(G) \ge 3$, let P be a diametral path such that $P: v_1, v_2, ..., v_k$. Suppose $I = \{v_1, v_2, ..., v_i\}$, $i \le k$ be a connected dominating set of G such that $I \subseteq V(P)$, so that $|I| = \gamma_c(G)$. Now we consider $D = \{v_{i-1}, v_{i-2}, ..., v_{i-m}\}$, $m \le i$ such that $V(P) \cap V(I) = D$ be a minimal connected dominating set of G^2. Hence $|D| = \gamma_c(G^2)$. Clearly, $\gamma_c(G^2) \le \gamma_c(G)$.

Suppose $diam(G) \le 2$ and if $G \cong K_{p_1,p_2}$, $p_1, p_2 \ge 2$ with $p_1 + p_2 = p = |V(G)|$. Then in this case, $|D| = p_1$ or p_2 such that $p_1 \le p_2$ or $p_2 \le p_1$ respectively. Now in G^2, since $diam(G) \le 2$, clearly, $|D| = 1 < |D'|$, and hence $\gamma_c(G^2) < \gamma_c(G)$.

Suppose $diam(G) \le 2$ and if $G \ncong K_{p_1,p_2}$. Then in this case, $|D| = 1 = |D'|$. Therefore $\gamma_c(G^2) = \gamma_c(G)$, gives the required result for equality. ∪

Now we develop the relation of connected domination of G^2 with the spanning trees of G.

Theorem 2.2: For any nontrivial connected graph G, $\gamma_c(G^2) = \{\max.\gamma_c(T^2) - 1\}$ where the maximum is taken over all spanning trees T of G.

Proof: Let G be a nontrivial connected graph and T be a spanning tree of G. Then any connected dominating set of T^2 is also a connected dominating set of G^2. Hence $\gamma_c(G^2) \le \gamma_c(T^2)$. Thus we have that $\gamma_c(G^2) \le \{\max.\gamma_c(T^2) - 1\}$, where maximum is taken over all the spanning trees T of G.

For equality, we consider the reverse inequality of the above. If G is a tree, then the Theorem holds trivially. So we assume that G is connected graph containing cycles. Let D be a minimum connected dominating set of G^2 and C be a cycle in G. If we can prove that D is also a connected dominating set of $(G-e)^2$ for some cycle edge $e \in E(C)$, then $\gamma_c(G-e)^2 \le |D| = \gamma_c(G^2)$. By applying this process a finite number of times, we have $\gamma_c(T^2) \le \gamma_c(C^2)$, where the maximum is taken over all spanning trees of T of G.

If $V(C) \subseteq V(D)$, then obviously $V(D) - e$ for any $e \in E(C)$ is also connected and the vertices in $V(C) - D$ are also within distance two to D.

If $V(C) \not\subseteq V(D)$, then we select an edge xy in C such that $dist(x,D) + dist(y,D) = \max.\{dist(u,D) + dist(v,D)\}$ such that $uv \in E(C)$ in G. Now we will show that D is connected dominating set of $[G - \{xy\}]^2$.

For any two adjacent vertices u and v in G we have $|dist(u,D) - dist(v,D)| \le 1$. Then if there exists a vertex w in $V(C)$ of G such that $dist(w,D) = \max.\{dist(v,D)\}$. Now we say that $w = x$ or $w = y$ .Without loss of generality suppose that $dist(x,D) = \max.\{dist(v,D) : v \in V(C)\}$.

Let z be another neighbor vertex of x different from y in $V(C)$. This gives immediately that $dist(z,D) \le dist(y,D)$ in G. Thus we get the distance between a vertex in $V(G)-D$ and D are not influenced by deleting the edge xy. This gives $dist(v,D)$ in $G-\{xy\}$, $xy \in E(C)$ is equal to $dist(v,D)$ such that $v \in V(G)$. Hence D is also a connected dominating set of $G-e$ for some cycle edge e.

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Now we give the following Proposition which is straight forward.

Proposition 2.1: For any (p,q) graph G with $p \ge 2$ vertices, if $rad(G) \le 2$ then $\gamma_c(G^2)=1$.

In the following Theorems we give lower bounds to $\gamma_c(G^2)$.

Theorem 2.3: For any nontrivial connected (p,q) -graph G with maximum degree Δ, then $\gamma_c(G^2) \le \min.\{1, p-(\Delta+2)\}$.

Proof: By Theorem 2.2, it is sufficient to show that $\gamma_c(G^2) \le \min.\{1, p-(\Delta+2)\}$ for any spanning tree T with maximum degree $\Delta = \Delta(T)$.

If $rad(T) \le 2$, then by Proposition 2.1, we get $\gamma_c(T^2)=1$. Now we assume that $rad(T) > 3$. Let P be any longest path in T with end vertices u and v. Then there exist two vertices x and y of P such that $dist(x,u)=2$ and $dist(y,v)=2$ in T. Let P' be a xy sub path of P and let $D' = V(P)-V(P')$. Let $D = V(T)-[D' \cup M(T)]$ where $M(T)$ is the set of all end vertices of $V(T)$. Hence D must contain a connected dominating set of T^2. Since $u,v \in D' \cap M(T)$ and $M(T) \ge \Delta$, we have

$$\gamma_c(T^2) \le |V(T)| - |D' \cup M(T)|$$
$$\le |V(T)| - |D'| - |M(T)| + |D' \cap M(T)|$$
$$\le p - 2^2 - \Delta + 2$$

$\le p-(\Delta+2)$ as required.
∪

Theorem 2.4: If G is connected graph, then $\gamma_c(G^2) \le 5\gamma(G^2)-4$.

Proof: Let G be a connected graph and let D be a dominating set of G^2. Then the induced sub graph $\langle D \rangle$ has at most $|D|$ components. Since D is a dominating set of G^2, we can connect two of these components to one component by adding at most four vertices to D. Hence we can construct a connected dominating set D' of G^2 such that $D' \supseteq D$ in at most $|D|-1$ steps by adding at most four times of $(|D|-1)$ vertices to D. Consequently, $\gamma_c(G^2) \le |D'| \le |D| + 4(|D|-1) = 5|D|-4$ and if we choose D such that $|D| = \gamma(G^2)$.

In the following Theorem, we obtain the inequality relation between domination and connected domination of G^2.

Theorem 2.5: For any connected graph G, $\gamma(G^2) \le \gamma_c(G^2)$. Equality holds for connected spanning sub graph H of G with $diam(H) \le 4$.

Proof: For $p \le 3$, the result is obvious. For $p \ge 4$, let $D_1 = \{v_1, v_2, v_3, ..., v_m\} \subseteq V(G^2)$ be the set of vertices in G^2 such that $diam(u,v) \le 4$, $u,v \in D_1$, which forms a minimal dominating set of G^2. Suppose $diam(u,v)=4$, $u,v \in D_1$ in G^2, then D_1 itself is a γ -set of G^2. Further, if $diam(u,v) > 4$, where $u,v \in D_1$ in G^2, then there exists at least one vertex $w \notin D_1$, such that the sub graph $\langle D_1 \cup \{w\} \rangle$ is connected. Clearly, $D_1 \cup \{w\} = D$ forms a minimal γ_c - set of G^2. Therefore, it follows that $|D_1| \le |D|$ and hence $\gamma(G^2) \le \gamma_c(G^2)$.

Suppose $diam(H) \le 4$ where H is spanning sub graph of G and is connected. Then in this case, there exist a vertex $v \in V(G^2)$ which covers all the vertices of G^2. Clearly, it follows that $\gamma(G^2) = |\{v\}| = \gamma_c(G^2)$.
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Theorem 2.6: For any nontrivial tree T with m end vertices, $\gamma_c(T^2) \geq \left\lfloor \frac{p-m}{3} \right\rfloor$.

Proof: Let $M = \{v_1, v_2, v_3, ..., v_k\}$ be the set of all end vertices in T with $|M| = m$. Since $V(T) = V(T^2)$, without loss of generality in T^2 there exists a vertex set $D = V - M = \{v_1, v_2, v_3, ..., v_n\}$ which are at distance at most two and they covers all the vertices in T^2 such that the sub graph $\langle D \rangle$ is connected. Clearly, D is a minimal connected dominating set of T^2. Further, if every tree with at least two end vertices and with $diam(T) \leq 2$, which generates $\left\lfloor \frac{p-|M|}{2+2-1} \right\rfloor \leq |D|$ and hence $\left\lfloor \frac{p-m}{3} \right\rfloor \leq \gamma_c(T^2)$. ∪

Theorem 2.7: For any connected (p,q)-graph G, $\gamma_c(G^2) \leq p - \alpha_0(G)$. Equality holds for K_p.

Proof: For $p = 2$, the result follows immediately. For $p \geq 3$, let $A = \{v_1, v_2, v_3, ..., v_k\}$, $\deg(v_i) \geq 2$, $1 \leq i \leq k$ be the minimal set of vertices which covers all the edges in G, such that $|A| = \alpha_0(G)$. Now without loss of generality in G^2, if $\deg(v_j) \leq 2$ in G^2, then the result follows immediately. Further, if $\deg(v_j) > 2$ in G^2, then there exists a vertex set $D = \{v_1, v_2, v_3, ..., v_m\}$, $\deg(v_n) \geq 3$, $1 \leq n \leq m$, which covers all the vertices of G^2 such that the sub graph $\langle D \rangle$ is connected. Clearly, D is a γ_c-set of G^2. It follows that $|D| \leq p - |A|$ and hence $\gamma_c(G^2) \leq p - \alpha_0(G)$.

Suppose G is isomorphic to K_p, then in this case, $|D| = 1$ and $|A| = p - 1 = p - |D|$. Therefore, $\gamma_c(G^2) = p - \alpha_0(G)$. ∪

Theorem 2.8: For any connected (p,q)-graph G, $\gamma(G) + \gamma_c(G^2) \leq p - 1$, for $p \geq 3$. Equality holds for C_3, P_3, C_4 and P_4.

Proof: For $p = 2$, clearly, $\gamma(G) + \gamma_c(G^2) \not\leq p - 1$. For $p \geq 3$, let $S = \{v_1, v_2, v_3, ..., v_n\}$ and $\deg(v_i) \geq 2$, $1 \leq i \leq n$, be the minimal set such that $N[v_i] = V(G)$. Clearly S is a minimal dominating set of G. Now without loss of generality in G^2, since $V(G) = V(G^2)$, there exists a set $D = \{v_1, v_2, v_3, ..., v_k\} \subseteq S$ which covers all the vertices in G^2. Suppose distance between two vertices $u, v \in D$ is at most two. Then the sub graph $\langle D \rangle$ itself is a connected and hence D is a connected dominating set of G^2. Further if $dist(u,v) \geq 3$ where $u, v \in D$, and clearly, the sub graph $\langle D \rangle$ is disconnected. Then there exists at least one vertex $w \notin D$ such that $D \cup \{w\}$ forms a connected dominating set in G^2. Since distance between any two vertices in G or G^2 is at least one, it follows that $|S| \cup |D \cup \{w\}| \leq p - 1$. Therefore $\gamma(G) + \gamma_c(G^2) \leq p - 1$.

For the equality, we have following cases.

Case 1: Suppose G is isomorphic to C_3 or P_3. Then in this case, $|S| = |D| = 1$ and $|S| \cup |D| = p - 1$. Clearly, $\gamma(G) + \gamma_c(G^2) = p - 1$.

Case 2: Suppose G is isomorphic to C_4 or P_4. Then in this case, $|S| = 2|D|$. Clearly, it follows that $\gamma(G) + \gamma_c(G^2) = p - 1$. ∪

Corollary 2.1: For any connected (p,q)-graph G, $\gamma_i(G) + \gamma_c(G^2) \leq p - 1$, for $p \geq 3$. Equality holds for C_3, P_3, C_4 and P_4.

Theorem 2.9: For any connected (p,q)-graph G with $p \geq 3$ vertices, $\gamma_c(G^2) \leq \left\lfloor \frac{p}{\Delta(G) - 1} \right\rfloor$.

Proof: For $p = 2$, $\gamma_c(G^2) \not\leq \left\lfloor \frac{p}{\Delta(G) - 1} \right\rfloor$.

For $p \geq 3$, if $A = \{v_1, v_2, v_3, ..., v_n\}$ be the set of all non-end vertices in G. Then there exists at least one vertex $v \in V(G)$ such that $\deg(v) = \Delta(G)$. Now without loss of generality in G^2, there exists a set

$D=\{v_1,v_2,v_3,...,v_k\}$ such that $\deg(v_i)\geq 2$, $1\leq i\leq k$ and the sub graph $\langle D\rangle$ is connected in G^2. It follows that D is γ_c - set of G^2. Since for any graph G there exists at least one vertex v_i, $\forall i$, $1\leq i\leq k$ such that $A\cap D=\{v_i\}$. Clearly, it follows that $|D|\leq\left\lfloor\frac{p}{\Delta(G)-1}\right\rfloor$.

Therefore $\gamma_c(G^2)\leq\left\lfloor\frac{p}{\Delta(G)-1}\right\rfloor$.

∪

Corollary 2.2: For any connected (p,q) - graph G with $p\geq 3$ vertices, $\gamma_c(G^2)\leq\left\lceil\frac{2q-p}{3}\right\rceil$.

The following Theorem relates domination and connected domination numbers of G with $\gamma_c(G^2)$.

Theorem 2.10: For any connected (p,q) -graph G, $\gamma_c(G)-\gamma(G)\leq p-\gamma_c(G^2)$.

Proof: Let $S=\{v_1,v_2,v_3,...,v_n\}$ be the minimal dominating set of G. Suppose the sub graph $\langle S\rangle$ is connected, then S it is a connected dominating set of G. Further if the sub graph $\langle S\rangle$ is disconnected, then there exists another vertex set $J=\{v_1,v_2,v_3,...,v_j\}$ where $J\subseteq V(G)-S$ whose vertices are at distance one to the vertices in S such that the sub graph $\langle S\cup J\rangle$ is connected. Clearly, $S\cup J$ is a connected dominating set of G. Since $V(G)=V(G^2)$, there exists a vertex set $D=\{v_1,v_2,v_3,...,v_k\}\subseteq S\cup J$, $\deg(v_i)\geq 2$, $1\leq i\leq k$, whose vertices are at distance at most one which covers all the vertices in G^2. Clearly, it follows that $|S\cup J|-|S|\subseteq p-|D|$.

Therefore $\gamma_c(G)-\gamma(G)\leq p-\gamma_c(G^2)$.

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Theorem 2.11: For any connected (p,q) - graph G, $\left\lceil\frac{\gamma_c(G^2)}{2}\right\rceil\leq p-diam(G)$, except for path P_p with $p>7$. Equality holds for path P_p with $p\leq 7$.

Proof: Suppose $G\cong P_p$ with $p>7$, then $\left\lceil\frac{\gamma_c(G^2)}{2}\right\rceil\nleq p-diam(G)$. Let $V(G)$ be the set of vertices in G such that there exists a diametral path between two vertices $u,v\in G$, where $dist(u,v)$ forms a $diam(G)$. Let $D=\{v_1,v_2,v_3,...,v_n\}$, $\deg(v_i)\geq 2$, $1\leq i\leq n$ in G^2, which are at distance at least two. These vertices covers all the vertices in G^2. Suppose the sub graph $\langle D\rangle$ connected, then D it self is a connected dominating set of G^2. Further, if the sub graph $\langle D\rangle$ is disconnected, then there exists at least one vertex $w\notin D$ which is at distance at most two to the vertices in G^2. Clearly, the sub graph $\langle D\cup\{w\}\rangle$ is connected and the set $D\cup\{w\}$ is a minimal γ_c -set of G^2. Since any two vertices $u,v\in G$ forms a diametral path of at least one in G, it follows

that $\left\lceil\frac{D\cup\{w\}}{2}\right\rceil\leq p-diam(G)$.

Hence, $\left\lceil\frac{\gamma_c(G^2)}{2}\right\rceil\leq p-diam(G)$.

Suppose G is isomorphic, path P_p with $p\leq 7$. Then in this case, $|D\cup\{w\}|=p-2$ and $diam(G)=p-1$. Clearly, it follows that,

$\left\lceil\frac{\gamma_c(G^2)}{2}\right\rceil=1=p-diam(G)$. ∪

Corollary 2.3: For any nontrivial tree T with $diam(T)\leq 4$, then $\gamma_c(T^2)=1$.

Further we obtain the connected domination number of squares of some standard graphs as:
$\gamma_c(K_p^2)=\gamma_c(W_p^2)=\gamma_c(K_{1,p}^2)=\gamma_c(K_{p_1,p_2}^2)=1$.

The following Theorem relates connected domination and total domination numbers of G^2.

Theorem 2.12: For any connected (p,q) - graph G with $p\geq 3$, $\gamma_c(G^2)+\gamma_t(G)\leq p$ .Equality holds for path P_3, P_6, C_3, C_6.

Proof: For $p=2$, clearly, $\gamma_c(G^2)+\gamma_t(G)\not\leq p$. For $p\geq 3$, if $S=\{v_1,v_2,v_3,...,v_n\}$ be the minimal set of vertices with $\deg(v_i)\geq 2, 1\leq i\leq n$ and $N[v_i]=V(G)$. And if the sub graph $\langle S\rangle$ is not disconnected then

S it self is a minimal total dominating set of G. Otherwise, $S\cup H$, where $H\subseteq V-S$, covers all the vertices in G and the sub graph $\langle S\cup H\rangle$ doesn't contain any isolated vertex. Clearly, the set $S\cup H$ forms a minimal total dominating set of G. Since $V(G)=V(G^2)$, there exists a vertex set $D=\{v_1,v_2,v_3,...,v_k\}\subseteq S$ in G^2 which covers all the vertices in G^2 and the sub graph $\langle D\rangle$ is connected.

Hence D forms a minimal γ_c-set of G^2. It follows that $|S\cup H|\cup|D|\leq p$ and hence $\gamma_t(G)+\gamma_c(G^2)\leq p$.

For equality, we have the following cases.

Case 1: Suppose G is isomorphic to C_3 or P_3. Then in this case $|S\cup H|=2|D|$ and $|D|=1$. Clearly, it follows that $\gamma_t(G)+\gamma_c(G^2)=p$.

Case 2: Suppose G is isomorphic to C_6 or P_6. Then in this case $|D|=2$ and $|S\cup H|=2|D|$. Clearly, it follows that $\gamma_t(G)+\gamma_c(G^2)=p$. ∪

The following result gives the relation of connected domination of G^2 in terms of vertices and maximum number of independent vertex of G.

Theorem 2.13: For any connected (p,q)-graph G, $\gamma_c(G^2)\leq p-\beta_0(G)$.

Proof: Suppose $F=\{v_1,v_2,v_3,...,v_n\}$ be the set of all end vertices in G then $F\cup B$ where B is a proper subset of $V(G)-F$, which are not adjacent to the vertices of F forms a minimal independent set of vertices such that $|F\cup B|=\beta_0(G)$. Now in G^2, since the distance of $(u,v)\geq 2$ for all $u,v\in V(G^2)$ and induced sub graph B is connected and vertices of B covers all the vertices in G^2. Clearly, B it self is a γ_c-set of G^2. Otherwise there exists at least one vertex $w\notin B$ in G^2, such that the sub graph $\langle B\cup\{w\}\rangle$ forms a connected sub graph in G^2. Therefore it follows that $|B\cup\{w\}|\leq p-|F\cup B|$ and hence $\gamma_c(G^2)\leq p-\beta_0(G)$. ∪

Theorem 2.14: Let G be a connected graph and H be any connected spanning sub graph of G. Then every connected dominating set of H^2 is also a connected dominating set of G^2 and hence $\gamma_c(G^2)\leq\gamma_c(H^2)$.

Proof: Suppose H is totally disconnected or disconnected with at least one component as an isolated vertex. Then we consider $V(G)=\{v_1,v_2,v_3,...,v_n\}$ and $S=\{v_i\}$, $1\leq i\leq n$ such that $S\subset V(G)$ in such a way that every vertex of $V-S$ are at a distance of at most two with respect to the corresponding vertices of S in G which gives a connected minimal connected dominating set in G^2. Let v_iv_j be an edge of G such that $i<j$ and $\forall i,j=1,2,...(n-1)$. Suppose H is a minimal connected spanning sub graph of G and $E(H)=E(G)-v_iv_j$. Since $V(G^2)=V(H^2)$ then S is also a minimal connected dominating set of both G^2 and H^2 which gives the equality $\gamma_c(G^2)=\gamma_c(H^2)$. If H is totally disconnected then $V(G^2)=V(H^2)$ such that $E(H^2)=\phi$ and $\gamma_c(H^2)=V(G^2)=n$, where n is the number of vertices in G. Now one can easily verify that $\gamma_c(G^2)<\gamma_c(H^2)$. Otherwise $\gamma_c(G^2)\leq\gamma_c(H^2)$. ∪

Finally we provide Nordhaus – Gaddum type result.

Theorem 2.15: For any connected (p,q)-graph G,

(i) If both G^2 and $\overline{G^2}$ are connected, then

(a) $\gamma_c(G^2)+\gamma_c(\overline{G^2})\leq p+1$.

(b) $\gamma_c(G^2)\cdot\gamma_c(\overline{G^2})\leq p$.

(ii) If both G^2 and $\overline{G}^2$ are connected, then

(a) $\gamma_c(G^2)+\gamma_c(\overline{G}^2)\leq p+1$.

(b) $\gamma_c(G^2)\cdot\gamma_c(\overline{G}^2)\leq p$.

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